A PRACTICAL ARITHMETIC

BY

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PREFACE.

Pupils can be trained to logical habits of mind and stimulated to a good degree of intellectual energy by solving problems suited to their capacities. By this course they acquire practical mastery over the everyday problems of common life as well as gain the very best mental discipline. The shortest and best way of learning Arithmetic is by solving problems. The rules which this book contains are not intended to be committed to memory, but to help the pupil to neat and intelligent methods. The rules *follow explanations of processes,* and no rule should be considered until the process which it summarizes is thoroughly understood. Dependence upon set rules and formulas is the worst possible mental slavery.

The examples are in the main *new,* and contain a great deal of accurate and valuable information of the latest date, while they are well-graded and progressive. This Arithmetic and the author's Elementary Arithmetic are intended for a two-book course sufficiently comprehensive to cover all the work that should be required of pupils in general. The Appendix contains subjects of value only for pupils in the States named, and other subjects of little *practical* value but convenient for reference.

Decimal fractions are introduced before common fractions. Experience proves that, when taught with reference to integral numbers, they present no difficulty. The difficulty of decimal fractions arises from comparing them with common fractions, and is avoided by teaching decimals first. The notation on the right of the decimal point is learned as easily as the notation on the left, if there is no break in presenting the notation on both sides.

Care has been taken to use only small common fractions. They are best adapted to give practice in the methods of working with fractions, and are the only ones used in ordinary business.

Domestic Exchange is put under Bank Discount where it belongs and not made a separate topic. Foreign Exchange is made one of the subjects in the Appendix.

Every effort has been made to avoid errors in problems and answers. The author will be very grateful to any one who will call his attention to any mistake that may be discovered.

The author thankfully acknowledges his obligations to many eminent school superintendents and teachers who have read the proofs of this book, and given him the benefit of valuable suggestions and criticisms.

G. A. WENTWORTH.

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VOCABULARY.

Abstract numbers. This phrase is employed to designate numbers used without reference to any particular unit, as 8, 10, 21. But *all numbers are in themselves abstract whether the kind of thing numbered is mentioned or is not mentioned.*

Addition. The process of combining two or more numbers so as to form a single number.

Aliquot parts of a given number. A number which is contained an integral number of times in the given number. Thus, 5, $6\frac{1}{4}$, $12\frac{1}{2}$, $16\frac{2}{3}$, are aliquot parts of 100.

Amount. The sum of two or more numbers. In Interest, the sum of principal and interest.

Analysis. The process of reasoning from the *given number* to *one*, and then from *one* to the *required number.*

Antecedent. The first term of a ratio.

Area of a surface. The number of square units the surface contains.

Arithmetic. The science that treats of numbers and the methods of using them.

Assets. All the property belonging to an estate, individual, or corporation.

Average of numbers. The number which can be put for each of them without altering their sum.

Average of payments. The average time at which several payments due at different dates may be made.

Bank. An establishment for the custody, loaning, and exchange of money; and often for the issue of money.

Bank discount. An allowance received by a bank for the loan of money, paid as interest at the time of lending.

Bills. Written statements of goods sold, or services rendered, giving the price of each item and total cost, as well as the dates, and the names of the parties concerned.

Bonds. Written contracts under seal to pay specified sums of money at specified times, issued by national governments, states, cities, and other corporations.

Cancellation. The striking out of a common factor from the dividend and divisor.

Commission. Compensation for the transaction of business, reckoned at some per cent of the money employed in the transaction.

Common denominator. A denominator common to two or more fractions.

Common factor. A factor common to two or more numbers.

Common fractions. Fractions expressed by two numbers, one under the other, with a line between them.

Common multiple of two or more numbers. A number exactly divisible by each of them.

Complex fraction. A fraction that has a fraction in one or both of its terms.

Composite number. The product of two or more integral factors, each factor being greater than unity.

Compound fraction. A fraction of an integer or of a fraction.

Compound interest. Interest not paid when due, but added to the principal at regular intervals.

Compound quantities. Quantities expressed in two or more denominations.

Concrete numbers. A phrase used to denote numbers applied to specified things; as 6 horses, 8 desks.

Consequent. The second term of a ratio.

Consignee. The person or firm to whom goods are sent.

Consignor. The person or firm who sends goods to another.

Corporation. An association of individuals authorized by law to transact business as a single person.

Couplet. The two terms of a ratio taken together.

Coupon. A certificate of interest attached to a bond, to be cut off when due and presented for payment.

Creditor. A person or firm to whom money is due.

Cube root of a number. One of its three equal factors.

Customs. Duties or taxes imposed by law on merchandise imported, and sometimes on merchandise exported.

Debtor. A person who owes money to another.

Decimal fractions. Fractions of which only the numerators are written, and the denominators are ten or some power of ten.

Decimal point. A dot placed after the *units'* figure to mark its place.

Decimal system. The common system of numbers founded on their relations to *ten*.

Denominator. The number which shows into how many equal parts a unit is divided.

Difference. The number found by subtraction.

Discount. Allowance made for the payment of money before it becomes due. Also, the difference between the market value and the face value when the market value is *below* the face value.

Dividend. In division, the given number which is equal to the product of a given factor (called divisor) and a required factor (called quotient). In business, the sum paid on each share of stock from the profits of the business.

Division. The operation by which, when a product and one of its factors are given, the other factor is found.

Divisor. The number by which a given dividend is to be divided.

Draft. A written order directing one person or firm to pay a specified sum of money to another.

Drawee of a draft. The person or firm on whom the draft is drawn. The drawee is expected to pay the draft.

Drawer of a draft. The person who signs the draft.

Duties. Taxes required by the government to be paid on goods imported, exported, or put on the market for consumption.

Equation. A statement that two expressions of number are equal.

Even numbers. Numbers exactly divisible by 2.

Evolution. The process of finding the root of a number.

Exchange. A system of paying debts, due to persons living at a distance, by transmitting drafts instead of money.

Exponent. A small figure placed at the right of a number to show how many times the number is taken as a factor.

Extremes. The first and last terms of a proportion.

Factors. The factors of a number are a set of numbers whose product is the given number; they are assumed to be integral, except in the extraction of roots. In commerce, agents employed by merchants to transact business.

Figures. Symbols used to represent numbers in the common system of notation. Also diagrams used to represent geometrical forms.

Firm. The name under which a company transacts business.

Fractions. One or more of the equal parts into which the unit is divided.

Gain. The selling price minus the cost price.

Grace. An allowance of three days, after the date a note becomes due, within which to pay the note.

Gram. The unit of weight in the metric system.

Greatest common measure of two or more numbers. The greatest number that will exactly divide each of them.

Improper fraction. A fraction whose numerator equals or exceeds its denominator.

Index. A figure written at the left and above the radical sign to show what root of the number under the radical sign is required. A fraction written at the right of a number, of which the numerator shows the required power of that number, and the denominator the required root of that power.

Instalment. A payment in part.

Insurance. A guarantee of a specified sum of money in the event of loss of property by fire, by storm at sea, or by other disaster; or in the event of loss of life.

Integral numbers. Numbers which denote whole things.

Interest. Money paid for the use of money.

Involution. The process of finding a power of a number.

Latitude of a place. The distance north or south from the equator, expressed in degree-measures.

Least common multiple of two or more numbers. The least number that is exactly divisible by each of them.

Liability. A debt, or obligation to pay.

Like numbers. Numbers applied to the same unit.

Line. Length without breadth or thickness. The path of a moving point.

Liter. The unit of capacity in the metric system.

Long division. The method of dividing in which the processes are written in full.

Longitude of a place. The distance east or west from a *standard meridian*, expressed in degree-measures.

Loss. The cost price minus the selling price.

Maturity of a note. The date at which a note legally becomes due.

Mean proportional. A number which is both the second and third terms of a proportion.

Means. The terms of a proportion between the extremes.

Meter. The unit of length in the metric system.

Minuend. The number from which the subtrahend is taken.

Mixed number. A whole number and a fraction.

Multiple of a number. The product obtained by taking the given number an integral number of times.

Multiplicand. The number to be multiplied by another.

Multiplication. The operation of taking a number of units a number of times.

Multiplier. The number by which the multiplicand is multiplied.

Net proceeds. The money that remains of the money received for property after all expenses and discounts are paid.

Notation. A system of expressing numbers by symbols.

Note. A written agreement to pay a specified sum of money at a specified time.

Numbers. Expressions applied to a unit to show how many times the unit is taken.

Numeration. A system of naming numbers.

Obligation. A debt, or liability to pay.

Odd numbers. Numbers not exactly divisible by 2.

Order of units. A name used to designate the number of things in a group, as *tens*, *hundreds*, *thousands*, etc.

Partial payment. Part payment on a note.

Partnership. An association of two or more persons to carry on business.

Par value. Face or nominal value.

Pendulum. A body suspended by a straight line from a fixed point, and moving freely about that point as a centre.

Percentage of a number. One or more hundredths of the number.

Perimeter. The length of the line or lines that bound a plane figure.

Period. A group of three figures.

Policy. A written contract of insurance.

Poll tax. A tax levied by the head or poll.

Power. The product of two or more equal factors.

Premium. Money paid for insurance computed at some rate per cent of the value insured. Also the excess of market value above par value.

Present worth. The present value of a debt due at some future day.

Prime number. A number that cannot be exactly divided by any number except itself and one.

Principal. Money drawing interest.

Problem. A question to be solved.

Product. The number found by multiplication.

Proof. The evidence by which the accuracy of any result is established.

Proper fraction. A fraction whose numerator is less than its denominator.

Proportion. A statement that two ratios are equal.

Quotient. The number found by division.

Rate per cent. Rate by the hundred.

Ratio. The *relative magnitude* of two numbers or of two quantities.

Reciprocal of a number. *One* divided by that number.

Reduction. The process of changing the *unit* in which a quantity is expressed without changing the *value* of the quantity.

Remainder. The number found by subtraction.

Root of a number. One of the equal factors of the number.

Rule. The statement of a prescribed method.

Security. Property used to guarantee the payment of any debt.

Share. One of a certain number of equal parts into which the capital of a company is divided.

Short division. The method of dividing in which the operations of multiplying and subtracting are performed mentally.

Similar fractions. Fractions that have a common denominator.

Simple fractions. Fractions whose numerators and denominators are integral numbers.

Solid. A magnitude which has length, breadth, and thickness.

Solution. The process by which the answer to a question is obtained.

Specific gravity of a substance. The ratio of the weight of a given volume of it to the weight of an equal volume of water.

Square root of a number. One of its two equal factors.

Stock. Capital invested in business.

Subtraction. The process of taking one number from another.

Sum. The number found by addition.

Surd. An indicated root, the value of which cannot be exactly expressed in figures.

Surface. A magnitude which has length and breadth.

Taxes. Money required of persons and corporations for the support of the government and other purposes.

United States money. The legal currency of the United States.

Units. The standards by which we count separate objects or measure magnitudes.

Verify. To establish by trial the truth of any statement.

Volume of a solid. The number of units of volume it contains; the unit of volume being a cube whose edge is a unit of length.

Short Processes.

NOTE. These processes should be learned as fast as they can be utilized in the ordinary work of Arithmetic. The time to learn each process is left to the discretion of the teacher.

1. To multiply by 25 ($\frac{1}{4}$ of 100),
Multiply by 100 and divide the product by 4.

2. To divide by 25,
Multiply by 4 and divide the product by 100.

3. To multiply by 2.5 or $2\frac{1}{2}$ ($\frac{1}{4}$ of 10),
Multiply by 10 and divide the product by 4.

4. To divide by 2.5 or $2\frac{1}{2}$,
Multiply by 4 and divide the product by 10.

5. To multiply by 50 ($\frac{1}{2}$ of 100),
Multiply by 100 and divide the product by 2.

6. To divide by 50,
Multiply by 2 and divide the product by 100.

7. To multiply by 75 ($\frac{3}{4}$ of 100),
Multiply by 100 and subtract from the product $\frac{1}{4}$ of it.

8. To divide by 75,
Divide by 100 and add to the quotient $\frac{1}{3}$ of it.

9. To multiply by $33\frac{1}{3}$ ($\frac{1}{3}$ of 100),
Multiply by 100 and divide the product by 3.

10. To divide by $33\frac{1}{3}$,
Multiply by 3 and divide the product by 100.

11. To multiply by $3\frac{1}{3}$ ($\frac{1}{3}$ of 10),
Multiply by 10 and divide the product by 3.

12. To divide by $3\frac{1}{3}$,
Multiply by 3 and divide the product by 10.

13. To multiply by $333\frac{1}{3}$ ($\frac{1}{3}$ of 1000),
Multiply by 1000 and divide the product by 3.

14. To divide by $333\frac{1}{3}$,
Multiply by 3 and divide the product by 1000.

15. To multiply by $16\frac{2}{3}$ ($\frac{1}{6}$ of 100),
Multiply by 100 and divide the product by 6.

16. To divide by $16\frac{2}{3}$,
Multiply by 6 and divide the product by 100.

17. To multiply by $166\frac{2}{3}$ ($\frac{1}{6}$ of 1000),
Multiply by 1000 and divide the product by 6.

18. To divide by $166\frac{2}{3}$,
Multiply by 6 and divide the product by 1000.

19. To multiply by $66\frac{2}{3}$ ($\frac{2}{3}$ of 100),
Multiply by 100 and subtract from the product $\frac{1}{3}$ *of it.*

20. To divide by $66\frac{2}{3}$,
Divide by 100 and add to the quotient $\frac{1}{2}$ *of it.*

21. To multiply by $12\frac{1}{2}$ ($\frac{1}{8}$ of 100),
Multiply by 100 and divide the product by 8.

22. To divide by $12\frac{1}{2}$,
Multiply by 8 and divide the product by 100.

23. To multiply by $14\frac{2}{7}$ ($\frac{1}{7}$ of 100),
Multiply by 100 and divide the product by 7.

24. To divide by $14\frac{2}{7}$,
Multiply by 7 and divide the product by 100.

25. To multiply by a number that is a little less than some multiple of 10, as 100, 1000, etc.,
Multiply the multiplicand by the multiple of 10 that differs little from the given multiplier. Then multiply the multiplicand by the difference between this multiple of 10 and the given multiplier, and find the difference of the two products.

Thus, to multiply by 998 (1000 − 2), multiply by 1000 and then by 2 and take the difference of the products.

PRACTICAL ARITHMETIC.

CHAPTER I.

Notation and Numeration.

1. Units. The standards by which we count or measure are called *units.*

The unit may be a single thing or a definite group of things. Thus, in counting the eggs in a nest the unit is an egg; in selling eggs by the dozen the unit is a dozen eggs; in selling bricks by the thousand the unit is a thousand bricks; in measuring short distances the unit is an inch, a foot, or a yard; in measuring long distances the unit is a rod or a mile.

2. Numbers. Expressions applied to a unit to show *how many times the unit is taken* are called *numbers.*

Thus, if we put an apple into an empty fruit-dish, then another, and then another, we shall have *three* apples in the dish. Here an apple is the unit, and *three* is the *number of times the unit is taken.*

3. Integral Numbers. Numbers applied to *whole units* are called *whole numbers, in'tegral numbers,* or *integers.*

4. Figures. The following symbols, called *figures,* or *digits,* are used to represent the numbers of Arithmetic:

0	1	2	3	4	5	6	7	8	9
Zero	One	Two	Three	Four	Five	Six	Seven	Eight	Nine

The first figure, 0, is called *zero, naught,* or *cipher,* and stands for the words *no number.* Each of the other figures stands for the number whose name is written below it.

5. Tens. The next number, *ten,* is expressed by writing 0 at the right of 1. Thus, ten is written 10. In this position 1 signifies not *one unit* but *one group of ten units.*

Figures signifying *tens* are written in the *second* place from the right.

In the same way twenty (2 tens) is expressed by 20; thirty (3 tens) by 30; forty (4 tens) by 40; fifty (5 tens) by 50; sixty (6 tens) by 60; seventy (7 tens) by 70; eighty (8 tens) by 80; ninety (9 tens) by 90.

6. Tens and Ones. Numbers containing tens and ones are expressed by writing the figures for the tens in the *second* place from the right, and the figures for the ones in the *first* place.

Eleven,	one ten and one,	is expressed by 11.
Twelve,	one ten and two,	is expressed by 12.
Thirteen,	one ten and three,	is expressed by 13.
Fourteen,	one ten and four,	is expressed by 14.
Fifteen,	one ten and five,	is expressed by 15.
Sixteen,	one ten and six,	is expressed by 16.
Seventeen,	one ten and seven,	is expressed by 17.
Eighteen,	one ten and eight,	is expressed by 18.
Nineteen,	one ten and nine,	is expressed by 19.
Twenty-one,	two tens and one,	is expressed by 21.
Twenty-two,	two tens and two,	is expressed by 22.
Forty-three,	four tens and three,	is expressed by 43.
Fifty-four,	five tens and four,	is expressed by 54.
Sixty-five,	six tens and five,	is expressed by 65.

And so on to ninety-nine (nine tens and nine) which is expressed by 99.

7. Hundreds. A group of 10 tens is called a *hundred,* and figures signifying hundreds are written in the *third* place from the right.

Thus, one hundred, two hundreds, three hundreds, *etc.*, are expressed by 100, 200, 300, *etc.*

8. To Write Hundreds, Tens, and Ones. We write first the hundreds, then the tens and ones.

Thus, two hundred seventy-six is written 276.

Express in figures the following numbers:

Seven hundred sixty-five.	Nine hundred.
One hundred twenty-three.	Five hundred eighty-one.
Six hundred ninety-four.	Four hundred thirty.
Nine hundred forty-six.	Seven hundred nine.
Two hundred twenty-nine.	Seven hundred ninety.
One hundred ten.	Seven hundred ninety-nine.
Six hundred twenty.	Two hundred thirty-five.
Five hundred nine.	Six hundred forty-nine.

9. To Read Hundreds, Tens, and Ones. We read first the hundreds, then the tens and ones.

Thus, the number 359 has 3 hundreds, 5 tens, and 9 ones, and is read three hundred fifty-nine; the number 807 has 8 hundreds, no tens, and 7 ones, and is read eight hundred seven.

NOTE. In reading 359, 807, or any other integral number, do not introduce the word *and;* that is, do not say three hundred *and* fifty-nine, eight hundred *and* seven; but simply three hundred fifty-nine, eight hundred seven.

Read and state the number of hundreds, tens, and ones in each of the following numbers:

507	469	101	260	206	301	808	888
321	694	929	300	185	340	671	999

10. Notation and Numeration. The method of writing numbers is called *notation,* and the method of reading numbers is called *numeration.*

11. Thousands. A group of 10 hundreds is called a *thousand,* and figures signifying thousands are written in the *fourth* place from the right.

Thus, one thousand, two thousands, three thousands, *etc.*, are expressed by 1000, 2000, 3000, *etc.*

12. Numbers expressed by Four Figures. Numbers expressed by four figures may be read as thousands, hundreds, tens, and ones; or as hundreds, tens, and ones.

The shortest way of reading numbers is the best way. The best way to read 1896 is eighteen hundred ninety-six. The best way to read 7005 is seven thousand five.

Read in the best way the following numbers:

1776	1924	1907	2359	5050	3627
7006	7076	2706	6010	5500	2036

13. Orders of Units. The ones of a number are called *units of the first order.* The tens of a number are called *units of the second order.* The hundreds of a number are called *units of the third order.* The thousands of a number are called *units of the fourth order.*

Figures in the *fifth* place signify ten-thousands, and ten-thousands are called *units of the fifth order.*

Figures in the *sixth* place signify hundred-thousands, and hundred-thousands are called *units of the sixth order.*

NOTE. The ones of a number are commonly called *units*, the word *units* standing for the phrase *units of the first order.* Thus, we say the number 459 has 4 hundreds, 5 tens, and 9 units.

14. Decimal System. Since *ten* units of any order are equal to *one* unit of the next higher order, this system of notation is called the *decimal system; decimal* being derived from the Latin word *decem,* meaning ten.

15. Periods. When the figures of a number are five or more, we separate them into groups of three figures each by commas, beginning at the right. The right-hand group is called the *period of units;* the second group is called the *period of thousands;* the third group is called the *period of millions;* the fourth group is called the *period of billions.*

The unit of any period is equal to 1000 units of the next lower period.

One million is equal to 1000 thousands, and is written 1,000,000; one billion is equal to 1000 millions, and is written 1,000,000,000.

The left-hand period may have one, two, or three figures; every other period has three figures, one for the hundreds, one for the tens, and one for the units of that period.

16. To Read an Integral Number expressed in Figures.

Read the number 26217320416.

We begin at the right and point off the figures into periods of three figures each. We then have

26,217,320,416.

We begin at the left and read each period as if it stood alone, adding the *name* of the period. The *fourth* period from the right is the period of *billions*. Hence the number is read :

Twenty-six billion, two hundred seventeen million, three hundred twenty thousand, four hundred sixteen.

NOTE 1. The name of the period of units is omitted in reading.

NOTE 2. The names of periods above billions are in order: trillions, quadrillions, quintillions, sextillions, septillions, octillions, *etc.* These periods are not often used.

EXERCISE 1.

Write in periods, and read the following:

1. 48257.	**11.** 9850005.	**21.** 121363450.
2. 81630.	**12.** 8604604.	**22.** 206306406.
3. 57063.	**13.** 4320500.	**23.** 300400500.
4. 80095.	**14.** 6203050.	**24.** 275275587.
5. 72108.	**15.** 5400406.	**25.** 999970209.
6. 112560.	**16.** 23807780.	**26.** 7887652476.
7. 120605.	**17.** 57708807.	**27.** 70221304240.
8. 113999.	**18.** 95380830.	**28.** 85000005644.
9. 270008.	**19.** 78058085.	**29.** 56070003203.
10. 506800.	**20.** 63005008.	**30.** 48526000400.

17. To Write Integral Numbers in Figures.

Write in figures the number two hundred sixty-three million, six hundred thirty-five thousand, two hundred one.

We consider first the *periods* of the number.

This number has the period of millions, the period of thousands, and the period of units.

We write first the period of millions, and put a comma after it; then the period of thousands, and put a comma after it; and then the period of units. Thus, 263,635,201.

NOTE. Every period except the one at the left must have three figures (§ 15). If any order of units of a period is lacking, we put a cipher in its place; and if any entire period is lacking, we put three ciphers in its place. Thus, four million, sixteen thousand, four is written 4,016,004; sixteen million, four hundred sixteen is written 16,000,416, the three ciphers taking the place of the missing period of thousands.

Therefore, to write integral numbers:

We begin at the left and write the hundreds, tens, and units of each period, putting zeros in all vacant places, and putting a comma between each period and the period that follows it.

EXERCISE 2.

Write in figures, arranged in periods, the following:

1. Eighteen thousand, five hundred ninety-seven.
2. Forty-seven thousand, nine hundred eighty.
3. Ninety-six thousand, nine hundred eight.
4. Two hundred six thousand, three hundred forty-four.
5. Four hundred sixty thousand, two hundred fifty.
6. Five hundred thousand, five hundred forty.
7. Nine hundred thousand, nine hundred ninety.
8. Eight hundred nine thousand, eight hundred nine.
9. Five hundred thirty thousand, nine hundred ninety.
10. Nine million, nine thousand, nine hundred.
11. Twenty-three million, twenty-three thousand, twenty.
12. Four million, two hundred thirty thousand, fifty.

13. Six million, three hundred two thousand, six.

14. Seventy-five million, eight hundred thousand, seven.

15. Eighty-seven million, fifty-eight thousand, eighty-five.

16. Sixty-three million, one hundred eight thousand, eight hundred one.

17. Forty-four million, forty-four thousand, forty-four.

18. Three hundred six million, two hundred six thousand.

19. Seven hundred fifty million, two hundred twenty-nine.

20. Three hundred twenty-six million, twenty-six.

21. Fifteen billion, two hundred thirty-four million, three hundred sixty thousand, four hundred eighty-one.

22. Fifty-four billion, three hundred million.

23. Five hundred six billion, two thousand, four.

24. Nine billion, eighteen thousand, six hundred.

25. Two hundred seventy billion, two hundred seventy.

26. Four hundred billion, forty.

27. Eight million, eight thousand, eight.

28. Two hundred million, two hundred seven.

29. Fifty million, one thousand, fifty-five.

30. Three million, six hundred seventy-nine.

31. Seven million, thirty thousand, thirty.

32. Nine million, two hundred ninety thousand.

33. One billion, five hundred sixty.

34. Four hundred billion, four hundred thousand.

18. The Roman System of Notation. The system of notation just explained is called the Arabic system. The Roman system employs seven letters, as follows:

Letters,	I	V	X	L	C	D	M
Values,	1	5	10	50	100	500	1000

Other numbers are expressed by combining two or more of these letters.

The first nine numbers are expressed by

I	II	III	IV	V	VI	VII	VIII	IX
1	2	3	4	5	6	7	8	9

The tens are expressed by

X	XX	XXX	XL	L	LX	LXX	LXXX	XC
10	20	30	40	50	60	70	80	90

Tens and ones are expressed by writing the expressions for units at the right of the expressions for tens. Thus,

XI	XII	XIII	XIV	XV	XVI	XVII	XVIII	XIX
11	12	13	14	15	16	17	18	19

XXII	XLIII	XXXIV	LVI	LXXVII	XXVIII	XCIX
22	43	34	56	77	28	99

The hundreds are expressed by

C	CC	CCC	CD	D	DC	DCC	DCCC	DCCCC
100	200	300	400	500	600	700	800	900

By writing M at the left of each of these expressions we have

MC	MCC	MCCC	MCD	MD	MDC	MDCC	MDCCC	MDCCCC
1100	1200	1300	1400	1500	1600	1700	1800	1900

Hundreds, tens, and ones are expressed by writing the hundreds, then the tens, and then the units.

Thus, eighteen hundred ninety-six is written: MDCCC for eighteen hundred, then XC for ninety, and VI for six, making MDCCCXCVI.

Exercise 3.

Read:

XXXVI; XL; XLVI; LVIII; LIX; LXXXI; XCI; XCIII; CIX; CCIX; CCXX; CLIX; MDCCCLXXXVI; MDCLXVI; MDCCLXXVI; MCDLIX; MDLXXXIX.

Express in the Roman system:

43; 55; 81; 77; 99; 113; 128; 514; 724; 630; 1020; 1040; 1088; 1131; 1218; 1492; 1776; 1899; 1319; 1556.

CHAPTER II.

ADDITION OF INTEGRAL NUMBERS.

19. If you put 4 cents with 5 cents, how many cents will there be in all?

How can you express this operation?

You can write the figure 5; then the figure 4 beneath it; draw a line underneath, and below the line write the figure 9; as shown in the margin.

$$\begin{array}{r} 5 \\ \underline{4} \\ 9 \end{array}$$

Or you can express the operation thus: $5 + 4 = 9$.

20. The sign $+$ is called *plus*, and means that the number after it is to be *counted with* the number before it; that is, *added* to the number before it.

The sign $=$ is called the sign of *equality*, and stands for the word *equals;* so that $5 + 4 = 9$ is read: 5 plus 4 equals 9.

21. Addition. The operation of finding a number equal to two or more numbers taken together is called *addition;* and the result is called their *sum.*

22. The sum of two or more numbers is the same in whatever order they are added. Thus,

$$3 + 2 + 5 = 10, \quad \text{or} \quad 5 + 3 + 2 = 10.$$

23. Abstract Numbers. Numbers not applied to any particular unit, as 7, 17, 25, are called *abstract numbers.*

24. Concrete Numbers. Numbers applied to a particular unit, as 7 horses, 17 apples, are called *concrete numbers.*

25. Like Numbers. Numbers applied to *the same unit*, expressed or understood, are called *like numbers.*

26. *Only like numbers, and units of the same order, can be added.*

EXERCISE 4. — ORAL.

Name the sums:

1	1	1	1	1	1	1	1	1	1
0	1	2	3	4	5	6	7	8	9
2	2	2	2	2	2	2	2	2	2
0	1	2	3	4	5	6	7	8	9
3	3	3	3	3	3	3	3	3	3
0	1	2	3	4	5	6	7	8	9
4	4	4	4	4	4	4	4	4	4
0	1	2	3	4	5	6	7	8	9
5	5	5	5	5	5	5	5	5	5
0	1	2	3	4	5	6	7	8	9
6	6	6	6	6	6	6	6	6	6
0	1	2	3	4	5	6	7	8	9
7	7	7	7	7	7	7	7	7	7
0	1	2	3	4	5	6	7	8	9
8	8	8	8	8	8	8	8	8	8
0	1	2	3	4	5	6	7	8	9
9	9	9	9	9	9	9	9	9	9
0	1	2	3	4	5	6	7	8	9

NOTE. This *addition table* should be copied on the board, and each pupil in turn required to name the sums as the teacher touches the examples at random with a pointer. The drill should be continued until every pupil can give quick and correct answers.

Name the sums, reading across the page:

11	21	31	41	51	61	71	81	91
7	8	3	2	5	1	4	7	9
12	22	32	42	52	62	72	82	92
2	3	1	4	6	5	7	9	8
13	23	33	43	53	63	73	83	93
1	4	2	6	3	5	8	7	9
14	24	34	44	54	64	74	84	94
2	1	4	3	6	5	7	9	8
15	25	35	45	55	65	75	85	95
3	2	1	5	4	6	8	9	7
16	26	36	46	56	66	76	86	96
6	5	3	1	2	4	9	7	8
17	27	37	47	57	67	77	87	97
5	6	4	1	8	7	2	9	3
18	28	38	48	58	68	78	88	98
4	5	8	2	7	6	9	3	1
19	29	39	49	59	69	79	89	99
6	8	9	5	4	7	3	2	1
16	66	86	46	56	26	76	36	96
7	7	7	7	7	7	7	7	7
18	48	38	28	88	68	78	58	98
7	7	7	7	7	7	7	7	7
19	29	79	49	99	69	39	89	59
7	7	7	7	7	7	7	7	7

Exercise 5. — Oral.

Count to 100 or more:

1. By 2's, thus, 0, 2, 4, 6, *etc.*; thus, 1, 3, 5, 7, *etc.*

2. By 3's, beginning 0, 3, 6; beginning 1, 4, 7; beginning 2, 5, 8.

3. By 4's, beginning 0, 4, 8; beginning 1, 5, 9; beginning 2, 6, 10; beginning 3, 7, 11.

4. By 5's, beginning 0, 5, 10; beginning 1, 6, 11; beginning 2, 7, 12; beginning 3, 8, 13; beginning 4, 9, 14.

5. By 6's, beginning 0, 6, 12; beginning 1, 7, 13; beginning 2, 8, 14; beginning 3, 9, 15; beginning 4, 10, 16; beginning 5, 11, 17.

6. By 7's, beginning 0, 7, 14; beginning 1, 8, 15; beginning 2, 9, 16; beginning 3, 10, 17; beginning 4, 11, 18; beginning 5, 12, 19; beginning 6, 13, 20.

7. By 8's, beginning 0, 8, 16; beginning 1, 9, 17; beginning 2, 10, 18; beginning 3, 11, 19; beginning 4, 12, 20; beginning 5, 13, 21; beginning 6, 14, 22; beginning 7, 15, 23.

8. By 9's, beginning 0, 9, 18; beginning 1, 10, 19; beginning 2, 11, 20; beginning 3, 12, 21; beginning 4, 13, 22; beginning 5, 14, 23; beginning 6, 15, 24; beginning 7, 16, 25; beginning 8, 17, 26.

Note. This exercise furnishes the best possible practice in mental addition. Each step must be taken without the aid of counters. The use of counters is fatal to all progress.

The rate of naming the successive numbers must be at first slow, and gradually quickened as facility is attained. *Each pupil in turn* should have sufficient practice to enable him to name the results as fast as he can count 1, 2, 3, 4, *etc.*

EXERCISE 6. — ORAL.

Find the sum of:

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**	**11.**	**12.**
2	3	5	6	3	5	3	2	3	2	3	5
4	2	7	3	8	0	1	7	5	1	6	6
3	6	3	5	7	9	7	8	7	9	7	7
7	7	2	4	2	6	8	3	6	8	8	8
8	6	4	9	6	4	9	5	3	2	2	9

13.	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**	**21.**	**22.**	**23.**	**24.**
3	2	5	5	4	5	3	1	6	9	6	4
3	7	3	6	8	5	6	8	8	5	4	5
4	7	2	4	7	3	7	8	7	4	3	6
5	3	1	7	3	6	3	7	9	3	7	7
2	2	3	7	5	4	4	4	4	6	2	8

25.	**26.**	**27.**	**28.**	**29.**	**30.**	**31.**	**32.**	**33.**	**34.**	**35.**	**36.**
4	3	6	7	5	8	2	9	8	7	4	1
4	7	2	5	5	2	9	6	2	8	7	5
3	5	1	8	9	2	9	5	7	5	8	8
7	5	8	3	3	7	4	4	3	5	5	7
2	6	8	4	2	4	5	8	6	4	2	9

37.	**38.**	**39.**	**40.**	**41.**	**42.**	**43.**	**44.**	**45.**	**46.**	**47.**	**48.**
3	7	6	8	7	4	5	2	9	1	6	9
2	8	8	4	2	9	3	8	8	9	6	9
7	5	9	2	6	8	2	9	7	8	6	9
2	6	7	8	1	2	5	5	3	2	8	9
6	3	3	4	7	3	4	7	5	9	7	6
1	4	2	4	8	4	5	1	4	1	3	4

NOTE. In adding, give *results* only. Thus, in example 1, say 15, 18, 22, 24; do *not* say 8 and 7 are 15, 15 and 3 are 18, 18 and 4 are 22, 22 and 2 are 24.

27. Find the sum of 4532, 2721, and 6855.

SOLUTION. Since only units of the same order can be added, we write ones under ones, tens under tens, *etc.;* and draw a line underneath.

We first find the sum of the ones and write this sum, 8, under the column of ones. We next find the sum of the tens and write the ones figure, 0, of this sum under the column of tens, but the tens figure, 1, we add with the digits of the next column, making $1 + 8 + 7 + 5 = 21$. We write the ones figure, 1, of this sum under the column of hundreds, but the tens figure, 2, we add with the digits of the next column, making $2 + 6 + 2 + 4 = 14$. We write the entire sum of the last column.

$$\begin{array}{r} 4532 \\ 2721 \\ \underline{6855} \\ 14108 \end{array}$$

Hence we have the following

RULE FOR ADDITION. *Write the numbers so that units of the same order shall stand in the same column.*

Add the right-hand column; write the units of this sum beneath, and add the tens, if any, to the next column. So proceed with each column.

Write the entire sum of the last column.

PROOF. *Add each column in the reverse order. If the results agree, the work may be assumed to be correct.*

EXERCISE 7. — WRITTEN.

Find the sum of:

1.	2.	3.	4.	5.	6.
1634	8976	8569	4848	2989	3787
7584	7438	3687	5155	3376	2578

7.	8.	9.	10.	11.	12.
2367	3627	6598	2654	8249	6337
5678	6587	2361	2876	7363	3785
1893	3981	3778	3789	9579	2879

13.	14.	15.	16.	17.	18.
8481	7557	5775	5437	2498	3448
2345	3452	7506	2975	3637	2700
4678	2534	6073	3000	4249	3136
3444	4567	2409	6894	5006	1999

19.	20.	21.	22.	23.	24.
3231	1323	5577	3705	5073	3750
2345	4352	3307	2608	8062	2430
4136	3614	3073	3555	3031	4880
3721	7198	2888	4878	2048	6220
1789	3255	1552	2492	4086	1890

25.	26.	27.	28.	29.
23,547	15,506	52,468	46,268	13,286
16,938	42,903	30,209	13,079	27,682
68,231	63,280	15,473	25,505	75,397
61,847	11,349	2,648	16,387	40,885

30.	31.	32.	33.
298,365	389,453	345,983	812,385
312,473	126,287	276,313	756,864
217,875	168,500	198,891	576,486
111,290	212,889	347,789	628,588

34.	35.	36.	37.
371,563	345,983	398,543	218,835
213,374	517,679	672,733	657,468
507,800	737,825	891,198	816,005
695,527	355,458	567,675	779,977
729,568	445,544	743,987	898,999

28. To add columns of digits correctly and rapidly is a real accomplishment, and the operation of addition should be continued until this can be done. The beginner, however, will need some **test** of the accuracy of his work. One test is to begin at the bottom of the right-hand column in adding, and write on a piece of waste paper the entire sum of each column; then to begin at the top of the left-hand column in adding, and write also the entire sum of each column; finally, to add the sums obtained in the first addition and the sums obtained in the second addition, and compare the results. If the results agree, the work may be assumed to be correct.

An example will show how this is done.

Beginning at the top of the left-hand column in adding, and writing the entire sum of each column, we have:	871254	**Beginning at the bottom of the right-hand column in adding, and writing the entire sum of each column, we have:**
	123456	
	789098	
28	357912	26
31	993286	28
23	3135006	17
17		23
28		31
26		28
3135006		3135006

On comparing the results we find that each sum is 3,135,006, and so conclude that the work is correct.

29. Since only units of the same order can be added, care must be taken in writing numbers to be added, that *all the units stand in a vertical column, all the tens in the next column (to the left), and all the hundreds in the next column, and so on.*

EXERCISE 8. — WRITTEN.

Write in columns and add:

1. 368; 784; 932; 227.
2. 878; 686; 897; 788.
3. 321; 254; 432; 542; 312.
4. 6842; 8284; 1643; 415; 243.
5. 3141; 9207; 164; 326; 541.
6. 958; 1803; 727; 468; 54.
7. 56; 927; 6529; 7892; 389.
8. 9; 5682; 4425; 8139; 27.
9. 1150; 2341; 5432; 476; 396.
10. 46,387; 42,386; 2679; 896; 685.
11. 32,461; 25,255; 3177; 4268; 7866.
12. 96,343; 74,817; 66,275; 37,840; 21,906.
13. 35,444; 47,781; 88,972; 44,960; 57,909.
14. 56,839; 29,948; 38,343; 43,433; 23,899.
15. 67,428; 39,001; 90,094; 53,718; 85,637.
16. 36 million 218 thousand 324; 22 million 8 thousand 63; 27 million 907 thousand 560; 19 million 27 thousand 812.
17. 5 million 18 thousand 589; 7 million 99 thousand 909; 6 million 66 thousand 666; 8 million 87 thousand 870.
18. Twenty million, twenty thousand, twenty; seventeen million, two hundred nine thousand, three hundred ninety; thirty-seven million, two hundred sixty-three thousand, eight hundred eighty-eight; ninety-seven million, seventy-nine thousand, five hundred sixty.
19. Five million, two hundred thousand, eight hundred six; three hundred seventy thousand, six hundred forty; twenty million, twenty thousand, twenty; eighty-seven million, eighty-seven thousand, eighty-seven.

Exercise 9. — Written.

1. Abraham lived 175 years; Isaac, 180; Jacob, 147; Joseph, 110; Moses, 120; Joshua, 110. Find the sum of their ages.

2. Adam lived 930 years; Seth, 912; Enos, 905; Cainan, 910; Methuselah, 969; Noah, 950. Find the sum of their ages.

3. The historical books of the Old Testament consist of 413 chapters; the prophecies, of 273; Job, of 42; books written by David and Solomon, of 201; books of the New Testament, of 260. How many chapters are there in the Bible?

4. The length of the Steamship Campania is 620 feet; of the Teutonic, 565 feet; of the New York, 560 feet; of the St. Paul, 554 feet. Find the sum of these lengths.

5. The number of vessels under the American flag on June 30, 1895, was as follows: foreign trade, 1193; domestic trade, 20,382; whale fisheries, 67; cod and mackerel fisheries, 1598. Find the total number.

6. The number of immigrants to the United States for the year ending June 30, 1895, was: females, 120,024; males, 159,924. Find the whole number of immigrants.

7. Of the number of immigrants given in Example 6, 47,972 were from Ireland; 31,948 from England; 36,351 from Germany; 31,855 from Austria and Hungary; 36,937 from Italy. Find the entire number from these countries.

8. By the latest census, the population of London was 4,349,000, and the population of Paris was 2,511,455. What is the population of London and Paris together?

9. By the latest census, New York has a population of 1,891,306; Chicago, 1,567,727; Philadelphia, 1,142,653. What is the population of these three cities together?

10. Berlin has a population of 1,579,244; Vienna, 1,364,548; St. Petersburg, 1,035,439. What is the population of these three cities together?

11. Canton, China, has a population of 1,800,000; Tokio, Japan, 1,389,684; Constantinople, 873,565. What is the population of these three cities together?

12. The three largest cities of Italy are Naples, 530,872; Rome, 423,217; Milan, 415,521. Find the total population of these three cities.

13. The cost of digging a cellar for an academy building was 419 dollars; the cost of the foundation walls, ready for the brick work, was 1687 dollars; the brick work and plastering, 21,569 dollars; the slating, gutters, and conductors, 2327 dollars; wood-work, glazing, and painting, 10,560 dollars. Find the entire cost of the building.

14. Boston received on the 13th of January, 1896, 28,373 bushels of corn; 30,774 bushels of oats; 27,883 bushels of wheat; 4450 bushels of barley. Find the total number of bushels of grain received that day.

15. A lumber merchant sold at one time 13,854 feet of pine boards; at another time, 27,396 feet; and at another time, 18,379 feet. How many feet did he sell?

16. A farmer sold five loads of hay. The first load weighed 2129 pounds; the second, 1997; the third, 2017; the fourth, 2007; the fifth, 1976. How many pounds did the five loads together weigh?

17. A miller bought in September 798 bushels of wheat; in October, 897 bushels; in November, 685 bushels; and in December, 550 bushels. How many bushels did he buy in the four months?

18. Lake Superior has 32,290 square miles of surface; Lake Michigan, 23,903; Lake Huron, 23,684; Lake Erie, 9439; Lake Ontario, 7654. Find the total area of these lakes in square miles.

19. The states which have the greatest number of miles of railway are: Illinois, 10,460; Pennsylvania, 9594; Texas, 9264; Kansas, 8841; Ohio, 8547; Iowa, 8478; New York, 8072. How many miles have these states in all?

20. The gold product of the United States for 1894 was valued at 39,500,000 dollars; of Australasia, 41,760,000 dollars; of Africa, 40,271,000 dollars; of Russia, 24,-133,000 dollars. Find the total value of the gold produced by these four countries in 1894.

21. The gold pieces coined in the mints of the United States from January 1 to June 30, 1895, were: double eagles, 755,753; eagles, 325,355; half eagles, 978,544; quarter eagles, 44. Find the whole number of gold coins.

22. The values of these coins are: double eagles, 15,-115,060 dollars; eagles, 3,253,550 dollars; half eagles, 4,892,720 dollars; quarter eagles, 110 dollars. Find their total value.

23. The silver pieces coined in the first half of 1895 were: standard dollars, 862,470; half dollars, 2,198,556; quarter dollars, 3,949,152; dimes, 1,650,470. Find the whole number of silver coins.

24. The values of these silver coins are: standard dollars, 862,470 dollars; half dollars, 1,099,278 dollars; quarter dollars, 987,288 dollars; dimes, 165,047 dollars. Find the total value of the silver coins.

CHAPTER III.

SUBTRACTION OF INTEGRAL NUMBERS.

30. There are 9 oranges in a fruit-dish. If 3 are taken away, how many will remain?

31. Subtraction. The process of taking one number from another is called *subtraction*.

32. The number taken away is called the *subtrahend;* the number from which the subtrahend is taken is called the *minuend;* and the number remaining, the *remainder* or *difference.*

Thus, in taking 3 oranges from 9 oranges, 9 is the minuend, 3 the subtrahend, and 6 the remainder. The subtrahend 3 and the remainder 6 together are equal to the minuend 9.

33. *The sum of the remainder and the subtrahend is always equal to the minuend.* Hence,

34. *To test the accuracy of the work in subtraction,* we add the remainder and the subtrahend. The sum will be equal to the minuend, if the work is correct.

35. The sign of subtraction is —. When placed between two numbers this sign means that the number after it is to be subtracted from the number before it. It is called the *minus* sign.

The expression 8 — 6 is read: eight *minus* six.

36. *The minuend, subtrahend, and remainder must all be like numbers. From units of any order units of the same order only can be subtracted, ones from ones, tens from tens, hundreds from hundreds, etc.*

EXERCISE 10. — ORAL.

Take the lower from the upper numbers:

1 1	2 1	3 1	4 1	5 1	6 1	7 1	8 1	9 1	10 1
2 2	3 2	4 2	5 2	6 2	7 2	8 2	9 2	10 2	11 2
3 3	4 3	5 3	6 3	7 3	8 3	9 3	10 3	11 3	12 3
4 4	5 4	6 4	7 4	8 4	9 4	10 4	11 4	12 4	13 4
5 5	6 5	7 5	8 5	9 5	10 5	11 5	12 5	13 5	14 5
6 6	7 6	8 6	9 6	10 6	11 6	12 6	13 6	14 6	15 6
7 7	8 7	9 7	10 7	11 7	12 7	13 7	14 7	15 7	16 7
8 8	9 8	10 8	11 8	12 8	13 8	14 8	15 8	16 8	17 8
9 9	10 9	11 9	12 9	13 9	14 9	15 9	16 9	17 9	18 9

NOTE. This *subtraction table* should be copied on the board, and each pupil in turn required to name the remainders as the teacher touches the examples at random with a pointer. The drill should be continued until every pupil can give quick and correct answers.

Take the lower from the upper numbers:

20	30	40	50	60	70	80	90	100
1	9	2	8	7	6	4	5	3
21	31	41	51	61	71	81	91	101
2	3	5	7	4	6	8	1	9
22	32	42	52	62	72	82	92	102
3	2	4	6	1	8	9	5	7
23	33	43	53	63	73	83	93	103
4	2	1	5	7	9	6	8	3
24	34	44	54	64	74	84	94	104
5	6	8	9	4	1	3	2	7
25	35	45	55	65	75	85	95	105
6	5	3	1	2	9	8	4	7
26	36	46	56	66	76	86	96	106
1	3	5	2	4	6	7	9	8
27	37	47	57	67	77	87	97	107
1	2	4	6	5	7	9	3	8
28	38	48	58	68	78	88	98	108
8	7	5	3	2	1	4	6	9

1. Subtract by 2's from 20 to 0; from 21 to 1.
2. Subtract by 3's from 20 to 2; from 21 to 0.
3. Subtract by 4's from 30 to 2; from 31 to 3.
4. Subtract by 5's from 32 to 2; from 34 to 4.
5. Subtract by 6's from 33 to 3; from 35 to 5.
6. Subtract by 7's from 43 to 1; from 46 to 4.
7. Subtract by 8's from 43 to 3; from 47 to 7.
8. Subtract by 9's from 54 to 0; from 57 to 3.

37. From 876 take 631.

Minuend, 876
Subtrahend, 631
Remainder, 245

SOLUTION. Since we cannot subtract units of one order from units of another order, we write ones under ones, tens under tens, and so on. Then 1 from 6 leaves 5, and we write 5 under the column of ones; 3 tens from 7 tens leave 4 tens, and we write 4 under the column of tens; 6 hundreds from 8 hundreds leave 2 hundreds, and we write 2 under the column of hundreds. The remainder, therefore, is 245.

EXERCISE 11. — WRITTEN.

Find the remainder:

1. 596 — 245.
2. 374 — 261.
3. 816 — 404.
4. 875 — 325.
5. 578 — 523.
6. 758 — 638.
7. 890 — 780.
8. 757 — 257.
9. 649 — 333.
10. 774 — 532.
11. 348 — 307.
12. 489 — 344.
13. 768 — 555.
14. 976 — 866.
15. 679 — 666.
16. 588 — 464.
17. 777 — 667.
18. 540 — 320.
19. 989 — 889.
20. 763 — 562.
21. 645 — 345.
22. 862 — 452.
23. 968 — 427.
24. 705 — 505.
25. 650 — 450.
26. 904 — 701.
27. 603 — 303.
28. 512 — 510.
29. 484 — 480.
30. 713 — 611.
31. 798 — 682.
32. 897 — 682.
33. 6981 — 4541.
34. 6297 — 4187.
35. 8247 — 8127.
36. 7428 — 7307.
37. 7926 — 6914.
38. 6927 — 3211.
39. 7842 — 6841.
40. 6283 — 5172.
41. 5498 — 5388.
42. 8456 — 7235.
43. 7259 — 6138.
44. 8945 — 8835.
45. 8791 — 6691.
46. 2896 — 2880.
47. 8962 — 3330.
48. 9145 — 8023.

38. From 783 take 469.

Minuend, 783
Subtrahend, 469
Remainder, 314

SOLUTION. Since we cannot take 9 from 3, we change 1 of the 8 *tens* to 10 ones and add them to the 3 ones, making 13; then 9 from 13 leaves 4. Now as we have taken 1 of the 8 *tens*, we have only 7 tens remaining, and 6 tens from 7 tens leave 1 ten; 4 hundreds from 7 hundreds leave 3 hundreds. The remainder, therefore, is 314.

From 359 take 186.

Minuend,	359
Subtrahend,	186
Remainder,	173

Solution. Here 6 from 9 leavès 3. Since we cannot take 8 tens from 5 tens, we change 1 of the 3 *hundreds* to 10 *tens* and add them to the 5 *tens*, making 15 *tens;* then 8 tens from 15 tens leave 7 tens. As we have taken 1 of the 3 *hundreds*, we have only 2 hundreds remaining; and 1 hundred from 2 hundreds leaves 1 hundred. The remainder, therefore, is 173.

From 5000 take 2765.

	(4)	(9)	(9)	(10)
Minuend,	5	0	0	0
Subtrahend,	2	7	6	5
Remainder,	2	2	3	5

Solution. As there are no ones, no tens, and no hundreds in the minuend, 1 of the 5 thousands is taken, leaving 4 thousands, and changed to 10 hundreds; then 1 of the 10 hundreds is taken, leaving 9 hundreds, and changed to 10 tens; then 1 of the 10 tens is taken, leaving 9 tens, and changed to 10 ones. That is, the 5000 is changed to 4 thousands, 9 hundreds, 9 tens, and 10 ones. Taking 2 thousands, 7 hundreds, 6 tens, 5 ones from 4 thousands, 9 hundreds, 9 tens, and 10 ones, we have 2235.

From these examples we have the following

Rule for Subtraction. *Write the subtrahend under the minuend, placing units of the same order in the same column, ones under ones, tens under tens,* etc.

Begin at the right and subtract each order of units of the subtrahend from the corresponding order of the minuend, and write the result beneath, step by step.

If any order of the minuend has fewer units than the same order of the subtrahend, increase the units of this order of the minuend by 10 and subtract; then diminish by one the units of the next higher order of the minuend and subtract.

Proof. *Add the remainder and the subtrahend. If the sum is equal to the minuend, the work is correct.*

Exercise 12. — Written.

Find the remainder and prove:

1. 945 — 896.	11. 407 — 348.	21. 682 — 497.
2. 761 — 674.	12. 544 — 489.	22. 611 — 363.
3. 904 — 816.	13. 855 — 768.	23. 701 — 304.
4. 925 — 875.	14. 964 — 787.	24. 821 — 234.
5. 623 — 578.	15. 665 — 467.	25. 452 — 267.
6. 820 — 758.	16. 863 — 588.	26. 713 — 409.
7. 980 — 890.	17. 482 — 294.	27. 927 — 468.
8. 733 — 649.	18. 866 — 577.	28. 847 — 449.
9. 832 — 774.	19. 720 — 540.	29. 560 — 283.
10. 855 — 768.	20. 962 — 763.	30. 650 — 384.

31. 4145 — 3981.	51. 52,431 — 37,658.
32. 4187 — 3299.	52. 42,315 — 18,597.
33. 8217 — 7368.	53. 48,115 — 43,567.
34. 7307 — 6429.	54. 91,617 — 76,928.
35. 3211 — 2933.	55. 52,222 — 18,765.
36. 6814 — 5842.	56. 82,336 — 36,798.
37. 7235 — 6856.	57. 73,451 — 27,654.
38. 6138 — 3349.	58. 62,134 — 28,456.
39. 4723 — 2934.	59. 37,273 — 11,498.
40. 5388 — 2389.	60. 62,130 — 38,685.
41. 3000 — 1234.	61. 72,006 — 38,569.
42. 5000 — 2737.	62. 60,003 — 35,742.
43. 4000 — 1349.	63. 85,004 — 25,687.
44. 7000 — 5738.	64. 94,052 — 76,584.
45. 6000 — 4985.	65. 50,000 — 34,073.
46. 5000 — 3126.	66. 36,007 — 27,129.
47. 8000 — 4778.	67. 90,005 — 24,847.
48. 9000 — 7879.	68. 51,020 — 38,236.
49. 7000 — 5416.	69. 70,040 — 28,567.
50. 8000 — 6993.	70. 79,000 — 29,325.

EXERCISE 13. — WRITTEN.

1. The longest river in the world is the Missouri. Its length from its source to the Gulf of Mexico is 4506 miles. The length of the Amazon is 3994 miles. How much longer is the Missouri than the Amazon?

2. The highest mountain in the world is Mt. Everest, Hindostan, which is 29,062 feet high. How many feet higher is Mt. Everest than Pikes Peak, Colorado, which is 14,147 feet high?

3. The greatest depth of water ever measured is 29,400 feet. The greatest height ever reached by a balloon is 37,000 feet. Find by how many feet the greatest depth ever reached is less than the greatest height ever reached.

4. Find the difference in height between Mt. Everest and the greatest height ever reached by a balloon.

5. The area of Delaware is 1960 square miles, and the area of Rhode Island is 1085. How many more square miles has Delaware than Rhode Island?

6. The population of Rhode Island in 1890 was 345,506, and that of Delaware 168,493. How many more inhabitants had Rhode Island than Delaware?

7. The population of Connecticut in 1890 was 746,258, and that of Maine 661,086. How many more inhabitants had Connecticut than Maine?

8. The area of New York is 47,620 square miles, and the area of Pennsylvania is 44,985 square miles. How many more square miles has New York than Pennsylvania?

9. The population of the state of New York in 1890 was 5,997,853, and that of Pennsylvania 5,258,014. How many more inhabitants had New York than Pennsylvania?

10. The number of tons of copper produced in the United States for the year 1895 was 171,067; the number of tons in 1894 was 159,623. How many more tons were produced in 1895 than in 1894?

11. The number of acres of barley in the United States for 1895 was 3,299,937, and the number of acres of rye was 1,890,345. How many more acres were sown to barley than to rye?

12. The product of barley for 1895 was 87,573,000 bushels, and of rye 27,210,000 bushels. How many more bushels of barley were raised than of rye?

13. The number of acres of winter wheat in the United States for 1895 was 22,609,332, and the number of acres of spring wheat was 11,438,000. How many more acres were sown to winter wheat than to spring wheat?

14. The number of bushels of winter wheat raised in the United States in 1895 was 261,242,000, and of spring wheat 205,861,000. How many more bushels of winter wheat were raised than of spring wheat?

15. The number of acres of corn in the United States for 1895 was 82,075,830, and the number of acres of oats was 27,878,406. How many more acres of corn were there than of oats?

16. The product of corn for 1895 was 2,151,139,000 bushels, and of oats 824,444,000 bushels. How many more bushels of corn were raised than of oats?

17. The value of horses and mules owned in the United States, January, 1895, was 687,658,414 dollars; the value of oxen, cows, and other cattle was 845,600,858 dollars. By how many dollars did the value of the cattle exceed the value of the horses and mules?

EXERCISE 14. — WRITTEN.

Review Problems.

1. A man has a hundred sheep. In one pasture he has 19; in another, 29; in another, 26. How many sheep has he besides the sheep in these three pastures?

2. A farm of 240 acres has 52 acres of corn, 22 acres of oats, 39 acres of wheat, and 26 acres of flax, and the rest wild land. How many acres of wild land has it?

3. In January a man had 1000 dollars in a bank. He afterwards drew out 312 dollars, and then 520 dollars. How many dollars had he still in the bank?

4. The distance from New York to Queenstown is 2890 miles. A steamship from New York for Queenstown made the first day 478 miles, the second day 496, and the third day 492. How many miles had the ship still to go?

5. The distance from Boston to Chicago by way of Albany is 1038 miles. The distance from Boston to Albany is 202 miles, and from Albany to Buffalo is 297 miles. How many miles is it from Buffalo to Chicago?

6. A man owns 8000 acres of wild land. If he sells 1280 acres to one man and 1600 acres to another man, how many acres will he still own?

7. A certain piece of land has 957 pine trees, 787 spruce trees, and a number of hemlock trees. If the whole number of pine, spruce, and hemlock trees is 2200, how many hemlock trees are there?

8. How many does the sum of 2583 and 4905 exceed the difference of 9421 and 2892?

9. The area of Maine is 29,895 square miles; of New Hampshire, 9005; of Vermont, 9135; of Massachusetts,

8040; of Connecticut, 4845; of Rhode Island, 1085. How many square miles does Maine lack of being equal in area to the other five New England states together?

10. The population of Massachusetts in 1890 was 2,238,943; of Maine, 661,068; of New Hampshire, 376,530; of Vermont, 332,242; of Connecticut, 746,258; of Rhode Island, 345,506. How much did the population of Massachusetts fall short of the entire population of the other New England states in 1890?

11. The area of Wales is 8125 square miles; of England, 50,535; of Scotland, 29,167; of Ireland, 31,762. How many more square miles are there in Scotland and Ireland together than in England and Wales?

12. At the battle of Waterloo in 1815 Napoleon had 48,950 infantry, 15,765 cavalry, 7732 artillery. Wellington had 69,686 men. How many more men had Napoleon than Wellington?

13. The greatest number of persons admitted to the Philadelphia Exposition in one day was 219,526; to the Paris Exposition, 397,150; to the Chicago Exposition, 729,203. Find the excess of Chicago's greatest number in one day over the sum of the greatest numbers of the other two Expositions in one day.

14. A farmer sold 8000 pounds of hay. The first load weighed 2112 pounds; the second, 1936; the third, 1987. How much must the fourth weigh to make just 8000 pounds?

15. If the minuend is 70,000 and the remainder 9889, what is the subtrahend?

16. If the subtrahend is 22,367 and the remainder 8733, what is the minuend?

CHAPTER IV.

MULTIPLICATION OF INTEGRAL NUMBERS.

39. A surveyor in using his four-foot rule to find the length of a stick of timber may count the *number of feet* measured each time he lays his rule on the stick; thus, 4, 8, 12, etc. If the stick is 40 feet long, he has by this method found the *sum* of 10 fours.

Or he may count the *number of times* he applies the rule to the stick, and then *multiply*, as we say, the number of feet in the length of the rule by the number of times he has applied the rule. Thus, 10 times 4 feet are 40 feet.

In this operation, 4 feet, *the number of units* taken each time, is called the *multiplicand;* 10, *the number of times* the multiplicand is taken, the *multiplier;* and 40 feet, the number obtained by taking 4 feet 10 times, the *product.* Hence, we have the following definitions:

40. Multiplication. The process of taking a *number of units* a *number of times* is called *multiplication.*

41. Multiplicand. The number of units taken is called the *multiplicand.*

42. Multiplier. The number that shows how many times the multiplicand is taken is called the *multiplier.*

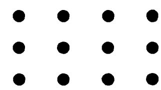
43. Product. The number of units obtained by taking the multiplicand a number of times is called the *product.*

44. *The multiplier always signifies a number of times, and is, therefore, an abstract number.*

45. *The multiplicand and product are like numbers.*

46. Factors. The numbers used in making a product are called *factors* of the product.

47. The product of two factors is the same whichever factor is taken as the multiplier. Thus, 3 times 4 = 4 times 3. The dots in the margin read across the page make 3 fours; read up and down the page they make 4 threes.



NOTE. The multiplicand always signifies a *number of units*, whether *the kind of units is stated or not.* The only difference between 15 and 15 horses is that in the first case the kind of units counted is not stated, and in the second case the kind of units counted is stated.

We may interchange the multiplicand and multiplier if we refer to the *numbers only.* Thus, in the example 3 times 4 horses, we cannot say 4 horses times 3, but we may interchange the 3 and 4, and have 4 times 3 horses. The product in either case is 12 horses. With this understanding, we may always use the smaller number as multiplier.

The sign of multiplication is ×. When the multiplier *precedes* the multiplicand, the sign × is read *times.*

Thus, 6 × 7 dollars = 42 dollars is read: 6 *times* 7 dollars equals 42 dollars.

When the multiplier *follows* the multiplicand the sign × is read *multiplied by.*

Thus, 7 dollars × 6 = 42 dollars is read: 7 dollars *multiplied by* 6 equal 42 dollars, and means 7 dollars *taken* 6 *times* equals 42 dollars.

48. The following table must be learned by heart:

$2 \times 1 = 2.$	$3 \times 1 = 3.$	$4 \times 1 = 4.$	$5 \times 1 = 5.$
$2 \times 2 = 4.$	$3 \times 2 = 6.$	$4 \times 2 = 8.$	$5 \times 2 = 10.$
$2 \times 3 = 6.$	$3 \times 3 = 9.$	$4 \times 3 = 12.$	$5 \times 3 = 15.$
$2 \times 4 = 8.$	$3 \times 4 = 12.$	$4 \times 4 = 16.$	$5 \times 4 = 20.$
$2 \times 5 = 10.$	$3 \times 5 = 15.$	$4 \times 5 = 20.$	$5 \times 5 = 25.$
$2 \times 6 = 12.$	$3 \times 6 = 18.$	$4 \times 6 = 24.$	$5 \times 6 = 30.$
$2 \times 7 = 14.$	$3 \times 7 = 21.$	$4 \times 7 = 28.$	$5 \times 7 = 35.$
$2 \times 8 = 16.$	$3 \times 8 = 24.$	$4 \times 8 = 32.$	$5 \times 8 = 40.$
$2 \times 9 = 18.$	$3 \times 9 = 27.$	$4 \times 9 = 36.$	$5 \times 9 = 45.$

$6 \times 1 = 6.$	$7 \times 1 = 7.$	$8 \times 1 = 8.$	$9 \times 1 = 9.$
$6 \times 2 = 12.$	$7 \times 2 = 14.$	$8 \times 2 = 16.$	$9 \times 2 = 18.$
$6 \times 3 = 18.$	$7 \times 3 = 21.$	$8 \times 3 = 24.$	$9 \times 3 = 27.$
$6 \times 4 = 24.$	$7 \times 4 = 28.$	$8 \times 4 = 32.$	$9 \times 4 = 36.$
$6 \times 5 = 30.$	$7 \times 5 = 35.$	$8 \times 5 = 40.$	$9 \times 5 = 45.$
$6 \times 6 = 36.$	$7 \times 6 = 42.$	$8 \times 6 = 48.$	$9 \times 6 = 54.$
$6 \times 7 = 42.$	$7 \times 7 = 49.$	$8 \times 7 = 56.$	$9 \times 7 = 63.$
$6 \times 8 = 48.$	$7 \times 8 = 56.$	$8 \times 8 = 64.$	$9 \times 8 = 72.$
$6 \times 9 = 54.$	$7 \times 9 = 63.$	$8 \times 9 = 72.$	$9 \times 9 = 81.$

It is also well to know the following table:

$10 \times 1 = 10.$	$11 \times 1 = 11.$	$12 \times 1 = 12.$	$10 \times 10 = 100.$
$10 \times 2 = 20.$	$11 \times 2 = 22.$	$12 \times 2 = 24.$	$10 \times 11 = 110.$
$10 \times 3 = 30.$	$11 \times 3 = 33.$	$12 \times 3 = 36.$	$10 \times 12 = 120.$
$10 \times 4 = 40.$	$11 \times 4 = 44.$	$12 \times 4 = 48.$	$11 \times 10 = 110.$
$10 \times 5 = 50.$	$11 \times 5 = 55.$	$12 \times 5 = 60.$	$11 \times 11 = 121.$
$10 \times 6 = 60.$	$11 \times 6 = 66.$	$12 \times 6 = 72.$	$11 \times 12 = 132.$
$10 \times 7 = 70.$	$11 \times 7 = 77.$	$12 \times 7 = 84.$	$12 \times 10 = 120.$
$10 \times 8 = 80.$	$11 \times 8 = 88.$	$12 \times 8 = 96.$	$12 \times 11 = 132.$
$10 \times 9 = 90.$	$11 \times 9 = 99.$	$12 \times 9 = 108.$	$12 \times 12 = 144.$

49. Find the product of 6×4587.

$$\begin{array}{r} 4587 \\ \underline{6} \\ 27522 \end{array}$$

SOLUTION. We write the multiplier 6 under the multiplicand, as in the margin, and begin at the right to multiply. 6 times 7 = 42, and we write the 2 ones in the place of ones, and reserve the 4 tens to be added to the product of the tens; then 6 times 8 tens = 48 tens, which, with the 4 tens reserved, make 52 tens, or 5 hundreds and 2 tens, and we write the 2 tens in the place of tens; then 6 times 5 hundreds = 30 hundreds, which, with the 5 hundreds reserved, make 35 hundreds, or 3 thousands and 5 hundreds, and we write the 5 hundreds in the place of hundreds; then 6 times 4 thousands = 24 thousands, which, with the 3 thousands reserved, make 27 thousands, and we write 27 to the left of the 5 hundreds.

EXERCISE 15. — WRITTEN.

Find the product of:

1. 2×479.
2. 3×307.
3. 4×654.
4. 5×578.
5. 6×698.
6. 7×384.
7. 8×278.
8. 9×217.
9. 5×896.
10. 3×987.
11. 6×289.
12. 8×832.
13. 7×788.
14. 5×909.
15. 6×678.
16. 7×887.
17. 9×239.
18. 4×698.
19. 3×575.
20. 8×876.
21. 2×6187.
22. 3×5283.
23. 4×8196.
24. 5×8753.
25. 6×3578.
26. 7×7538.
27. 8×5783.
28. 9×9127.
29. 6×1987.
30. 7×2789.
31. 4×7829.
32. 7×6408.
33. 3×8046.
34. 4×7809.
35. 6×9087.
36. 7×7089.
37. 7×3847.
38. 8×2395.
39. 2×9887.
40. 9×8832.
41. $2 \times 76{,}195$
42. $3 \times 67{,}593$
43. $4 \times 59{,}387$.
44. $5 \times 68{,}734$.
45. $6 \times 73{,}486$.
46. $7 \times 24{,}386$.
47. $8 \times 43{,}286$.
48. $9 \times 34{,}682$.
49. $3 \times 95{,}738$.
50. $7 \times 48{,}037$.
51. $6 \times 84{,}307$.
52. $4 \times 90{,}374$.
53. $5 \times 89{,}207$.
54. $7 \times 70{,}289$.
55. $9 \times 39{,}281$.
56. $8 \times 41{,}879$.
57. $2 \times 78{,}978$.
58. $8 \times 65{,}278$.
59. $7 \times 52{,}873$.
60. $9 \times 28{,}537$.

EXERCISE 16. — ORAL.

Find the cost of 9 pints of chestnuts at 5 cents a pint.

SOLUTION. If 1 pint of chestnuts costs 5 cents, 9 pints will cost 9×5 cents, or 45 cents.

1. Find the cost of 9 quarts of milk at 6 cents a quart.
2. Find the cost of 6 quarts of beans at 9 cents a quart.
3. Find the cost of 7 quarts of berries at 8 cents a quart.
4. Find the cost of 8 quarts of vinegar at 7 cents a quart.
5. Find the cost of 7 pounds of mutton at 9 cents a pound.

6. Find the cost of 9 tons of coal at 7 dollars a ton.

7. Find the cost of 11 cords of wood at 6 dollars a cord.

8. Find the cost of 12 two-cent postage stamps.

9. How much milk will 9 eight-quart cans hold?

10. At 9 dollars a week, how much will a man earn in 9 weeks?

11. At 9 cents a pound, what will 8 pounds of lard cost?

12. At 6 cents a pound, what will 7 pounds of sugar cost?

13. At 20 cents a dozen, what will 3 dozen eggs cost?

14. If a peck of oats weighs 8 pounds, what will 4 pecks weigh?

15. At 4 cents each, what will 9 oranges cost?

16. How many days are there in 7 weeks?

17. If a horse goes 6 miles an hour, how far will he go in 4 hours?

18. Find the cost of 7 yards of velvet at 3 dollars a yard.

19. A fly has 6 legs. How many legs have 8 flies?

20. How many acres in 5 fields of 12 acres each?

21. Find the cost of 5 bushels of charcoal at 11 cents a bushel.

22. Find the cost of 12 yards of cloth at 10 cents a yard.

23. Find the cost of 8 quarts of berries at 8 cents a quart.

24. At 8 cents a head, what will 7 heads of lettuce cost?

25. At 9 cents a bunch, what will 6 bunches of celery cost?

26. At 7 dollars a load, what will 8 loads of bricks cost?

27. At 6 dollars a pair, what will 9 pairs of boots cost?

50. When the multiplier is 10, 100, 1000, etc., the product is obtained by simply annexing as many zeros to the multiplicand as are found in the multiplier. Thus:

$$4587 \times 10 = 45{,}870.$$

Find the product of 400×360.

The	multiplicand	is	36×10;	that is,	360
the	multiplier	is	4×100;	that is,	400
the	product	is	144×1000;	that is,	144,000

Hence, *When the multiplicand and multiplier end in zeros, or either ends in zeros, we multiply without regard to the zeros, but annex to the product as many zeros as there are at the end of the multiplicand and of the multiplier.*

Exercise 17. — Written.

Find the product of:

1. Multiply 37 by 10; by 100; by 1000; by 10,000.
2. Multiply 84 by 10; by 100; by 1000; by 10,000.
3. Multiply 92 by 10; by 100; by 1000; by 10,000.
4. Multiply 630 by 10; by 100; by 1000; by 10,000.
5. Multiply 750 by 20; by 200; by 2000; by 20,000.
6. Multiply 6100 by 3; by 30; by 300; by 3000.
7. Multiply 8700 by 8; by 80; by 800; by 8000.
8. Multiply 3800 by 9; by 90; by 900; by 9000.
9. Multiply 7900 by 7; by 70; by 700; by 7000.
10. Multiply 8730 by 6; by 60; by 600; by 6000.

51. Find the cost of 1105 barrels of flour at 5 dollars a barrel.

Solution. If one barrel of flour costs 5 dollars, 1105 barrels will cost 1105 times 5 dollars. But 1105 times 5 dollars is equal to 5 times 1105 dollars, and this product is 5525 dollars. *Ans.*

$$\begin{array}{r} 1105 \\ \underline{5} \\ 5525 \end{array}$$

Exercise 18. — Written.

Find the cost of:

1. 1122 cords of wood at 6 dollars a cord.
2. 1232 barrels of flour at 5 dollars a barrel.
3. 2873 barrels of apples at 3 dollars a barrel.
4. 2575 firkins of butter at 4 dollars a firkin.
5. 2300 barrels of cranberries at 9 dollars a barrel.
6. 2425 bushels of beans at 3 dollars a bushel.
7. 3618 barrels of vinegar at 7 dollars a barrel.
8. 1200 barrels of linseed oil at 20 dollars a barrel.
9. 3725 barrels of sugar at 9 dollars a barrel.
10. 4886 sheep at 6 dollars a head.
11. 5280 thousand bricks at 7 dollars a thousand.
12. 8689 cords of pine wood at 3 dollars a cord.
13. 1350 cases of shoes at 60 dollars a case.
14. A bushel of oats weighs 32 pounds. How much will 2000 bushels weigh?
15. A bushel of corn weighs 56 pounds. How much will 600 bushels weigh?
16. A bushel of wheat weighs 60 pounds. How much will 458 bushels weigh?
17. A mile contains 1760 yards. How many yards are there in 90 miles?
18. An acre contains 160 square rods. How many square rods are there in 80 acres?
19. If a railway train goes on the average 40 miles an hour, how far will it go in 24 hours?
20. A ton is 2000 pounds. How many pounds are there in 50 tons?
21. If a barrel of kerosene oil holds 42 gallons, how many gallons are there in 600 barrels?
22. A clock strikes 156 times a day. How many times does it strike in 90 days?

52. Find the product of 649 times 4587.

SOLUTION. The multiplier is 600 + 40 + 9. The product is obtained by multiplying by 9, then by 40, and then by 600, and adding the products.

	(1)	(2)
	4587	4587
	649	649
9 times the multiplicand =	41283	41283
40 times the multiplicand =	183480	18348
600 times the multiplicand =	2752200	27522
649 times the multiplicand =	2976963	2976963

Since zeros at the right of the partial products do not affect the result of the addition, they may be omitted as in (2). *Care must be taken, however, to put the right-hand figure of each partial product directly under the figure of the multiplier used to obtain it.*

Find the product of 2007 times 4587.

SOLUTION. The products corresponding to the zeros in the multiplier will be zero, and therefore they need not be written.

```
   4587
   2007
  32109
 9174
9206109
```

Find the product of 4587 times 2007, and compare the product with the product obtained in the last example.

From the examples given we have the following

RULE FOR MULTIPLICATION. *Write the multiplier under the multiplicand, units under units, tens under tens, etc., and draw a line beneath.*

Begin at the right and multiply each order of units of the multiplicand by each figure of the multiplier.

Place the right-hand figure of each product under the figure of the multiplier used to obtain it, and add the partial products.

PROOF. *Interchange the multiplier and multiplicand and multiply. If the results agree, the work may be assumed to be correct.*

EXERCISE 19. — WRITTEN.

Find the product of:

1. 25×3684.
2. 36×5873.
3. 57×3597.
4. 54×5328.
5. 34×6324.
6. 37×8465.
7. 73×7124.
8. 46×7059.
9. 53×2854.
10. 75×5481.
11. 28×8163.
12. 87×1854.
13. 91×2345.
14. 83×4895.
15. 19×4652.
16. 39×7564.
17. 62×6781.
18. 92×3765.
19. 85×4796.
20. 43×9037.
21. 47×8249.
22. 77×8009.
23. 65×2846.
24. 57×2859.
25. 49×3785.
26. 97×3854.
27. 79×8764.
28. 98×9852.
29. 46×6234.
30. 372×5746.
31. 328×1875.
32. 564×2836.
33. 259×4781.
34. 476×2958.
35. 367×4856.
36. 471×4938.
37. 587×7548.
38. 428×4137.
39. 286×2397.
40. 384×6257.
41. 359×2975.
42. 792×7085.
43. 478×2086.
44. 509×3692.
45. 873×5947.
46. 467×5508.
47. 809×5973.
48. 237×8787.
49. 468×8035.
50. 574×9123.
51. 387×3876.
52. 807×3063.
53. 567×4689.
54. 329×4987.
55. 418×5073.
56. 389×9876.
57. 376×6739.
58. 987×2875.

Exercise 20. — Written.

1. Find the cost of 38 cows at 27 dollars a head.

2. Find the cost of 37 acres of land at 67 dollars an acre.

3. Find the cost of 12 passenger cars at 3728 dollars a car.

4. There are 640 acres in a square mile. How many acres are there in 654 square miles?

5. There are 160 square rods in an acre. How many square rods are there in 640 acres?

6. There are 320 rods in a mile. How many rods are there in 976 miles?

7. There are 1760 yards in a mile. How many yards are there in 29 miles?

8. There are 5280 feet in a mile. How many feet are there in 17 miles?

9. There are 144 square inches in a square foot. How many square inches are there in 89 square feet?

10. There are 128 cubic feet in a cord. How many cubic feet are there in 187 cords of wood?

11. A bushel of corn weighs 56 pounds. How many pounds will 625 bushels weigh?

12. How many oranges are there in 47 boxes if each box contains 189 oranges?

13. How many gallons of vinegar in 78 casks if each cask contains 42 gallons?

14. How many bushels of buckwheat will 989 acres produce if each acre produces 27 bushels?

15. How many bushels of spring wheat will 197 acres produce if each acre produces 18 bushels?

16. How many bushels of winter rye will 278 acres produce if each acre produces 17 bushels?

17. How many bushels of barley will 287 acres produce if each acre produces 27 bushels?

18. How many bushels of corn will 1018 acres produce if each acre produces 27 bushels?

19. How many bushels of oats will 2087 acres produce if each acre produces 29 bushels?

20. How many bushels of potatoes will 320 acres produce if each acre produces 129 bushels?

21. How many pounds of hay will 299 acres produce if each acre produces 2178 pounds?

22. If a ton of stone measures 13 cubic feet, how many cubic feet will there be in 139 tons?

23. A cubic foot of hard coal weighs 84 pounds. How many pounds will 27 cubic feet weigh?

24. A cubic foot of silver weighs 656 pounds. How many pounds will 16 cubic feet weigh?

25. A cubic foot of gold weighs 1204 pounds. How many pounds will 12 cubic feet weigh?

26. A cubic foot of cast iron weighs 447 pounds. How many pounds will 28 cubic feet weigh?

27. A cubic foot of wrought iron weighs 486 pounds. How many pounds will 27 cubic feet weigh?

28. A cubic foot of lead weighs 709 pounds. How many pounds will 128 cubic feet weigh?

29. A cubic foot of platinum weighs 1365 pounds. How many pounds will 16 cubic feet weigh?

30. A cubic foot of solid ice weighs 58 pounds. How many pounds will 128 cubic feet weigh?

31. If a man takes 2265 steps in walking a mile, how many steps will he take in walking 17 miles?

32. There are 4840 square yards in an acre. How many square yards are there in 27 acres?

33. At 29 dollars a ton, find the cost of 137 tons of steel rails.

34. Find the number of men in an army consisting of 79 regiments, if the regiments average 897 men.

35. A barrel of flour weighs 196 pounds. How many pounds will 175 barrels weigh?

36. If an oil well produces 327 barrels of oil a day, how many barrels will it produce in 28 days?

37. How many pounds of cotton will 640 acres produce if each acre produces 460 pounds?

38. How many dozen eggs will 78 barrels hold if each barrel holds 83 dozen?

39. If 250 pounds of charcoal are used in making a ton of gunpowder, how many pounds will be used for 1280 tons of gunpowder?

40. If it takes 158 tons of steel rails to lay one mile of railway, how many tons of steel rails are there in one track from Albany to Buffalo, a distance of 297 miles?

41. Rhode Island has an area of 1085 square miles. If it has a population of 319 to the square mile, what is the population of Rhode Island?

42. Massachusetts has an area of 8040 square miles. If it has a population of 279 to the square mile, what is the population of Massachusetts?

43. Belgium, the most densely populated country in the world, has 551 inhabitants to the square mile, and 11,373 square miles. Find its population.

CHAPTER V.

DIVISION OF INTEGRAL NUMBERS.

53. In Multiplication two factors are given to find the product. In Division the product and one factor are given to find the other factor.

54. The questions,

How many 3's in 15?

How many times is 3 contained in 15?

What is the number found if 15 is divided by 3?

are all answered by finding *the number by which we must multiply 3 to obtain 15.*

55. Division. If the product and one factor are given, the process of finding the other factor is called *division.*

56. Dividend. The given product, that is, *the number to be divided,* is called the *dividend.*

57. Divisor. The given factor, that is, *the number by which we divide,* is called the *divisor.*

58. Quotient. The required factor, that is, *the number found by division,* is called the *quotient.*

EXERCISE 21. — ORAL.

1. How many 2's in 4? 6? 8? 10? 12? 14? 16? 18?
2. How many 3's in 6? 9? 12? 15? 18? 21? 24? 27?
3. How many 4's in 8? 12? 16? 20? 24? 28? 32? 36?
4. How many 5's in 10? 15? 20? 25? 30? 35? 40? 45?
5. How many 6's in 12? 18? 24? 30? 36? 42? 48? 54?

6. How many 7's in 14? 21? 28? 35? 42? 49? 56? 63?
7. How many 8's in 16? 24? 32? 40? 48? 56? 64? 72?
8. How many 9's in 18? 27? 36? 45? 54? 63? 72? 81?
9. How many 11's in 22? 33? 44? 55? 66? 77? 88? 99?
10. How many 12's in 24? 36? 48? 60? 72? 84? 96? 108?

59. To divide 35 apples by 7 apples is to find the *number of times* we must take 7 apples to obtain 35 apples.

If the divisor and dividend denote the same kind of units, the quotient is an abstract number.

60. To divide 35 apples by 7 is to find the *number of apples* in each part, when 35 apples are divided into 7 equal parts.

If the divisor is an abstract number, the quotient denotes units of the same kind as the dividend.

61. What is one of the parts called, if a number is divided into 2 equal parts? 3? 4? 5? 6? 7? 8? 9?

One-half is written $\frac{1}{2}$; one-third, $\frac{1}{3}$; one-fourth, $\frac{1}{4}$; one-fifth, $\frac{1}{5}$; one-sixth, $\frac{1}{6}$; one-seventh, $\frac{1}{7}$; one-eighth, $\frac{1}{8}$; one-ninth, $\frac{1}{9}$; and so on.

To divide 35 apples by 7 is to find $\frac{1}{7}$ of 35 apples.

62. Division is indicated by the *sign of division* $\div$, or by writing the dividend over the divisor with a line between them. Thus, $42 \div 6$ and $\frac{42}{6}$ mean the same, and each is read: forty-two *divided by* six.

EXERCISE 22. — ORAL.

What is the quotient of:

1. $24 \div 6$?	6. $63 \div 9$?	11. $54 \div 9$?
2. $32 \div 4$?	7. $54 \div 6$?	12. $64 \div 8$?
3. $36 \div 9$?	8. $63 \div 7$?	13. $56 \div 8$?
4. $27 \div 3$?	9. $81 \div 9$?	14. $36 \div 4$?
5. $42 \div 7$?	10. $72 \div 8$?	15. $88 \div 11$?

Exercise 23. — Oral.

1. If 7 pounds of lard cost 63 cents, what is the price of one pound?

2. If 8 barrels of flour cost 40 dollars, how much will one barrel cost?

3. If 9 oranges cost 27 cents, how many cents will one orange cost?

4. If there are 56 seats in 7 rows, how many seats in each row?

5. At 9 dollars a week, in how many weeks will a man earn 45 dollars?

6. At 7 dollars a head, how many sheep can be bought for 42 dollars?

7. At 7 cents a quart, how many quarts of berries can be bought for 21 cents?

8. At 6 dollars a ton, how many tons of coal can be bought for 54 dollars?

9. At 4 dollars a cord, how many cords of wood can be bought for 28 dollars?

10. At 3 dollars a day, how many days will it take to earn 36 dollars?

11. At 8 cents a pound, how many pounds of loaf sugar can be bought for 56 cents?

12. At 9 cents a quart, how many quarts of blueberries can be bought for 54 cents?

13. At 12 cents a quart, how many quarts of cranberries can be bought for 48 cents?

14. At 8 cents a pound, how many pounds of rice can be bought for 48 cents?

15. If 55 dollars are paid for 11 cords of hard wood, what is the price per cord?

16. If 8 cans hold 64 quarts of milk, how many quarts will 1 can hold?

17. If a plow costs 9 dollars, how many plows can be bought for 54 dollars?

63. If a man has 42 dollars and buys sheep at 4 dollars a head, how many sheep can he buy, and how many dollars will he have left?

Solution. At 4 dollars a head, as many sheep can be bought for 42 dollars as 4 is contained times in 42; 4 is contained 10 times in 42, and there is 2 over. Hence, he can buy 10 sheep and have 2 dollars left.

When the divisor does not exactly divide the dividend, the part of the dividend left from the division is called the *remainder.*

Exercise 24. — Oral.

1. If a boy has 37 cents, how many oranges at 3 cents each can he buy, and how many cents will he have left?

2. How many lengths of 6 yards each can be cut from a piece of cloth 43 yards long, and how many yards will be left?

3. A man has 57 dollars. How many lambs can he buy at 5 dollars each, and how many dollars will he have left?

4. How many baseball teams of 9 men each can be made from 40 men, and how many men will be left?

5. How many boat crews of 8 men each can be made from 60 men, and how many men will be left?

6. A man has 57 dollars. How many barrels of flour at 6 dollars a barrel can he buy, and how many dollars will he have left?

Short Division.

64. When the divisor is so small that the work can be performed mentally, the process is called **Short Division.**

Divide 6975 by 3.

SOLUTION. The divisor is written at the left of the dividend, and the quotient under the dividend, as in the margin.

$$3\overline{)6975}$$
$$2325$$

Here the divisor is contained in 6 twice, in 9 three times, and in 7 twice with remainder 1; this 1 is equal to 10 of the next lower order, and with the 5, the next order of the dividend, makes 15. Then 3 is contained in 15 five times.

Wording. 3 in 6, **2**; in 9, **3**; in 7, **2**; in 15, **5**.

Divide 4,236,158 by 7.

$$7\overline{)4236158}$$
$$605165 \text{ with remainder } 3.$$

In this example, 7 is not contained in 3, so 0 is the second figure of the quotient; then the next figure, 6, of the dividend is joined to the 3, making 36, and the division is continued. When the division is finished, there is a remainder 3.

Divide 54,123 by 9.

$$9\overline{)54123}$$
$$6013 \text{ with remainder } 6.$$

Each quotient figure has the same name, that is, it denotes the same *order of units* as the right-hand figure of that part of the dividend used in obtaining it. Thus, 54 in this example is 54 thousands, and the first figure of the quotient is 6 thousands.

Divide 23,087 dollars by 5.

$$5\overline{)23087} \text{ dollars.}$$
$$4617 \text{ dollars with 2 dollars remaining.}$$

In this example, we are to divide 23,087 dollars into *five equal parts*, and find the *number of dollars* in each part. The answer is 4617 *dollars*, with 2 dollars over. The *complete quotient* may be written $4617\frac{2}{5}$ dollars.

65. To divide a number by 10 is to find the *tens* of the number. The tens of a number are found by cutting off the units figure of the number. To divide a number by 100 is to find the *hundreds* of the number. The hundreds of a number are found by cutting off the tens and units figures of the number. Thus, 35,764 is 3576 tens 4 units, or 357 hundreds 64 units.

To divide a number by 200 we first divide by 100 by cutting off two figures from the right of the dividend, and then divide by 2.

Divide 5,786,342 by 200.

$$\begin{array}{r l} 2\cancel{0}\cancel{0})\underline{57863\cancel{4}\cancel{2}} & \\ 28931 & \textit{with remainder } 142. \end{array}$$

In this example, we cut off the two zeros at the right of the divisor and two figures at the right of the dividend; then we divide by 2.

The remainder from division is 1, and to this we annex the figures 42 cut off from the right of the dividend, and have 142 for the true remainder.

66. *When a divisor ends in one or more zeros,* cut off the zeros and an equal number of figures from the right of the dividend, perform the division with the numbers left. The figures cut off from the dividend, or the figures cut off joined to the remainder from division, if there is a remainder, express the true remainder.

67. If we add the remainder to the product of the divisor and quotient we obtain the dividend.

Thus, in dividing 698 by 3 we have 232 for a quotient, and 2 for a remainder. The divisor is 3 and the dividend is 698. Now 3×232 is 696, and $696 + 2$ is 698 (the dividend). Hence,

To test the accuracy of the work of division, find the product of the divisor and quotient, and to this product add the remainder, if any. The result will be equal to the dividend if the work is correct.

EXERCISE 25. — WRITTEN.

Find the quotient of:

1. 736 ÷ 2.
2. 735 ÷ 3.
3. 952 ÷ 4.
4. 915 ÷ 5.
5. 714 ÷ 6.
6. 826 ÷ 7.
7. 984 ÷ 8.
8. 954 ÷ 9.
9. 568 ÷ 8.
10. 875 ÷ 5.
11. 816 ÷ 6.
12. 854 ÷ 7.
13. 984 ÷ 4.
14. 492 ÷ 7.
15. 653 ÷ 2.
16. 887 ÷ 8.
17. 952 ÷ 5.
18. 954 ÷ 6.
19. 3954 ÷ 6.
20. 9656 ÷ 8.
21. 9785 ÷ 5.
22. 9783 ÷ 9.
23. 8457 ÷ 3.
24. 8720 ÷ 8.
25. 8736 ÷ 7.
26. 6307 ÷ 7.
27. 2832 ÷ 4.
28. 3681 ÷ 9.
29. 3654 ÷ 9.
30. 3716 ÷ 4.
31. 5472 ÷ 6.
32. 7483 ÷ 6.
33. 8721 ÷ 7.
34. 9212 ÷ 9.
35. 8413 ÷ 6.
36. 5387 ÷ 4.
37. 37,632 ÷ 30.
38. 42,631 ÷ 20.
39. 54,777 ÷ 40.
40. 36,286 ÷ 50.
41. 56,216 ÷ 70.
42. 38,425 ÷ 60.
43. 92,460 ÷ 70.
44. 85,563 ÷ 80.
45. 46,376 ÷ 90.
46. 27,485 ÷ 200.
47. 87,585 ÷ 300.
48. 96,464 ÷ 400.
49. 83,690 ÷ 500.
50. 58,775 ÷ 600.
51. 75,230 ÷ 700.
52. 89,567 ÷ 800.
53. 98,254 ÷ 900.
54. 82,610 ÷ 700.

EXERCISE 26. — WRITTEN.

1. If a man divides 264 pounds of oatmeal into 8-pound packages, how many packages will there be?

2. How many times must you take 7 dollars to make 567 dollars? How many times must you take 9 dollars?

3. If a blacksmith uses 7 nails for each horseshoe, and used 168 nails on a certain day, how many horseshoes did he nail on that day?

4. How many tons of coal at 7 dollars a ton can be bought for 1995 dollars?

5. A mile is 5280 feet. How many times will a wheel turn in going a mile if it goes over 12 feet each turn?

6. A square yard is 9 square feet. How many square yards are there in 7506 square feet?

7. A man gave 25,645 dollars to his five children in equal shares. How much did he give to each child?

8. James Stone paid 75,008 dollars for wild land at 8 dollars an acre. How many acres did he buy?

9. At 4 dollars a cord, how many cords of birch wood can be bought for 7068 dollars?

10. How many five-dollar bills will amount to 500 dollars?

11. A farmer raised 2030 bushels of oats from 70 acres. How many bushels an acre did he raise?

12. A mile is 63,360 inches. How many steps does a boy take in walking a mile, if his steps average 20 inches?

13. At 6 dollars a barrel, how many barrels of flour can be bought for 1140 dollars?

14. If 9 carloads of freight weigh 147,114 pounds, what is the average weight of the carloads?

15. If sound moves 10,080 feet in 9 seconds, how many feet does it move in one second?

16. There are 60 minutes in an hour. How many hours are there in 1440 minutes?

Long Division.

68. The process of Long Division is the same as that of Short Division, except that the work is written in full, and the quotient is written *over* the dividend, the first quotient figure being written over the right-hand figure of the partial dividend used in obtaining it.

NOTE. The quotient may be written *at the right* of the dividend. The advantage, however, of writing the quotient *over* the dividend will be seen in the chapters on United States Money, and Decimal Fractions.

69. Divide 41,998 by 78.

The beginner will find it helpful to form a table of products of the divisor by the numbers 1, 2, 3, ..., as follows:

$1 \times 78 = 78$	$4 \times 78 = 312$	$7 \times 78 = 546$
$2 \times 78 = 156$	$5 \times 78 = 390$	$8 \times 78 = 624$
$3 \times 78 = 234$	$6 \times 78 = 468$	$9 \times 78 = 702$

SOLUTION. As 78 is more than 41, it is necessary to take *three* figures of the dividend for the first partial dividend. Of the products in the table that do not exceed 419 the greatest is 390, that is, 5×78. Hence, the first quotient figure is 5, and this is written over the 9 in the dividend; then 390 is subtracted from 419. To the remainder 29, the next figure, 9, of the dividend is annexed. Of the products that do not exceed 299, the greatest is 234, that is, 3×78. Hence 3 is the next figure of the quotient, and the next remainder is 65; and to this the 8 of the dividend is annexed. Of the products that do not exceed 658, the greatest is 624, that is, 8×78. Hence, the next figure of the quotient is 8, and the remainder is 34.

```
     538
78)41998
   390
    299
    234
     658
     624
      34 remainder.
```

70. After a little practice, the operation of division can be performed without the aid of a table of products. Each quotient figure may be estimated by taking for a trial divisor the nearest number of tens, or hundreds, etc., represented by the divisor, and by taking for a trial dividend the nearest number of tens, or hundreds, etc., represented by the partial dividend.

71. Divide 2,791,163 by 394.

SOLUTION. The first partial dividend is 2791. As the nearest number of *hundreds* represented by the divisor, 394, is 4, we take 4 for a trial divisor. As the nearest number of hundreds represented by the partial dividend, 2791, is 28, we take 28 for the trial dividend. 4 is contained 7 times in 28. We write the 7 over the 1, and multiply the divisor 394 by 7. We subtract the product 2758 from 2791 and have for a remainder 33, to which we annex the 1 of the dividend. As 331 is less than 394, the next quotient figure is 0. To 331 we annex the next figure, 6, of the dividend. 4 is contained 8 times in 33. We therefore write 8 for the next quotient figure, and find the product of 8×394 to be 3152. The remainder obtained by subtracting 3152 is 164, to which the 3 of the dividend is annexed. 4 is contained 4 times in 16. The product of 4×394 is 1576, and this subtracted from 1643 leaves 67 for the final remainder.

```
      7084
394)2791163
    2758
      3316
      3152
       1643
       1576
         67 remainder.
```

The complete quotient may be written $7084\frac{67}{394}$.

NOTE. In order to have the work correct, *the product must be less than the partial dividend*, and *the remainder less than the divisor*, in each step.

From these examples we have the following

RULE FOR LONG DIVISION. *Write the divisor to the left of the dividend with a curved line between them.*

Take for the first partial dividend the fewest left-hand figures that will contain the divisor, and write the quotient over the right-hand figure of this partial dividend.

Multiply the divisor by this quotient, and place the product under the partial dividend used.

Subtract this product, and to the remainder annex the next figure of the dividend.

Divide as before, and continue this process until all the figures of the dividend have been used.

NOTE. If there is a remainder after the last division, it may be written with the divisor under it as a part of the complete quotient.

PROOF. *Find the product of the divisor and quotient, and to this product add the remainder, if any. If the work is correct, the result will be equal to the dividend.*

EXERCISE 27. — WRITTEN.

Find the quotient of:

1. 1728 ÷ 12.
2. 4389 ÷ 21.
3. 5276 ÷ 31.
4. 8569 ÷ 41.
5. 7191 ÷ 51.
6. 9394 ÷ 61.
7. 5254 ÷ 71.
8. 3402 ÷ 81.
9. 7553 ÷ 91.
10. 4593 ÷ 73.
11. 3987 ÷ 94.
12. 8789 ÷ 43.
13. 7684 ÷ 69.
14. 9988 ÷ 97.
15. 6548 ÷ 68.
16. 8429 ÷ 79.
17. 5498 ÷ 89.
18. 89,713 ÷ 76.
19. 82,596 ÷ 77.
20. 53,691 ÷ 88.
21. 80,963 ÷ 39.
22. 43,657 ÷ 23.
23. 35,372 ÷ 35.
24. 30,704 ÷ 29.
25. 33,765 ÷ 33.
26. 63,208 ÷ 69.
27. 20,748 ÷ 67.
28. 37,465 ÷ 53.
29. 20,087 ÷ 75.
30. 93,847 ÷ 19.
31. 84,793 ÷ 26.
32. 74,938 ÷ 87.
33. 57,530 ÷ 59.
34. 50,375 ÷ 64.
35. 40,675 ÷ 101.
36. 67,540 ÷ 201.
37. 54,700 ÷ 205.
38. 59,376 ÷ 308.
39. 93,567 ÷ 409.
40. 94,362 ÷ 871.
41. 92,436 ÷ 529.
42. 43,269 ÷ 864.
43. 46,327 ÷ 927.
44. 76,395 ÷ 945.
45. 834,561 ÷ 408.
46. 341,586 ÷ 248.
47. 543,816 ÷ 357.
48. 861,345 ÷ 395.
49. 370,406 ÷ 843.
50. 978,217 ÷ 498.
51. 604,730 ÷ 189.

Exercise 28. — Written.

1. A farmer raised 221 bushels of wheat from 17 acres. How many bushels an acre did he raise?

2. A man sold 59 acres of land for 8732 dollars. How many dollars an acre did he get?

3. If 17,068 dollars is divided into 17 equal parts, how many dollars will there be in each part?

4. If it takes 18,430 dollars to pay a regiment of men 19 dollars apiece, how many men are there in the regiment?

5. A mile contains 63,360 inches. How many steps of 24 inches each will a boy have to take to walk a mile?

6. A boy walked a mile (63,360 inches) in 3168 steps. How many inches did his steps average?

7. A steamship made 528 miles in 24 hours. How many miles did she average an hour?

8. If a steamship averages 23 miles an hour, how many hours will it take her to go 2990 miles?

9. There are 153,120 feet in 29 miles. How many feet are there in one mile?

10. There are 9009 cubic inches in 39 gallons. How many cubic inches are there in one gallon?

11. There are 13,600 square rods in 85 acres. How many square rods are there in one acre?

12. There are 47,520 yards in 27 miles. How many yards are there in one mile?

13. There are 2000 pounds in a ton. The bell of the Kremlin in Moscow, the largest bell in the world, weighs 448,000 pounds. How many tons does it weigh?

14. There are 640 acres in a square mile. How many square miles are there in the District of Columbia which contains 38,400 acres?

15. A bushel of wheat weighs 60 pounds. How many bushels are there in a carload of 18,660 pounds of wheat?

16. How many times must a pole 11 feet long be applied to measure a distance of a mile (5280 feet)?

17. A certain book contains 95,337 words, and averages 297 words on a page. How many pages has the book?

18. There are 63 gallons in a hogshead. A certain cistern holds 6111 gallons. How many hogsheads does it hold?

19. In how many hours will a cistern holding 12,222 gallons be filled by a pipe that discharges into it 194 gallons an hour?

20. There are about 1221 millions of inhabitants on the earth. If this number die in 33 years, how many die a year on the average?

21. A cubic foot of solid ice weighs 58 pounds. How many cubic feet of ice are there in a freight car that contains 29,116 pounds of ice?

22. If the earth in its revolution round the sun moves 1,641,600 miles a day, how far does it move in one second, a day containing 86,400 seconds?

23. If one pound of sugar is obtained from 18 sugar canes, how many pounds will be obtained from 162,162 canes?

24. The largest State in the Union is Texas, the smallest is Rhode Island. Texas has 262,290 square miles and Rhode Island has 1085. How many States of the size of Rhode Island can be made of Texas?

25. If sound moves 92,960 feet in 83 seconds, how many feet does it move in one second?

26. The greatest height ever reached by a balloon is 37,000 feet. How many miles has a balloon ascended?

27. The total number of ocean sailing vessels afloat in 1896, measuring over 50 tons, was 25,570, and the total tonnage of these vessels was 9,323,995 tons. Find the average number of tons to a vessel, giving the answer to the nearest ton.

72. If any *three* of the four terms, dividend, divisor, quotient, and remainder are given, we can readily find the *fourth* term.

Exercise 29. — Written.

1. If the divisor is 207, dividend 4776, quotient 23, find the remainder.

2. If the divisor is 72, quotient 127, and the remainder 39, find the dividend.

3. If the divisor is 87, dividend 2389, and remainder 40, find the quotient.

4. If the quotient is 37, dividend 1916, and remainder 29, find the divisor.

73. The Parenthesis. If numbers are included in a parenthesis ( ), the first step is to reduce these numbers to a single number as the signs direct. Thus,

$$(9 + 7 - 1) \div 5 = 15 \div 5. \qquad (4 \times 6 - 9) \div 5 = 15 \div 5.$$
$$48 \div (4 \times 3) = 48 \div 12. \qquad 48 \div (24 \div 4) = 48 \div 6.$$

Note. Instead of a parenthesis we sometimes use a bracket [ ], a brace { }, or a vinculum $\overline{\quad}$. Thus, $(8-3)$, $[8-3]$, $\{8-3\}$, $\overline{8-3}$, all have the same meaning.

In reducing expressions containing the signs $+$, $-$, $\times$, $\div$, we first perform the operations indicated by the signs $\times$ and $\div$ in *the order in which they stand;* then the operations indicated by $+$ and $-$. Thus,

$$48 \div 8 \times 2 - 3 \times 2 + 6 \times 5 \div 2 = 12 - 6 + 15 = 21.$$

EXERCISE 30. — WRITTEN.

Reduce to a single expression:

1. $(13 + 7 - 5) \times 6.$
2. $(9 \times 5) \div (2 + 7).$
3. $(99 \div 11) + (3 + 6 - 2).$
4. $(56 - 16) \times (11 - 9).$
5. $(72 \div 8 + 3) - (6 \times 5 \div 10).$
6. $(81 - 7 \times 6) + (3 \times 5) - 9 \div 3.$
7. $79 - 5 \times 4 + 3 \times 2 - 5 \times 4.$
8. $(51 - 3 + 1) \div 7 + (3 \times 5 - 9) \div 3.$

74. Principles of Division. The value of the quotient depends upon the *relative* values of the dividend and divisor.

Suppose we have $36 \div 6 = 6.$

If we multiply the dividend 36 by 2, what effect will this have on the quotient?

If we divide the dividend 36 by 2, what effect will this have on the quotient?

If we multiply the divisor 6 by 2, what effect will this have on the quotient?

If we divide the divisor 6 by 2, what effect will this have on the quotient?

If we multiply both the dividend and divisor by 2, what effect will this have on the quotient?

If we divide both the dividend and divisor by 2, what effect will this have on the quotient?

From the answers to the last two questions, we have the following important principle of Division:

Multiplying the dividend and divisor by the same number, or dividing both by the same number, does not change the quotient.

EXERCISE 31. — WRITTEN.

Review Problems.

1. The United States produced in 1890, 6,940,898 bales of cotton. Of this number Texas produced 1,594,305 bales. How many bales did the other States produce?

2. A contractor purchased a million bricks. After 523 thousand have been delivered, how many carloads of 9000 each remain to be delivered?

3. A bushel of wheat weighs 60 pounds, and a bushel of oats 32 pounds. How many bushels of oats will weigh as much as 176 bushels of wheat?

4. How many 50-pound sacks of flour can be made from 25 barrels of flour, each containing 196 pounds?

5. A farmer has 253 bushels of apples. How many barrels does he need for his apples, a bushel being 4 pecks and a barrel 11 pecks?

6. Seventeen boys and two girls agreed to share 200 oranges equally so far as possible, without cutting any of the oranges; and if any remained, to divide them equally between the two girls. How many did each boy receive, and how many did each girl receive?

7. Given the numbers 76, 309, 4426, 9375. Subtract each number from the one following. Add the several remainders and the first number. Is the sum equal to the last number?

8. If a book contains 320 pages of 32 lines each, and the lines have on the average 11 words, how many words are there in the book?

9. How many days will it take a ship to sail from Liverpool to the Cape of Good Hope, 7000 miles, if it averages 125 miles a day?

10. By what number must 1621 be multiplied to give 614,359 for the product?

11. A drover bought 17 cows of a farmer at 45 dollars a head, and gave the farmer 300 dollars and 93 sheep. What were the sheep reckoned at a head?

12. If a fast train travels at the rate of 40 miles an hour, and a slow train at the rate of 25 miles an hour, how many hours will be saved by taking the fast train for a journey of 1000 miles?

13. A certain number divided by 243 gives 1306 for the quotient and 92 for the remainder. Find the number.

14. Light travels at the rate of one hundred eighty-six thousand miles a second. If we reckon the distance from the sun to the earth at 93 million miles, how many seconds does it take light from the sun to reach us?

15. The diameter of the earth at the poles is 41,707,620 feet, and at the equator 41,847,426 feet. How many feet longer is the diameter at the equator than at the poles?

16. The number of foreign-born whites in the United States by the census of 1890 was 9,121,867, and the number of colored persons was 7,638,360. Find the excess of foreign-born whites over the colored population of the United States in 1890.

17. What number must be added to three hundred eighty-two thousand six hundred fifty-four to make a million?

18. The free gold in the United States treasury on January 20, 1896, was 53,704,160 dollars. How much did it lack of 100,000,000 dollars?

19. A long ton of coal is 2240 pounds and a short ton is 2000 pounds. How many short tons are there in a cargo containing 250 long tons?

75. Arithmetical Analysis. The process of reasoning from the *given number* to *one*, and then from *one* to the *required number* is called *analysis*.

If 8 tons of coal cost 32 dollars, what will 9 tons cost?

ANALYSIS. If 8 tons cost 32 dollars, 1 ton costs $\frac{1}{8}$ of 32 dollars, or 4 dollars; if 1 ton costs 4 dollars, 9 tons will cost 9 times 4 dollars, or 36 dollars.

EXERCISE 32. — ORAL.

1. If 7 barrels of flour cost 35 dollars, what will 9 barrels cost?
2. If 4 cords of wood cost 16 dollars, what will 12 cords cost?
3. If 7 pounds of mutton cost 56 cents, what will 8 pounds cost?
4. If 3 men can dig a cellar in 12 days, how many men will be required to dig it in 1 day? in 6 days?
5. If 6 men can build a certain wall in 9 days, how many men will be required to build it in 3 days?
6. If 7 men can do a piece of work in 8 days, in how many days can 1 man do it? can 8 men do it?
7. If 7 sheep cost 28 dollars, what will 11 sheep cost?
8. If 6 pounds of rice cost 48 cents, what will 9 pounds cost?
9. If 9 tons of coal cost 63 dollars, what will 8 tons cost?
10. If 3 yards of velvet cost 27 dollars, what will 8 yards cost?
11. If 2 dozen eggs cost 24 cents, what will 7 dozen cost?
12. If 12 yards of cloth cost 48 dollars, what will 7 yards cost?
13. If 6 vests cost 42 dollars, what will 9 vests cost?
14. If 9 barrels of extra flour cost 54 dollars, what will 8 barrels cost?

Exercise 33. — Written.

1. If 15 acres of corn yield 480 bushels, how many bushels will 137 acres yield at the same rate?

2. If 127 barrels of flour cost 508 dollars, what will 239 barrels cost?

3. If 139 barrels of sugar cost 1807 dollars, what will 69 barrels cost?

4. Twenty-five barrels of flour weigh 4900 pounds. Find the weight of 87 barrels.

5. If 20 men can do a certain piece of work in 11 days, how many days will it take 22 men to do it?

6. How long will it take 19 men to do a piece of work that 17 men can do in 133 days?

7. How long will it take 23 men to do a piece of work that 19 men can do in 437 days?

8. How many men will it take to do a piece of work in 84 days that 49 men can do in 96 days?

9. How many men will it take to do a piece of work in 26 days that 39 men can do in 76 days?

10. If 17 horses cost 1802 dollars, what will 19 horses cost at the same rate?

11. A man bought some sheep at the rate of 3 for 18 dollars. How many did he buy for 1206 dollars?

12. A boy had 75 cents. He bought 3 dozen oranges, and had 3 cents left. What did he pay for the oranges apiece?

13. If 19 horses can be bought for 1520 dollars, how many horses can be bought for 1360 dollars?

14. If 13 hogsheads of molasses can be bought for 273 dollars, how many hogsheads can be bought for 609 dollars?

15. If 180 tons of coal will pay for 30 tons of steel rails at 30 dollars a ton, how many tons of coal can be bought for 100 dollars?

Review Questions.

What are units? numbers? integral numbers? abstract numbers? concrete numbers? like numbers?

What is the figure 0 called? How do we write ten? one hundred? one thousand? What do figures written in the first place represent? in the second place? in the third place? in the fourth place?

What is notation? What is numeration? What order of units are the ones of a number? the tens? the hundreds? the thousands? How many units of any order make *one* unit of the next higher order? Why is the common system of notation called the *decimal* system? In separating a row of figures into periods do we begin at the right or the left? How many figures do we put in each period? What is the period on the right called? the next period? the third period? the fourth period? How many units of any period make one unit of the next higher period? After we have separated the figures of a number into periods, which period do we read first? In writing an integral number in figures, what do we consider first? Which period do we write first?

What is addition? What is the result called? What kind of numbers only can be added? What is the sign of addition? Does it make any difference in what order numbers are added?

What is subtraction? What is the greater number called? the smaller number? the result? What kind of numbers must the minuend, subtrahend, and remainder be? What is the sign of subtraction? How do we prove that the work of subtraction is correct?

What is multiplication? the multiplicand? the multiplier? the product? What kind of a number must the multiplier be? What kind of a number will the product be? What are the factors of a product? Does it make any difference in what order the factors are multiplied? What is the sign of multiplication? How do we prove that the work of multiplication is correct?

What is division? the dividend? the divisor? the quotient? If the dividend and divisor are concrete numbers of the same kind, what kind of a number will the quotient be, and what will the quotient signify? If the dividend is a concrete number and the divisor an abstract number, what kind of a number will the quotient be, and what will the quotient signify? What is the sign of division? How do we prove that the work of division is correct? What effect does it have on the quotient, if we multiply both dividend and divisor by the same number? If we divide both by the same number?

CHAPTER VI.

UNITED STATES MONEY.

76. The legal currency of the United States has the *decimal* system of notation. Thus,

10 mills (m.)	= 1 cent (ct.).
10 cents	= 1 dime (d.).
10 dimes or 100 cents	= 1 dollar.

77. The *unit* of United States money is the dollar.

78. The *dollar sign*, **$**, is written before the number.

Thus, $25 means and is read twenty-five dollars.

79. When dollars and cents are written, a dot, called the *decimal point*, is placed between the dollars and the cents.

80. Since *ten* dimes make a dollar, figures in the *first place* at the right of the decimal point signify *tenths* of a dollar, or dimes.

Since *one hundred* cents make a dollar, figures written in the *second place* at the right of the decimal point signify *hundredths* of a dollar, or cents.

Since *one thousand* mills make a dollar, figures written in the *third place* at the right of the decimal point signify *thousandths* of a dollar, or mills.

81. In reading an expression of United States money, we read the number at the left of the decimal point as dollars; the number in the *first two places* at the right of the decimal point as cents; and the number in the *third place* at the right of the decimal point as mills.

Thus, $33.278 is read thirty-three dollars and twenty-seven cents, eight mills.

82. In writing cents (*hundredths* of a dollar), we must have the number of cents occupy *two* places. Hence,

If the number of cents is less than ten, we write a cipher in the first place at the right of the decimal point.

Thus, the expression for ten dollars and eight cents is $10.08.

Exercise 34. — Oral.

Read:

1. $1.25.	**5.** $2.05.	**9.** $0.20.	**13.** $5.863.
2. $3.63.	**6.** $7.08.	**10.** $0.50.	**14.** $7.765.
3. $4.87.	**7.** $9.07.	**11.** $0.80.	**15.** $6.274.
4. $5.69.	**8.** $6.03.	**12.** $0.06.	**16.** $8.075.

Exercise 35. — Written.

Write in figures:

1. Five dollars and eighty-seven cents.

2. Seventy-eight dollars and twenty-five cents.

3. Seventy-six dollars and seventy cents.

4. Ninety dollars and ninety-nine cents.

5. Eighty-eight dollars and eight cents.

6. One hundred twenty-seven dollars and one cent.

7. One hundred forty dollars and four cents.

8. Four thousand dollars and five cents.

9. One thousand six dollars and nine cents.

10. Forty thousand dollars and forty cents, five mills.

11. Five hundred thousand dollars and six cents, three mills.

12. One thousand one dollars and one cent, one mill.

Note 1. The coins of the United States are

Gold.		Silver.	
Double-eagle.	$20.00	Dollar	$1.00
Eagle	10.00	Half-dollar	0.50
Half-eagle	5.00	Quarter-dollar	0.25
Quarter-eagle	2.50	Dime	0.10

The 5-cent coin is made of nickel and is called a nickel; the smallest coin in value is made of bronze and is called a cent.

NOTE 2. The mill is not coined, and is used only in computation.

NOTE 3. Bank bills and United States Treasury notes are largely used in place of coins. These are issued in denominations of $1, $2, $5, $10, $20, $50, $100, $500, and $1000.

83. Reduction. The process of changing the *unit* without changing the *value* of a quantity is called *reduction.*

Thus, $2 and 200 cents have different units but the same value, and either may be reduced to the other.

The sign ¢ is often used for the word cents.

Find the number of cents in $2.59.

SOLUTION. Since $1 = 100 cents, $2 = 2 × 100 cents, or 200 cents, and 200 cents + 59 cents = 259 cents.

Therefore, $2.59 = 259 cents. Hence,

To reduce a number of dollars to cents,

Multiply the number of dollars by 100.

To reduce dollars and cents to cents,

Multiply the number of dollars by 100, and add the number of cents to the product.

EXERCISE 36. — WRITTEN.

Reduce to cents:

1. $3.	5. $161.08.	9. $2000.09.
2. $2.50.	6. $275.25.	10. $5000.10.
3. $4.16.	7. $415.50.	11. $7250.50.
4. $5.28.	8. $250.40.	12. $8110.07.

Find the number of dollars in 940 cents.

SOLUTION. Since 100 cents make $1, 900 cents make $9, and 940 cents make $9 and 40 cents.

Therefore, 940 cents = $9.40. Hence,

To reduce a number of cents to dollars,

Divide the number by 100.

The quotient will be the number of dollars, and the remainder will be the number of cents left over.

Exercise 37. — Written.

Reduce to dollars and cents:

1. 836 cents.	5. 8007 cents.	9. 840,007 cents.
2. 404 cents.	6. 6000 cents.	10. 320,010 cents.
3. 500 cents.	7. 5150 cents.	11. 100,001 cents.
4. 560 cents.	8. 3030 cents.	12. 102,008 cents.

84. Since numbers expressing mills, cents, dimes, and dollars increase from right to left precisely as integral numbers, they may be added, subtracted, multiplied, and divided in the same way as integral numbers.

Addition of U. S. Money.

Add $7.375, $19.50, $150.667, $270.08.

Solution. We write dollars under dollars, cents under cents, mills under mills, so that the decimal points stand in a vertical line. Since 10 units of any order make 1 unit of the next higher order, we add as in integral numbers, taking care *to put the decimal point in the sum directly under the vertical line of decimal points in the numbers added.*

```
 $7.375
  19.50
 150.667
 270.08
$447.622
```

Exercise 38. — Written.

Add:

1.	2.	3.	4.
$36.17	$148.63	$843.02	$8.437
196.07	697.82	516.89	19.70
802.27	14.70	269.07	38.969
417.93	968.36	300.98	47.674

5. A farmer sold a cow for $25.75, a ton of rye straw for $15.50, ten bushels of potatoes for $6.25, a hundred pounds of butter for $31, and five barrels of apples for $11.25. How much money did he receive in all?

6. A man sold a horse for $87.50, a wagon for $66.67, and a harness for $19.50. How much money was received for all?

7. A man bought a barrel of flour for $4.75, a barrel of sugar for $9.37, 10 gallons of kerosene for $0.90, 2 pounds of tea for $1.30, 3 pounds of coffee for $1.13. Find the amount of his bill.

Subtraction of U. S. Money.

Subtract $18.375 from $1029.25.

SOLUTION. Since the subtrahend has mills, and the minuend has no mills, we put a zero in the place of mills in the minuend.

$1029.250
18.375
$1010.875

We write units of the same order in columns so that the decimal points stand in a vertical line; and subtract as in integral numbers, taking care *to put the decimal point in the remainder directly under the vertical line of decimal points.*

EXERCISE 39. — WRITTEN.

Find the remainder:

1.	3.	5.	7.
$94.28	$100.17	$200.00	$300.000
29.48	69.39	87.56	190.125

2.	4.	6.	8.
$87.63	$200.01	$100.00	$278.000
63.87	199.29	65.78	90.667

9. A man has a deposit in a bank of $10,000. After he draws out $3829.76, how much will be left?

10. Charles has $18.37, and George has $45. How much more has George than Charles?

11. A man pays $6400 for a farm, and $1586.75 for stock and tools. How much more does he pay for the farm than for the stock and tools?

12. A man bought a barrel of flour for $4.25. He gave a ten-dollar bill in payment. How much money ought he to receive?

13. If a man pays $1337.75 on a note of $2000, how much is still due?

Multiplication of U. S. Money.

Multiply $87.36 by 19.

$87.36	
19	SOLUTION. We multiply as in integral numbers, and *point off in the product as many places for cents as there are places for cents in the multiplicand.*
78624	
8736	
$1659.84	

EXERCISE 40. — WRITTEN.

Find the product of:

1. $77.83 × 8	3. $84.75 × 29	5. $123.13 × 125	7. $33.875 × 420
2. $76.98 × 9	4. $57.48 × 37	6. $879.17 × 836	8. $19.335 × 500

9. A farmer sold 160 acres of land at $37.50 an acre. How much did he get for the land ?

10. At 72 cents a pound, what will 17 pounds of tea cost ?

11. At $3.20 a cord, what will be the cost of 179 cords of pine wood ?

12. A miller sold 737 barrels of flour at $3.17 a barrel. How much did he receive ?

13. Find the cost of 37 geese at 75 cents each.

14. At $0.56 a bushel, find the cost of 600 bushels of wheat.

15. Find the cost of 15 barrels of pork at $16.375 a barrel.

16. Find the cost of 95 calves at $6.75 each.

17. If coal is $3.625 a ton, find the cost of 324 tons.

Division of U. S. Money.

1. Divide \$379.625 by 125.

$$\begin{array}{r} \$3.037 \\ 125\overline{)\$379.625} \\ \underline{375} \\ 462 \\ \underline{375} \\ 875 \\ \underline{875} \end{array}$$

SOLUTION. We divide as in integral numbers, writing the first figure of the quotient over the right-hand figure of the first partial dividend, and *put the decimal point in the quotient directly over the decimal point in the dividend.*

2. Divide \$46.25 by \$0.125.

$$\begin{array}{r} 370 \\ 125\overline{)46250} \\ \underline{375} \\ 875 \\ \underline{875} \end{array}$$

SOLUTION. We reduce the dividend and the divisor to mills, the lowest denomination of either number, and then divide as in integral numbers. The answer is 370 *times*.

EXERCISE 41. — WRITTEN.

Find the quotient of:

1. \$149.59 ÷ 7.	**3.** \$236.43 ÷ 9.	**5.** \$302.08 ÷ 128.
2. \$147 ÷ \$7.	**4.** \$234 ÷ \$9.	**6.** \$7296 ÷ \$57.

7. A miller sold 325 barrels of flour for \$1381.25. How much did he get for his flour a barrel?

8. How many bags of corn at \$1.25 a bag can be bought for \$46.25?

9. If a man pays \$297.96 for 52 weeks' board, how much does he pay a week?

10. If \$380.38 is divided into 19 equal parts, how many dollars will there be in each part?

11. At 56 cents a bushel, how many bushels of wheat can be bought for \$11.20?

Exercise 42. — Oral.

Review Problems.

1. If 7 hens cost $3.50, what is the price of one hen?

2. If a dealer buys boots at $4.75 a pair and sells them at $6 a pair, how much does he make per pair?

3. If a pound of tea costs 65 cents and a pound of coffee 35 cents, how much must be paid for a pound of tea and two pounds of coffee?

4. A boy paid 90 cents for paper, 10 cents for pencils, and $1.50 for a book. How much did he pay in all?

5. A man has $10. How much will he have left after he has paid $4.25 for a barrel of flour and $3.75 for 75 pounds of sugar?

6. At 4 cents each, how many oranges can I buy for $1?

7. At 3 cents each, how many lemons can I buy for $3.24?

8. If I give a two-dollar bill in payment for 15 yards of cloth at 10 cents a yard, how much change is due me?

9. If calico is worth 7 cents a yard, how many yards can be bought for 10 dozen eggs worth 21 cents a dozen?

10. If butter is worth 20 cents a pound, how many pounds of sugar at 5 cents a pound must be given for 10 pounds of butter?

11. If I buy a barrel of sugar for $10.80, and give a ten-dollar bill and a two-dollar bill in payment, how many oranges at 2 cents each can I buy with the change due me?

12. A farmer sold 6 pounds of butter at 20 cents a pound, and 10 pounds of lard at 10 cents a pound. He took his pay in sugar at 5 cents a pound. How many pounds of sugar did he get?

13. If you can buy 5 oranges for 15 cents, how many oranges can you buy for $1.23?

Exercise 43. — Written.

Review Problems.

1. Divide $4490.69 among 37 persons, and find the share of each person.

2. Find the cost of 200 bushels of potatoes at $0.625 a bushel.

3. A man buys $125 worth of hay, and $75 worth corn. He pays down $139.87. How much is still due?

4. A man earns $19.50 a week. He pays $4.50 a week for his board, and $2.50 a week for other expenses. In how many weeks will he save $600?

5. A man has $10. How much of this money will remain after he has paid for 4 dozen eggs at 28 cents a dozen, 6 pounds of beefsteak at 22 cents a pound, and 3 bushels of potatoes at 60 cents a bushel?

6. A drover bought 425 lambs at an average price of $4.25 a head. He sold them at an average price of $5 a head. How much did he make on the whole lot?

7. A man sold a hogshead of molasses at 40 cents a gallon. The molasses cost him $18.90, and he gained on the whole hogshead $6.30. How many gallons were there in the hogshead?

8. A man bought a piano for $276, and agreed to pay for it in monthly payments of $17.25 each. If he keeps his promise, in how many months will he pay for the piano?

9. A man has $25,000 to invest. He buys 100 shares of bank stock at $99.50 a share, 100 shares of railway stock at $110 a share, and with the balance he buys mining stock at $81 a share. How many shares of mining stock does he buy?

Bills.

A Bill is a written statement of goods sold, or services rendered, giving the price of each item and total cost, as well as the dates, and the names of the parties concerned.

The party who owes is called a *Debtor*, and the party to whom a debt is owed is called a *Creditor*.

(Specimen of a Bill.)

Boston, Mass., March 9, 1896.

Mr. George Brown,

BOUGHT OF JAMES BATES.

1896					
Jan.	*15*	*10 lbs. Coffee*	*@ 35¢*	*$3*	*50*
	22	*11 lbs. Lard*	*@ 9¢*	*0*	*99*
Feb.	*5*	*25 lbs. Sugar*	*@ 5¢*	*1*	*25*
	12	*2 lbs. Tea*	*@ 65¢*	*1*	*30*
				$7	*04*

(Specimen of a Receipted Bill.)

Boston, Mass., March 17, 1896.

Mr. John Jones,

TO JAMES BROWN, DR.

1896							
Jan.	*22*	*To 40 tons Coal*	*@ $4.75*	*$190*	*00*		
	29	*To 20 cords Wood*	*@ 3.25*	*65*	*00*		
						$255	*00*
		Cr.					
Jan.	*29*	*By 40 bbls. of Apples*	*@ $3.50*	*140*	*00*		
Feb.	*10*	*By 50 bu. of Potatoes*	*@ 0.80*	*40*	*00*	*180*	*00*
		Balance due				*$75*	*00*

1896, March 26. *Received payment,*

James Brown.

EXERCISE 44. — WRITTEN.

Make out receipted bills for the following accounts, supplying dates:

1. Mr. John James bought of H. Merrill & Co. 8 rolls of wall-paper at 32 cents a roll, 21 yards of border at 12 cents a yard, 62 feet of moulding at 5 cents a foot, 3 curtains at 56 cents each.

2. H. Anderson sold to Mr. John Hoyt 12 tons of furnace coal at $6.25, 5 tons of stove coal at $6.50, 3 cords of pine wood at $3.25, 4 cords of hard wood at $5.25.

3. Mr. C. Haines bought of F. Button 25 bags of oats at 64 cents, 30 bags of corn at 85 cents, 12 bags of fine feed at 65 cents, 7 tons of hay at $16.25.

4. Mr. Thomas Leavitt bought of Gardner & Hilliard 31 pounds of sugar at 5 cents, 12 pounds of coffee at 35 cents, 14 pounds of rice at 9 cents, 12 pounds of crackers at 16 cents.

5. Mrs. George S. Walker bought of T. Brown 24 yards of calico at 8 cents, 16 yards of muslin at 12 cents, 14 yards of linen at 37 cents, 13 yards of dress goods at 63 cents.

6. E. Russell sold to James York 6 barrels of flour at $5.25, 8 gallons of molasses at 42 cents, 33 pounds of coffee at 35 cents, 4 pounds of tea at 67 cents, 219 pounds of sugar at 5 cents; and he took in exchange 20 bushels of potatoes at 75 cents, 14 dozen eggs at 22 cents, 12 barrels of apples at $2.25.

7. G. Perkins sold to James Arthur 9 pounds of ham at 15 cents, 18 pounds of steak at 22 cents, 16 pounds of mutton at 12 cents, 8 pounds of veal at 12 cents; and took in exchange 3 dozen eggs at 23 cents, 6 pounds of butter at 33 cents, 9 bushels of potatoes at 45 cents.

CHAPTER VII.

DECIMAL FRACTIONS.

85. How many dimes make a dollar?

How many *tenths* of a dollar make a dollar?

How many *tenths of any unit whatever* make the unit?

86. How many cents make a dime? How many cents make a dollar?

How many *hundredths* of a dollar make a *tenth* of a dollar? How many *hundredths* of a dollar make a dollar?

How many *hundredths of any unit whatever* make a *tenth* of the unit? How many *hundredths* make a unit?

87. How many mills make a cent?

How many *thousandths* of a dollar make a *hundredth* of a dollar? How many *thousandths* of a dollar make a dollar?

How many *thousandths of any unit whatever* make a *hundredth* of the unit? How many *thousandths* make the unit?

88. For *any* unit, then, figures in the *first* place at the right of the decimal point signify *tenths* of the unit; in the *second* place, *hundredths;* in the *third* place, *thousandths.*

Figures in the *fourth* place signify *ten-thousandths;* in the *fifth* place, *hundred-thousandths;* and so on.

And 10 in any place is equal to one in the next place to the left.

89. Units expressed by figures at the right of the decimal point are called *decimal units;* and a number containing decimal units is called a *decimal fraction,* or more briefly, a *decimal.*

NOTE. The meter-stick gives the best illustration of *decimal units*. The meter-stick is the *integral unit*, and the *tenth*, the *hundredth*, and the *thousandth* which are marked off on the stick are the *decimal units*.

A clear notion of the meaning of *decimals* is gained by using the meter in measuring given lengths, as the length of the room, of the platform, of the window-sill, etc., and writing the results.

Suppose the length of the room is found to be 8 meters, with a remainder less than a meter. In measuring this remainder, the *tenth* of the meter is used as the unit of measure. Suppose this remainder is found to be 3 tenths of a meter, with a remainder. Suppose this last remainder is found to be 5 hundredths and 4 thousandths of a meter.

The length of the room, then, is 8 meters, 3 tenths of a meter, 5 hundredths of a meter, and 4 thousandths of a meter. This is written 8.354 meters, and is read eight *and* three hundred fifty-four thousandths meters.

90. To Read Decimals. Read precisely as if the decimal were an integral number, and give it the *name of the lowest decimal place*. It is best to pronounce the word *and* at the decimal point, and omit it in all other places. Thus, 100.023 is read one hundred *and* twenty-three thousandths.

Ambiguity in reading, from having zeros at the end of a decimal, is avoided by a pause; thus, 0.300 is read three hundred . . . thousandths, while 0.00003 is read three . . . hundred-thousandths.

NOTE. When a number has no integral part, a cipher is placed at the left of the decimal point. In reading such an expression, the decimal only is read. Thus, 0.9 meter is read nine-tenths of a meter.

EXERCISE 45. — ORAL.

Read:

1. 85.4.	**5.** 0.368.	**9.** 4.3127.	**13.** 12.3601.
2. 8.54.	**6.** 0.683.	**10.** 9.2871.	**14.** 19.0032.
3. 0.63.	**7.** 9.213.	**11.** 6.3141.	**15.** 25.00081.
4. 0.06.	**8.** 7.389.	**12.** 8.7854.	**16.** 29.15625.

91. Write in figures two hundred one hundred-thousandths.

SOLUTION. Since five decimal places are required to express hundred-thousandths and 201 occupies three places, the first *two* places of decimals must be filled with ciphers. Hence, it is written 0.00201.

EXERCISE 46. — WRITTEN.

Write in figures:

1. Nine tenths. Three and twenty-seven hundredths. Four and seven tenths. Four and seven hundredths. Five and sixty-four hundredths.

2. Eight hundred four and ninety-four hundredths. Eight hundred four and ninety-four thousandths.

3. Two thousand seventeen and seven hundred eighty-nine thousandths. Two thousand seventeen and seven hundred eighty-nine ten-thousandths.

4. Four and one thousand nine ten-thousandths. Four and one thousand nine hundred-thousandths.

5. One hundred one and one hundred one thousandths. One hundred one and one hundred one ten-thousandths.

6. Two hundred twenty-four thousandths. Two hundred twenty-four hundred-thousandths.

7. Thirty-seven and nineteen hundred-thousandths. Twenty-five and two hundred six ten-thousandths.

8. Seven and seven thousandths. Seventy and seven hundred nine hundred-thousandths. Two hundred two and two thousand one hundred-thousandths.

NOTE. Decimals extend to the right without limit and are named in order: tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths, millionths, ten-millionths, hundred-millionths, billionths, and so on. In most problems, however, three decimal places give a sufficiently correct answer, and five places (hundred-thousandths) give a great degree of accuracy. Thus, a hundred-thousandth of a ton is less than a third of an ounce, and a hundred-thousandth of a mile is a trifle more than half of an inch.

92. The number 3.42700 means 3 whole units + 4 tenths of the unit + 2 hundredths of the unit + 7 thousandths of the unit + 0 ten-thousandths + 0 hundred-thousandths. This number, therefore, has the same value as 3.427. Hence,

Annexing ciphers to a decimal does not alter the value of the decimal.

Addition of Decimals.

93. Add 18.6275, 346.8, 1.27, 0.56253.

```
 18.6275
346.8
  1.27
  0.56253
---------
367.26003
```

Solution. We write the numbers as in Addition of U. S. Money so that the decimal points stand in a vertical line. We then add as in integral numbers, and *place the decimal point in the sum directly under the line of decimal points.*

Exercise 47. — Written.

Find the sum of:

1. 5.4; 60.73; 5.097; 25.608.
2. 0.125; 5.03; 0.537; 7.039.
3. 112.63; 0.0317; 2.973; 70.007.
4. 230.07; 27.107; 5.0513; 7.2685.
5. 11.3879; 5.1464; 0.03003; 13.269.
6. 110.7; 13,201.36; 0.01763; 82.75641.
7. 8.173; 29.27; 0.0024; 170.96.
8. 19.083; 3.2005; 4.07806; 236.
9. 2.352; 0.0008; 5.0856; 9.6823.
10. 0.0285; 7.921; 9.28; 50.704.
11. 7.07; 5.0909; 1.9090; 19.009.
12. 3.4159; 341.59; 34.159; 3415.9.
13. A merchant has four pieces of cotton cloth. One piece contains 39.5 yards; another, 39.375 yards; a third, 42.125 yards; and a fourth, 41.25 yards. How many yards are there in the four pieces?

Subtraction of Decimals.

94. From 14.13 take 5.9764.

SOLUTION. We write the numbers as in Subtraction of U. S. Money.

$$\begin{array}{r} 14.1300 \\ \underline{5.9764} \\ 8.1536 \end{array}$$

Since there are two more figures in the decimal part of the subtrahend than in the minuend, we annex two ciphers to the minuend. This does not alter the value of the minuend (§ 92).

We then subtract as in integral numbers, and *place the decimal point in the remainder directly under the line of decimal points.*

EXERCISE 48. — WRITTEN.

From

1. 2.876 take 0.59.
2. 1.5376 take 0.85.
3. 18.7 take 2.476.
4. 0.36 take 0.274.
5. 1.896 take 0.0378.
6. 0.006 take 0.0009.
7. 204.01 take 89.009.
8. 2 take 1.375.
9. 18.37 take 9.0189.
10. 1000 take 8.0999.
11. 24.503 take 7.9.
12. 36 take 24.869.

13. A meter is 39.3704 inches. A pendulum that beats seconds is 39.1392 inches long. Find the difference of their lengths.

14. The United States five-dollar gold coin contains 116.1 grains of pure gold. The English sovereign contains 113.0003 grains of pure gold. Find how many more grains of pure gold there are in the United States $5 gold-piece than in the English sovereign.

15. The value of the English sovereign is $4.8665. Express to the nearest cent the difference in value between a sovereign and a $5 gold-piece.

16. A man did 0.37 of a piece of work the first day, and 0.33 of it the second day. What part of the work was left for him to do the third day?

Multiplication of Decimals.

95. *A change in the position of the decimal point* of a number expressed in figures affects the value of the number.

Thus, if in 79.253 we move the decimal point one place to the right, so that the number becomes 792.53, we increase the value of each figure ten-fold; the 7 tens become 7 hundreds, the 9 units become 9 tens, the 2 tenths become 2 units, the 5 hundredths become 5 tenths, and the 3 thousandths become 3 hundredths. The value of the entire number, therefore, is increased ten-fold. But multiplying a number by 10 increases its value ten-fold. Hence, moving the decimal point of a number one place to the right has the same effect as multiplying the number by 10. In the same way, moving the decimal point two places to the right multiplies the number by 100, and so on. Hence,

96. To multiply a decimal by 10, 100, 1000, etc.,

Move the decimal point in the multiplicand as many places to the right, annexing zeros if necessary, as there are zeros in the multiplier.

Again, if in 79.253 we move the decimal point one place to the left, so that the number becomes 7.9253, we decrease the value of each figure ten-fold; the 7 tens become 7 units, the 9 units become 9 tenths, the 2 tenths become 2 hundredths, the 5 hundredths become 5 thousandths, and the 3 thousandths become 3 ten-thousandths. The value of the resulting number, therefore, is one-tenth that of the original number. But multiplying a number by 0.1 means to find one-tenth of the number. Hence, moving the decimal point of a number one place to the left has the same effect as multiplying the number by 0.1. In the same way, moving the decimal point two places to the left has the same effect as multiplying the number by 0.01, and so on. Hence,

97. To multiply a decimal by 0.1, 0.01, 0.001, etc.,

Move the decimal point in the multiplicand as many places to the left, prefixing zeros if necessary, as there are decimal places in the multiplier.

98. 1. Multiply 123.826 by 3.

SOLUTION. Here 3 × 6 thousandths = 18 thousandths, or 1 hundredth and 8 thousandths; the 8, therefore, is written under the thousandths; then 3 × 2 hundredths = 6 hundredths, which, with the 1 hundredth, make 7 hundredths, and the 7 is written under the hundredths; then 3 × 8 tenths = 24 tenths, or 2 units and 4 tenths, and the 4 is written under the tenths; then 3 × 3 units = 9 units, which, with the 2 units, make 11 units, and so on.

$$\begin{array}{r} 123.826 \\ 3 \\ \hline 371.478 \end{array}$$

2. Multiply 123.826 by 0.3.

SOLUTION. The multiplier 0.3 = 3 × 0.1. We therefore multiply first by 3, and this product by 0.1. But multiplying by 0.1 simply moves the decimal point in the product one place to the left. Hence, the product will have three decimal places for the decimal in the multiplicand, and one more place for the decimal in the multiplier.

$$\begin{array}{r} 123.826 \\ 0.3 \\ \hline 37.1478 \end{array}$$

3. Multiply 0.123826 by 0.32.

SOLUTION. The multiplier 0.32 = 32 × 0.01. We therefore multiply first by 32, and this product by 0.01. But multiplying by 0.01 simply moves the decimal point in the product two places to the left. Hence, the product has six decimal places for the decimal in the multiplicand, and two more places for the decimal in the multiplier. As the product has only seven figures, we prefix a cipher to make eight decimal places.

$$\begin{array}{r} 0.123826 \\ 0.32 \\ \hline 247652 \\ 371478 \\ \hline 0.03962432 \end{array}$$

Hence, in the multiplication of decimals,

We multiply as if the numbers were integral, and point off from the right of the product as many figures for decimals as there are decimal places in the multiplicand and multiplier together.

NOTE. If the product does not contain as many figures as there are decimal places in the multiplicand and multiplier together, we *prefix* enough zeros to the product to make the number equal to the number of the decimal places in the multiplicand and multiplier.

EXERCISE 49. — WRITTEN.

Find the value of:

1. 132×2.475.	10. 9.007×106.8.
2. 13.2×2.475.	11. 70×387.45.
3. 1.32×2.475.	12. 0.07×387.45.
4. 0.132×2.475.	13. 70.07×387.45.
5. 0.236×12.13.	14. 42×0.065.
6. 1.121×71.12.	15. 4.2×0.065.
7. 9.06×0.045.	16. 4.02×0.565.
8. 10.01×10.09.	17. 2000×0.075.
9. 0.008×751.1.	18. 8000×0.0755.

19. The circumference of a circle is 3.1416 times its diameter. Find to the nearest hundredth of an inch the circumference of a circle whose diameter is 39.37 inches.

20. A cubic foot of water weighs 62.5 pounds. Gold is 19.26 times as heavy as water. Find the weight of a cubic foot of gold.

21. The average yield of corn per acre in the United States for 1895 was 26.2 bushels, and the farm price per bushel was 26.4 cents. What was the average value of the corn crop per acre to the farmer?

22. The average yield of wheat per acre in the United States for 1895 was 13.7 bushels, and the farm price per bushel was 50.9 cents. What was the average value of the wheat crop per acre to the farmer?

23. The average yield of oats per acre in the United States for 1895 was 26.6 bushels, and the farm price per bushel was 19.9 cents. What was the average value of the oat crop per acre to the farmer?

24. The average yield of barley per acre in the United States for 1895 was 26.4 bushels, and the farm price per bushel was 33.7 cents. What was the average value of the barley crop per acre to the farmer?

Division of Decimals.

99. *If the divisor is an integral number,* each quotient figure is of the *same order of units* as the right-hand figure of the partial dividend used in obtaining it; and, therefore, *the decimal point is written in the quotient as soon as the decimal point in the dividend is reached.*

If the divisor contains decimal places, we may multiply the divisor and dividend by 10, 100, 1000, etc., so as to make the divisor an integral number, without altering the quotient (§ 74).

1. Divide 7.8528 by 8.

$$\begin{array}{r} 8\underline{)7.8528} \\ 0.9816 \end{array}$$

2. Divide 78.528 by 0.8.

$$\begin{array}{r} 8\underline{)785.28} \\ 98.16 \end{array}$$

3. Divide 28.3696 by 1.49.

(1)

$$\begin{array}{rl} & 19.04 \\ 149) & \overline{2836.96} \\ & \underline{149} \\ & 1346 \\ & \underline{1341} \\ & \quad 596 \\ & \quad \underline{596} \end{array}$$

(2)

$$\begin{array}{rl} & 19.04 \\ 1.49) & \overline{28.36_{\wedge}96} \\ & \underline{149} \\ & 1346 \\ & \underline{1341} \\ & \quad 596 \\ & \quad \underline{596} \end{array}$$

In form (1) we multiply both dividend and divisor by 100. Writing the first figure of the quotient over the right-hand figure of the first partial dividend brings the decimal point of the quotient directly over the decimal point in the dividend.

The same result is obtained by counting to the right from the decimal point in the dividend as many places as there are decimal places in the divisor, inserting some mark as shown in form (2), and putting the decimal point in the quotient directly over this mark.

100. If the divisor is not contained in the dividend without a remainder, ciphers may be annexed to the dividend (§ 92), and the division continued.

Divide 3.9842 by 3.7164 to three decimal places.

```
               1.072
3.7164)3.9842^000
          37164
           267800
          260148
             76520
             74328
              2192
```

As the divisor has *four* decimal places, we count four places to the right of the decimal point in the dividend and insert a caret. We then divide as in integral numbers, and put the decimal point directly over the caret.

NOTE. If we wish to find the value of a decimal correct to the nearest *tenth*, *hundredth*, *thousandth*, etc., we add 1 to the last required decimal figure if the next decimal figure would be 5 or more. Thus, the value of the answer 1.072 correct to the nearest tenth is 1.1; correct to the nearest hundredth is 1.07; correct to the nearest thousandth is 1.072.

101. If the divisor is a whole number, and *ends in one or more zeros*, cut off the zeros from the divisor, and put a caret in the dividend as many places *to the left* of the decimal point as there are zeros cut off, prefixing zeros to the dividend if necessary.

Divide 42.08 by 8000.

$$8\cancel{0}\cancel{0}\cancel{0})\underline{0_{\wedge}042.08}$$
$$0.00526$$

Here we cut off the three zeros from the divisor, and put a caret three places to the left of the decimal point in the dividend; that is, we *divide* both the divisor and the dividend by 1000.

Exercise 50. — Written.

Find the value of :

1. 57.50 ÷ 5.75.
2. 34.88 ÷ 4.36.
3. 36.08 ÷ 3.28.
4. 18.72 ÷ 1.56.
5. 2.538 ÷ 0.047.
6. 25.6 ÷ 0.0016.
7. 25.6 ÷ 16000.
8. 10.01 ÷ 0.02.
9. 10.01 ÷ 200.
10. 15.625 ÷ 0.064.
11. 15.625 ÷ 6400.
12. 1.066 ÷ 1300.
13. 1361.5 ÷ 50,000.
14. 40 ÷ 0.025.
15. 0.04 ÷ 0.05.
16. 729 ÷ 3600.
17. 10.24 ÷ 320.
18. 600 ÷ 0.625.
19. 6.256 ÷ 0.375.
20. 0.375 ÷ 250.
21. 16.025 ÷ 250.
22. 327.50 ÷ 0.025.
23. 241.802 ÷ 0.319.
24. 317.10 ÷ 10.5.
25. 405.15 ÷ 0.111.
26. 45.625 ÷ 0.125.
27. 0.0125 ÷ 2.5.
28. 0.0169 ÷ 0.013.
29. 13.0375 ÷ 800.
30. 80.019 ÷ 0.009.
31. 800.19 ÷ 9000.
32. 200 ÷ 3.125.
33. 3.00625 ÷ 25.
34. 15 ÷ 0.0025.
35. 277.808 ÷ 3880.
36. 6.51021 ÷ 3.207.
37. 87.912 ÷ 40.7.
38. 7704.256 ÷ 8.302.
39. 156.25 ÷ 5000.
40. 15.21 ÷ 1170.
41. 1050 ÷ 43.75.
42. 8468 ÷ 0.292.
43. 130.4 ÷ 4000.
44. 2847.432 ÷ 40.
45. 31 ÷ 500.
46. 640 ÷ 16,000.
47. 723.6 ÷ 1440.
48. 276.766 ÷ 37,100.
49. 12,876.36 ÷ 120,000.
50. 309.45 ÷ 150,000.

EXERCISE 51. — WRITTEN.

1. If $2.88 is paid for 12 dozen eggs, what is the price per dozen?

2. The value of 20 francs is $3.86. Find the value of 1 franc.

3. The pound sterling is worth $4.8665. There are 20 shillings in a pound. Find to the nearest mill the value of an English shilling in United States money.

4. If the average rate of a railway train is 36.7 miles per hour, how many hours will it take the train to go 1101 miles?

5. How many yards of muslin at 12 cents a yard can be bought for $4.56?

6. How many yards of velvet at $4 a yard can be bought for $23?

7. A yard is 36 inches and a meter is 39.3704 inches. Find to the nearest ten-thousandth the value of a meter in yards.

8. A bankrupt's assets are $12,000, and he is able to pay 75 cents on a dollar. Find the amount of his debts.

NOTE. The word *assets* means the value of his property.

9. Find to the nearest hundredth the weight of a cubic foot of air, water being 830 times as heavy as air, and a cubic foot of water weighing 1000 ounces.

10. A gallon contains 231 cubic inches. Olive oil weighs 0.915 as much as water. Find to the nearest hundredth the number of cubic inches of olive oil that will weigh as much as a gallon of water.

11. An ounce of pure gold is worth $20.67. When silver is selling at 62.5 cents per ounce, how many ounces of silver are worth as much as 1 ounce of gold?

Exercise 52. — Written.

Review Problems.

1. A man had in a bank the first day of May, 1896, $1152.35. He deposited in the bank at different times during the month $165, $210.50, and $300; and drew out $1000.90. What was his balance at the end of the month?

2. A man sold 0.625 of his farm of 160 acres. What part of his farm had he left? How many acres had he left?

3. A, B, and C harvest a field of corn. A does 0.375 of the work, B does 0.33 of it. What part of the work does C do?

4. Pure gold is worth $20.67 an ounce. A pound of gold is 12 ounces. Find the value of a pound of pure gold.

5. A bushel of turnips weighs 60 pounds, and 0.905 of the weight of turnips is water. Find the weight of water in a bushel of turnips.

6. A bushel of potatoes weighs 60 pounds, and 0.155 of the weight of potatoes is starch. Find the weight of starch in a load of 40 bushels of potatoes.

7. The distance from Paris to Berlin is 1308 kilometers. A kilometer is 0.6214 of a mile. How many miles is it from Paris to Berlin?

8. The diameter of a wheel is 0.31831 of its circumference. Find the diameter of a wheel 13 feet in circumference.

9. A stone roller, 4.13 feet round, makes 18.9 turns in rolling from one end of a tennis-court to the other. Find the length of the tennis-court.

10. A cubic foot of water weighs 62.5 pounds, and a cubic foot of mercury weighs 849 pounds. How many times as heavy as water is mercury?

NOTE. In this and the following examples, answers to the nearest hundredth are sufficiently correct.

11. A cubic foot of platinum, the heaviest substance known, weighs 1365 pounds. A cubic foot of water weighs 62.5 pounds. How many times as heavy as water is platinum?

12. How many times as heavy as water is common sand, a cubic foot of sand weighing 103 pounds?

13. Granite is 2.72 times as heavy as water. Find the weight of a cubic foot of granite.

14. Wrought iron is 7.77 times as heavy as water. Find the weight of a cubic foot of wrought iron.

15. Platinum is 21.841 times as heavy as water, and lead is 11.35 times as heavy as water. How many times as heavy as lead is platinum?

16. Silver is 10.5 times as heavy as water, and cork is 0.24 as heavy as water. How many times as heavy as cork is silver?

17. A *board foot* is a board 1 foot long, 1 foot wide, and 1 inch thick; and 12 feet of boards are equal to 1 cubic foot. Find the number of cubic feet in 1000 feet of boards.

18. From the number of cubic feet in 1000 board feet found in example 17, find the weight of 1000 feet of dry white pine boards, knowing that dry white pine is 0.4 as heavy as water.

19. Find the weight of 1000 feet of dry white oak boards, knowing that dry white oak is 0.83 as heavy as water.

20. In the United States for 1895 the average yield of hay per acre was 1.06 tons, and the average value of the crop per acre was $8.851. Find the average price of the hay per ton.

21. In the United States for 1895 the average yield of potatoes per acre was 100.6 bushels, and the average value of the crop per acre was $26.76. Find the average price of the potatoes per bushel.

22. How many lengths of fence each 12.75 feet long will be required to enclose a field having four sides, if two of the sides are each 395.25 feet long and the other two sides are each 357 feet long?

23. James Brown bought of John Smith 7 pounds of coffee at 35 cents; 3 pounds of tea at 65 cents; 2 boxes of raisins at $3.25; 2 barrels of flour at $4.25. He gave a $20 bill in payment. How much change is due him?

24. A farmer bought for $110 an equal number of cows, sheep, and pigs, paying $22.25 for each cow, $3.50 for each sheep, and $1.75 for each pig. How many of each did he buy?

25. A pendulum of a certain clock swings 1.375 times a second. In how many seconds will it swing 660 times?

26. If 0.741 of the weight of rice is starch, and 0.155 of the weight of potatoes is starch, find the number of bushels of potatoes, 60 pounds to the bushel, required to furnish as much starch as 100 pounds of rice.

27. Reckoning the distance round the equator of the earth as 25,000 miles, and the time of the revolution of the earth about its axis as 24 hours, find how many miles a man standing on the equator is carried round in an hour.

28. A bicycle wheel is 28 inches in diameter, and its circumference is 3.1416 times as much. Find the number of times the wheel turns in going a mile, 63,360 inches.

Review Questions.

What system of notation is used in United States money? What is the unit of United States money? What is the sign for United States money? How many places of decimals do cents occupy? If the number of cents is less than ten, with what must the first decimal place be filled? What part of a dime is one cent? What part of a dollar is one cent? What part of a dollar is one mill? How are dollars reduced to cents? How are dollars reduced to mills? How are cents reduced to dollars? How are mills reduced to dollars? How are dollars and cents arranged for addition and subtraction? How many decimal places must there be in the product? In dividing dollars and cents into equal parts, where is the decimal point in the quotient placed? What are the gold coins of the United States? the silver coins? What coins have we that are neither gold nor silver?

How many tenths make an entire unit? How many hundredths make an entire unit? How many thousandths make an entire unit? How many hundredths make a tenth? How many thousandths make a hundredth? At what rate does the value of a figure increase from right to left? Which decimal place do tenths occupy? hundredths? thousandths? ten-thousandths? hundred-thousandths? How do we write a decimal? How do we read a decimal? How do we read a number that has an integral part and a decimal part? By what word do we connect the integral and decimal parts of a number? What effect does it have on a decimal to annex ciphers to it? In adding and subtracting decimals, how do we arrange the decimal points? In subtraction of decimals, if the minuend has fewer decimal places than the subtrahend what do we do? What effect does it have on the value of a number if the decimal point is moved one place to the right? one place to the left? two places to the right? two places to the left? What effect does it have on the value of a decimal if it is multiplied by 10, 100, etc.? What effect does it have on the value of a decimal if it is multiplied by 0.1, 0.01, 0.001, etc.? In the multiplication of decimals, how many decimal placcs must the product have? In division, if the divisor is not an integral number, how can we make it an integral number without altering the quotient? If the divisor is an integral number, where must we put the decimal point in the quotient? If the quotient is written directly over the dividend, how will the decimal points of the dividend and quotient be arranged? What, then, is the advantage of writing the quotient over the dividend?

CHAPTER VIII.

FACTORS, MEASURES, MULTIPLES.

102. Factors of a Number. The *factors* of a number are the numbers whose product is that number.

103. Prime Numbers. A *prime number* is a number that has *no integral factors,* except itself and one.

Thus, 2, 3, 5, 7, 11, 13, 17, 19 are prime numbers.

104. Composite Numbers. A *composite number* is a number that is the product of two or more integral factors.

Thus, 10, 21, 143 are composite numbers, as 10 is 2×5; 21 is 3×7; 143 is 11×13.

NOTE. In speaking of the integral factors of a number we exclude the number itself and one.

105. Prime Factors. A *prime factor* is a factor that is a prime number.

106. *A composite number can have but one set of prime factors.*

Thus, 12 cannot be expressed as the product of any *set of prime factors* except $2 \times 2 \times 3$. It is the product of 2×6, and of 3×4, but one of the factors of 2×6 and one of 3×4 is *composite.*

107. A number that can be divided by another *without a remainder* is said to be *exactly divisible* by the other number; and the divisor is called an *exact divisor.*

108. Even Numbers. An *even number* is a number that is exactly divisible by 2.

109. Odd Numbers. An *odd number* is a number that is *not* exactly divisible by 2.

110. *To find the prime factors of a number,* we try to divide the number by one of the prime numbers, 2, 3, 5, 7, 11, etc., in the order named, until we find a prime number that will exactly divide the given number; we repeat the same process with the quotient obtained, and so on until the quotient is a prime number.

NOTE. We need not try 2 for a divisor unless the last digit of the number is 0, 2, 4, 6, or 8.

We need not try 3 for a divisor unless the sum of the digits of the number is exactly divisible by 3.

We need not try 5 for a divisor unless the last digit of the number is 0 or 5.

1. Find the prime factors of 144.

$$\begin{array}{r|r} 2 & 144 \\ \hline 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

That is, $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$.

NOTE. Divide *as many times as possible* by the *smallest* prime number that will exactly divide the given number before taking the next larger prime number for a divisor.

2. Find the prime factors of 233.

The last digit is not 0, 2, 4, 6, or 8; therefore 2 is not a factor. The sum of the digits is not exactly divisible by 3; therefore 3 is not a factor. The last digit is not 0 or 5; therefore 5 is not a factor. We need not try, therefore, 2, 3, or 5 as divisors. We find by trial that 7, 11, 13, or 17 is not a factor. We need not try any higher prime number, as the quotient when 17 is tried is *less* than 17. Therefore, no prime number greater than 17 can be a factor; and we have found by trial that no prime number less than 17 is a factor. Therefore, 233 is a prime number.

111. From these two examples we have the following

RULE. *Divide the given number by any prime number that exactly divides it; then the quotient by any prime number that exactly divides it; and so on until the quotient is itself a prime number. The several divisors and the last quotient are the prime factors.*

If no prime factor is found before the quotient becomes equal to or less than the divisor, the number is a prime number.

112. Exponents. To avoid the necessity of writing long rows of equal factors, a small figure called an *exponent* is written at the right of a number to show the number of times it is taken as a factor.

Thus, $2 \times 2 \times 2 \times 2 \times 3 \times 3$ is written $2^4 \times 3^2$.

113. Powers. The expression 2^3 is called the *third power*, or *cube*, of 2, and 3^2 is called the *second power*, or *square*, of 3.

EXERCISE 53. — ORAL.

Find the prime factors of:

1. 6.	**7.** 15.	**13.** 25.	**19.** 33.
2. 9.	**8.** 16.	**14.** 26.	**20.** 36.
3. 8.	**9.** 18.	**15.** 27.	**21.** 38.
4. 10.	**10.** 20.	**16.** 28.	**22.** 39.
5. 12.	**11.** 21.	**17.** 30.	**23.** 40.
6. 14.	**12.** 22.	**18.** 24.	**24.** 42.

EXERCISE 54. — WRITTEN.

Find the prime factors of:

1. 112.	**4.** 117.	**7.** 555.
2. 121.	**5.** 495.	**8.** 324.
3. 132.	**6.** 520.	**9.** 539.

10. 289.	**19.** 1575.	**28.** 1170.
11. 860.	**20.** 3318.	**29.** 4260.
12. 437.	**21.** 6480.	**30.** 1001.
13. 1155.	**22.** 1017.	**31.** 3650.
14. 1220.	**23.** 2135.	**32.** 1760.
15. 6006.	**24.** 1932.	**33.** 1365.
16. 1435.	**25.** 3432.	**34.** 5070.
17. 2520.	**26.** 1144.	**35.** 1830.
18. 1084.	**27.** 1690.	**36.** 8175.

114. Measures of a Number. The *measures* of a number are the *exact divisors* of the number.

Thus, a man with five-dollar bills can make up a sum of $20, but not of $18. A wood chopper with the two-foot mark on his axe-handle can measure off lengths of 2, 4, 6, or 8 feet, but not of 3, 5, or 7 feet. A man with scales and a four-ounce weight can weigh out 16 ounces of tea, but not 22 ounces. A man with a four-quart measure can measure out 4, 8, or 12 quarts of molasses, but not 5, 6, or 7 quarts.

115. Common Measures. A common measure of two or more numbers is a number that exactly divides *each* of them.

Thus, $5 is a common measure of $35 and $40, being contained exactly 7 times in $35 and 8 times in $40. 3 feet is a common measure of 21 feet, 15 feet, and 12 feet. 1 yard is a common measure of 2 yards, 3 yards, and 5 yards. 4 is a common measure of 12, 16, and 20.

116. Greatest Common Measure. The greatest common measure of two or more numbers is *the greatest number* that exactly divides each of them.

Thus, the measures of 84 are 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, and the measures of 36 are 1, 2, 3, 4, 6, 9, 12, 18.

We see from the two series of measures that 1, 2, 3, 4, 6, 12 are the only measures of both 84 and 36, and that 12 is the greatest; therefore, 12 is the greatest common measure of 84 and 36.

The letters G. C. M. stand for the words Greatest Common Measure.

NOTE. Greatest Common Divisor is sometimes used instead of Greatest Common Measure, and then G. C. D. is used instead of G. C. M.

117. If two integral numbers have no common measure except 1, they are said to be *prime to each other.*

Thus, 27 and 32 are prime to each other, though both are composite numbers.

118. 1. Find the G. C. M. of 84, 126, 210.

Resolve each of the numbers into its prime factors.

$$\begin{array}{r|l} 2 & 84 \\ \hline 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|r} 2 & 126 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|r} 2 & 210 \\ \hline 3 & 105 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

$84 = 2^2 \times 3 \times 7.$ $126 = 2 \times 3^2 \times 7.$ $210 = 2 \times 3 \times 5 \times 7.$

The factor 2 occurs once in all the numbers.
The factor 3 occurs once in all the numbers.
The factor 7 occurs once in all the numbers.
No other factor occurs in *all* the numbers.
Therefore, the G. C. M. is $2 \times 3 \times 7 = 42$.

2. Find the G. C. M. of 40 and 72.

$40 = 2 \times 2 \times 2 \times 5.$ $72 = 2 \times 2 \times 2 \times 3 \times 3.$

Hence, 2 occurs *three* times as a factor in 40, and *three* times as a factor in 72. No other factor is common to 40 and 72. Therefore, $2 \times 2 \times 2 = 8$ is the G. C. M. Therefore,

119. For finding the G. C. M. of two or more numbers we have the following

RULE. *Separate the numbers into their prime factors and find the product of the prime factors that are common to all the numbers.*

The common factors of two or more numbers may be taken out at the same time as follows:

2	84	126	210
3	42	63	105
7	14	21	35
	2	3	5

As all the numbers are *even*, 2 is a common factor. As 3 is an exact divisor of 42, 63, and 105, 3 is a common factor.

As 7 is an exact divisor of 14, 21, and 35, 7 is a common factor.

The quotients 2, 3, and 5 have no common factor. Therefore, the only *common* factors are 2, 3, and 7; and the G. C. M. is $2 \times 3 \times 7 = 42$.

Exercise 55. — Written.

Find the G. C. M. of:

1. 9, 12, 18.
2. 24, 30, 36.
3. 12, 18, 24.
4. 24, 80, 96.
5. 28, 42, 56.
6. 42, 84, 126.
7. 44, 77, 110.
8. 25, 35, 110.
9. 21, 28, 77.
10. 60, 120, 150.
11. 32, 48, 128.
12. 45, 72, 81.
13. 51, 105, 243.
14. 36, 84, 132.
15. 36, 81, 135.
16. 42, 54, 60.
17. 75, 300, 450.
18. 144, 576, 720.
19. 13, 91, 143.
20. 14, 98, 112.

120. Find the G. C. M. of 115 and 161.

```
115)161(1
    115
     46)115(2
         92
         23)46(2
            46
```

Solution. We divide the greater number by the smaller, and the last divisor by the last remainder, and so on until there is no remainder. The final divisor is the greatest common measure.

This method of finding the G. C. M. can be employed when the numbers cannot readily be separated into their prime factors.

Exercise 56. — Written.

Find the G. C. M. of:

1. 374 and 2295.	6. 6004 and 3318.
2. 333 and 592.	7. 2871 and 4234.
3. 820 and 697.	8. 1820 and 3367.
4. 1086 and 905.	9. 315 and 2268.
5. 1220 and 2013.	10. 6870 and 8473.

Exercise 57. — Written.

1. A man has two trees which he wishes to cut into logs of *equal length*. If the trees are 84 feet and 96 feet long and are cut into the longest logs possible, what is the length of the logs?

2. A man has 152 quarts of blueberries and 140 quarts of blackberries which he wishes to put into boxes, each box to hold the same number of quarts and the largest number possible. How many quarts can he put in each box?

3. What is the length of the longest chain that will measure exactly the length and the width of a field 484 rods long and 420 rods wide?

4. A man has two rolls of bank bills all of the same denomination. In one roll he has $280, and in the other $275. What is the denomination of the bills if they are the largest possible?

5. A chest of tea containing 70 pounds and another containing 72 pounds were put into equal packages of the same weight. Find the weight of each package if the tea was put into the largest packages possible.

6. A grocer has 1314 pounds of white sugar and 1533 pounds of brown sugar. He wishes to put this sugar into barrels without mixing the sugar, the same number of pounds and the greatest number possible in each barrel. How

many pounds can he put in each barrel, and how many barrels of sugar will he have?

7. A man steps more than 20 inches and less than 30 inches at each step. He takes an exact number of steps in walking 203 inches and an exact number in walking 261 inches. What is the length of his step?

8. From some small shot weighing 6015 grains a part of the shot was taken which weighed 1650 grains. Find the greatest weight each shot can have.

121. Multiples. If a number is multiplied by an integer, the product is called a *multiple* of the number.

Thus, \$20 is a multiple of \$5, since 4 times \$5 is \$20; 18 feet is a multiple of 3 feet, since 6 times 3 feet is 18 feet.

122. A series of multiples of a number is found by multiplying the number by the integers, 1, 2, 3, 4, 5, etc.

Since a composite number is the product of *only one set* of prime numbers (§ 106), every multiple of a number *contains all the prime factors of the number.*

123. Common Multiple. A multiple of two or more numbers is called a *common multiple* of the number.

Thus, 6 × \$2 = \$12; 4 × \$3 = \$12; 3 × \$4 = \$12; 2 × \$6 = \$12. Therefore, \$12 is a common multiple of \$2, \$3, \$4, and \$6.

124. Least Common Multiple. The *smallest* common multiple of two or more numbers is called their *least common multiple;* and it is the smallest number that is exactly divisible by each of them.

Thus, the multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, etc.; and the multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36, etc. The common multiples of 3 and 4, are 12, 24, and 36, etc.; and the smallest of these is 12. Therefore, the least common multiple of 3 and 4 is 12.

The letters L. C. M. stand for the words Least Common Multiple.

125. The L. C. M. of two or more numbers contains *all the prime factors* of each of these numbers. Every prime factor, therefore, must occur in the L. C. M. the *greatest number of times* it occurs as a factor *in any one* of them.

A prime factor, however, of *one* of the given numbers need not be *repeated* because it occurs in *another* of the given numbers.

Thus, $\$20 = 2 \times 2 \times 5 \times \$1$,
and $\$30 = 2 \times 3 \times 5 \times \$1$.

The L. C. M. is found by taking as factors 2 *twice*, 3 *once*, and 5 *once*, and it is, therefore, $2 \times 2 \times 3 \times 5 \times \$1 = \$60$.

126. Find the L. C. M. of 84, 168, 252, and 420.

Resolve each of the numbers into its prime factors.

$$\begin{array}{r|r} 2 & 84 \\ \hline 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|r} 2 & 168 \\ \hline 2 & 84 \\ \hline 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|r} 2 & 252 \\ \hline 2 & 126 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|r} 2 & 420 \\ \hline 2 & 210 \\ \hline 3 & 105 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

The factor 2 occurs *three* times in 168; the factor 3 occurs *twice* in 252; the factor 5 occurs *once* in 420; and the factor 7 occurs *once* in all the given numbers.

Therefore, the L. C. M. is $2^3 \times 3^2 \times 5 \times 7 = 8 \times 9 \times 5 \times 7 = 2520$.

127. Hence, for finding the L. C. M. of two or more numbers we have the following

RULE. *Separate each number into its prime factors; and find the product of these factors, taking each factor the greatest number of times it occurs in any one of the given numbers.*

1. Find the L. C. M. of 18, 24, 27, 45.

Arrange the numbers in line, and divide by the smallest prime factor that will divide *two* or more of the numbers.

$$\begin{array}{r|rrrr} 2 & 18 & 24 & 27 & 45 \\ \hline 3 & \cancel{9} & 12 & 27 & 45 \\ \hline 3 & & 4 & 9 & 15 \\ \hline & & 4 & 3 & 5 \end{array}$$

We first divide by 2, and write the quotients and undivided numbers in a line below. In the first line of quotients we cancel 9, as it is an exact divisor of 27, and, therefore, 27 contains all the factors of 9. We next divide by 3, and the quotients by 3, and obtain the numbers 4, 3, 5 in the last line. No *two* of the numbers 4, 3, and 5 have a common factor. Hence, the L. C. M. is $2 \times 3 \times 3 \times 4 \times 3 \times 5 = 1080$.

2. Find the L. C. M. of 3, 9, 27, 54.

$$\cancel{3},\quad \cancel{9},\quad \cancel{27},\quad 54.$$

We cancel the 3, which is contained in 9; then the 9, which is contained in 27; then the 27, which is contained in 54, and have 54 for the L. C. M. of the numbers.

3. Find the L. C. M. of 13, 15, 26, 39.

$$\begin{array}{r|cccc} 3 & \cancel{13} & 15 & 26 & 39 \\ \hline & & 5 & 26 & \cancel{13} \end{array}$$

We cancel the 13 which is contained in 26 and divide by 3, getting 5, 26, 13. We cancel the 13 of this line, which is contained in 26, and have for the L. C. M. $3 \times 5 \times 26 = 390$.

Exercise 58. — Written.

Find the L. C. M. of:

1. 5, 10, 15.
2. 9, 12, 18.
3. 24, 30, 36.
4. 9, 24, 40.
5. 2, 3, 5, 7.
6. 2, 5, 13, 26.
7. 5, 20, 25, 100.
8. 2, 19, 38, 76.
9. 3, 9, 27, 81.
10. 3, 9, 36, 45.
11. 13, 26, 39, 52.
12. 7, 13, 21, 26.
13. 6, 18, 54, 108.
14. 5, 9, 12, 15.
15. 12, 15, 18, 24.
16. 12, 18, 30, 45.
17. 9, 12, 22, 33.
18. 8, 21, 28, 35.
19. 6, 7, 8, 9.
20. 36, 45, 54, 63.
21. 13, 26, 39, 169.
22. 8, 12, 18, 20.
23. 11, 44, 132, 198.
24. 11, 22, 55, 110.
25. 14, 18, 20, 21.
26. 20, 28, 56, 70.
27. 17, 34, 51, 85.
28. 45, 50, 60, 63, 84.
29. 9, 10, 14, 15, 18.
30. 21, 24, 26, 28, 30.

Exercise 59. — Written.

1. Find the shortest distance that can be exactly measured by a three-foot rule, a four-foot rule, or a ten-foot pole.

2. Find the smallest sum of money that can be counted out in five-cent pieces, ten-cent pieces, or in twenty-five-cent pieces.

3. What is the least number of yards of carpet in a roll that can be cut into lengths of 12 yards, 15 yards, or 20 yards?

4. What is the least number of oranges that can be divided equally among 21, 24, or 30 boys?

5. Find the least number of acres in a farm that can be divided exactly into lots of 12, 15, or 18 acres.

6. What is the least quantity of milk that can be exactly measured by two-quart, four-quart, six-quart, or eight-quart measures?

7. I have just enough money to buy a whole number of dozens of oranges at 40 cents a dozen, or a whole number of baskets of peaches at $1.25 a basket. How much have I?

8. Find the capacity of the smallest cistern that can be filled in an exact number of minutes by either of two pipes, one discharging 36 gallons and the other 42 gallons a minute.

9. Find the capacity of the smallest cistern that can be filled in an integral number of minutes by either pipe, or by both pipes together, one discharging 30 gallons and the other 20 gallons a minute.

10. Find the least number of oranges that arranged in groups of 6, 7, 8, or 9 have just five over in each case.

Hint. Add 5 to the L. C. M. of 6, 7, 8, 9.

128. Cancellation. Cancellation is the process of shortening work in division by removing or *cancelling* equal factors from the dividend and divisor.

Divide $9 \times 18 \times 24$ by $3 \times 6 \times 12$.

$$\frac{\overset{3}{\cancel{9}} \times \overset{3}{\cancel{18}} \times \overset{2}{\cancel{24}}}{\cancel{3} \times \cancel{6} \times \cancel{12}} = \frac{3 \times 3 \times 2}{1 \times 1 \times 1} = 18.$$

The factor 3 in the divisor is cancelled, and 3 in 9 in the dividend. The factor 6 in the divisor is cancelled, and 6 in 18 in the dividend. The factor 12 in the divisor is cancelled, and 12 in 24 in the dividend. The resulting dividend is $3 \times 3 \times 2$, and the divisor $1 \times 1 \times 1$. The quotient, therefore, is 18.

Exercise 60. — Written.

Find the quotient of:

1. $\dfrac{7 \times 6 \times 16}{3 \times 8 \times 14}$.

2. $\dfrac{11 \times 27 \times 30}{9 \times 15 \times 3}$.

3. $\dfrac{32 \times 35 \times 9}{3 \times 16 \times 7}$.

4. $\dfrac{9 \times 14 \times 39}{13 \times 7 \times 18}$.

5. $\dfrac{625 \times 3 \times 54}{75 \times 5 \times 18}$.

6. $\dfrac{1728 \times 3 \times 7}{12 \times 12 \times 12}$.

7. $\dfrac{29 \times 15 \times 6}{87 \times 6 \times 5}$.

8. $\dfrac{84 \times 13 \times 5}{91 \times 4 \times 15}$.

9. If a farmer sells 25 bushels of wheat at 60 cents a bushel and takes his pay in cloth at 40 cents a yard, how many yards of cloth does he get?

10. Three pieces of cloth containing 20 yards each, worth \$5 a yard, were exchanged for 5 pieces of cloth containing 40 yards each. What was the second kind of cloth worth per yard?

11. James Brown exchanged 9 firkins of butter weighing 56 pounds each, at 25 cents a pound, for straw matting having 40 yards in a roll, worth 15 cents a yard. How many full rolls did he get?

CHAPTER IX.

Common Fractions.

129. What is the name of one of the parts when a unit is divided into:

1. Two equal parts?	6. Eight equal parts?
2. Three equal parts?	7. Ten equal parts?
3. Four equal parts?	8. Twelve equal parts?
4. Five equal parts?	9. Twenty equal parts?
5. Six equal parts?	10. One hundred equal parts?

130. A unit contains how many:

1. Halves?	7. Ninths?	13. Twentieths?
2. Thirds?	8. Sevenths?	14. Twenty-fourths?
3. Fourths?	9. Tenths?	15. Thirtieths?
4. Fifths?	10. Twelfths?	16. Fortieths?
5. Sixths?	11. Elevenths?	17. Fiftieths?
6. Eighths?	12. Fifteenths?	18. Hundredths?

131. When a unit is divided into twelve equal parts, what is the name of:

1. One part?	4. Two parts?	7. Eight parts?
2. Three parts?	5. Five parts?	8. Ten parts?
3. Six parts?	6. Four parts?	9. Twelve parts?

132. Equal parts of a unit are called *fractional parts* of the unit.

Exercise 61. — Oral.

1. How many halves of an orange are there in 2 oranges? How many quarters? How many eighths?

2. How many halves of an apple are there in 3 apples? How many quarters? How many thirds?

3. How many tenths of a dollar in $1? $2? $3?

4. How many twelfths of a foot in 3 feet? 4 feet?

5. How many halves of a pie make a whole pie? How many thirds? How many fourths? How many sixths? How many eighths? How many ninths?

6. Which is greater, a half of a pie or a third of a pie? A fourth of a pie or a third of a pie? A fourth of a pie or a sixth of a pie?

7. If a whole pie is cut into equal parts, on what does the size of each part depend?

8. How many times as large as a fourth of a circle is a half of the circle? How many times as large as a sixth is a third?

9. John has three fourths of a dollar and James has three fifths of a dollar. Which one has the greater sum?

10. A man makes a journey in 4 days, going the same number of miles each day. What part of the journey does he make in one day?

11. A boy has two thirds of a dozen oranges. What part of a dozen must he buy to have a whole dozen?

12. If 2 pints fill exactly one fourth of a measure, how many pints will the measure hold?

Note. The teacher should *illustrate* these examples in order to give to each pupil a clear notion of fractional parts, and of their comparative magnitude.

133. In 3 quarters of a yard, the *unit* counted is a *quarter of a yard.*

134. A unit that is a fractional part of a whole unit is called a *fractional unit.*

135. Fractions. Numbers that count fractional units are called *fractions.*

136. Common Fractions. A fraction expressed by two numbers one under the other with a line between them is called a *common fraction.*

137. Simple Fractions. If the two numbers are whole numbers, the fraction is called a *simple fraction.*

Thus, three fifths, written $\frac{3}{5}$, is a simple fraction.

138. The lower number is called the *denominator* (name-giver), the upper number is called the *numerator* (number-giver); and the numerator and denominator together are called the *terms* of the fraction.

139. The fraction $\frac{3}{5}$ means 3 times $\frac{1}{5}$, where $\frac{1}{5}$ is the *fractional unit* and 3 is the *number* of them.

140. *The fractional unit is expressed by 1 divided by the denominator,* and the *number* of fractional units taken is expressed by the *numerator.*

Name the fractional unit and the integral unit of:

1. $\frac{3}{4}$ of an inch.
2. $\frac{4}{5}$ of a dollar.
3. $\frac{3}{8}$ of a bushel.
4. $\frac{2}{3}$ of a yard.
5. $\frac{5}{7}$ of a week.
6. $\frac{7}{12}$ of a foot.
7. $\frac{9}{16}$ of a pound.
8. $\frac{3}{4}$ of an acre.

141. To read a common fraction, read the numerator and then the denominator.

Thus, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{1}{6}$, $\frac{8}{11}$ are read one half, two thirds, three fourths or three quarters, two fifths, one sixth, eight elevenths.

Read:

$\frac{3}{5}$, $\frac{7}{8}$, $\frac{4}{3}$, $\frac{11}{2}$, $\frac{3}{10}$, $\frac{12}{13}$, $\frac{5}{12}$, $\frac{1}{20}$, $\frac{17}{50}$, $\frac{85}{100}$.

Express in figures:

1. Two thirds.
2. Five sevenths.
3. Seven ninths.
4. Eight tenths.
5. Nine twelfths.
6. Eleven sixteenths.
7. Seventeen twentieths.
8. One twenty-fifth.
9. Thirteen seventieths.
10. Thirty hundredths.

142. A **proper fraction** is a fraction whose numerator is less than its denominator; as $\frac{7}{9}$.

143. An **improper fraction** is a fraction whose numerator is not less that its denominator; as $\frac{9}{9}$, $\frac{17}{4}$.

NOTE. When the numerator is greater than the denominator, more than one unit must be regarded as divided into equal parts.

Thus, $\frac{9}{4}$ means that three units have been divided each into four equal parts, and that all the parts of two units and one part of the third unit are taken.

144. How many integral units must be divided into equal parts so that we may have $\frac{9}{4}$ of the unit? $\frac{3}{2}$? $\frac{15}{5}$? $\frac{7}{9}$? $\frac{20}{3}$? $\frac{28}{7}$? $\frac{47}{10}$? $\frac{25}{8}$?

145. A **mixed number** is a whole number and a fraction; as $4\frac{3}{7}$, 5.35. These are read four *and* three sevenths, five *and* thirty-five hundredths.

NOTE. Every mixed number means that some entire units are taken, and the fraction of another unit.

146. Name the proper fractions, the improper fractions, and the mixed numbers of the following expressions: $\frac{5}{11}$, $3\frac{1}{7}$, $\frac{9}{5}$, $4\frac{2}{15}$, $\frac{8}{8}$, $\frac{3}{17}$, $\frac{9}{23}$, $11\frac{2}{9}$, $3\frac{15}{17}$, $\frac{19}{100}$, $\frac{19}{3}$, $\frac{13}{13}$, $\frac{107}{100}$, $18\frac{13}{27}$, $\frac{50}{41}$, $27\frac{11}{100}$, $\frac{23}{80}$, $\frac{11}{7}$, $\frac{19}{20}$, $3\frac{4}{5}$, $\frac{3}{19}$, $\frac{11}{5}$.

Reduction of an Improper Fraction to a Whole or a Mixed Number.

147. Reduce $\frac{21}{4}$ to a whole or a mixed number.

There are $\frac{4}{4}$ in 1 unit, and in $\frac{21}{4}$ there are as many units as there are 4's in 21, or $5\frac{1}{4}$. Hence,

148. To reduce an improper fraction to a whole or a mixed number,

Divide the numerator by the denominator.

The quotient will be the whole number, and the remainder, if any, will be the numerator of the fractional part, of which the denominator is the denominator of the improper fraction.

Exercise 62. — Oral.

Reduce to a whole or a mixed number:

1. $\frac{11}{4}$.	7. $\frac{42}{7}$.	13. $\frac{100}{10}$.	19. $\frac{36}{7}$.
2. $\frac{17}{3}$.	8. $\frac{15}{11}$.	14. $\frac{33}{4}$.	20. $\frac{17}{12}$.
3. $\frac{25}{5}$.	9. $\frac{23}{6}$.	15. $\frac{43}{12}$.	21. $\frac{19}{10}$.
4. $\frac{31}{9}$.	10. $\frac{25}{12}$.	16. $\frac{55}{11}$.	22. $\frac{52}{5}$.
5. $\frac{29}{8}$.	11. $\frac{26}{2}$.	17. $\frac{22}{9}$.	23. $\frac{50}{3}$.
6. $\frac{41}{10}$.	12. $\frac{46}{5}$.	18. $\frac{16}{5}$.	24. $\frac{39}{4}$.

Exercise 63. — Written.

Reduce to a whole or a mixed number:

1. $\frac{68}{13}$.	11. $\frac{57}{20}$.	21. $\frac{721}{50}$.	31. $\frac{242}{69}$.
2. $\frac{77}{20}$.	12. $\frac{93}{25}$.	22. $\frac{157}{35}$.	32. $\frac{65}{57}$.
3. $\frac{75}{15}$.	13. $\frac{93}{31}$.	23. $\frac{87}{28}$.	33. $\frac{655}{28}$.
4. $\frac{40}{19}$.	14. $\frac{117}{20}$.	24. $\frac{81}{27}$.	34. $\frac{543}{32}$.
5. $\frac{17}{17}$.	15. $\frac{95}{42}$.	25. $\frac{153}{55}$.	35. $\frac{144}{36}$.
6. $\frac{91}{16}$.	16. $\frac{53}{53}$.	26. $\frac{248}{65}$.	36. $\frac{1728}{12}$.
7. $\frac{107}{14}$.	17. $\frac{71}{65}$.	27. $\frac{198}{71}$.	37. $\frac{396}{25}$.
8. $\frac{83}{18}$.	18. $\frac{129}{100}$.	28. $\frac{246}{82}$.	38. $\frac{551}{17}$.
9. $\frac{52}{13}$.	19. $\frac{354}{11}$.	29. $\frac{1597}{100}$.	39. $\frac{827}{20}$.
10. $\frac{31}{17}$.	20. $\frac{196}{13}$.	30. $\frac{129}{94}$.	40. $\frac{923}{36}$.

Reduction of a Whole or a Mixed Number to an Improper Fraction.

149. Reduce 9 to sevenths.

Since 1 unit contains 7 *sevenths* ($\frac{7}{7}$), 9 units contain 9 times 7 *sevenths* ($\frac{7}{7}$), or 63 *sevenths* ($\frac{63}{7}$). Hence,

150. To reduce a whole number to an improper fraction,

Multiply the whole number by the denominator of the required fraction, and under this product write the denominator.

EXERCISE 64. — ORAL.

Reduce to an improper fraction:

1. $3 = \frac{}{2}$.	5. $4 = \frac{}{7}$.	9. $8 = \frac{}{9}$.	13. $5 = \frac{}{10}$.
2. $4 = \frac{}{1}$.	6. $11 = \frac{}{1}$.	10. $13 = \frac{}{3}$.	14. $25 = \frac{}{4}$.
3. $12 = \frac{}{7}$.	7. $8 = \frac{}{6}$.	11. $9 = \frac{}{7}$.	15. $100 = \frac{}{4}$.
4. $8 = \frac{}{2}$.	8. $7 = \frac{}{8}$.	12. $8 = \frac{}{7}$.	16. $9 = \frac{}{9}$.

17. How many halves are there in \$1? \$2? \$5? \$4? \$3? \$6?

18. How many thirds are there in 1? 2? 8? 3? 5? 10? 7? 6? 9?

19. How many fourths are there in 1? 3? 2? 6?

151. Reduce $3\frac{3}{4}$ to fourths.

Since 1 is equal to 4 *fourths* ($\frac{4}{4}$), 3 is equal to 3×4 *fourths* ($\frac{4}{4}$), or 12 *fourths* ($\frac{12}{4}$); and 12 *fourths* ($\frac{12}{4}$) and 3 *fourths* ($\frac{3}{4}$) are 15 *fourths* ($\frac{15}{4}$). Hence,

152. To reduce a mixed number to an improper fraction,

Multiply the whole number by the denominator of the fraction, and to the product add the numerator; under this sum write the denominator.

Exercise 65. — Oral.

Reduce to an improper fraction:

1. $2\frac{3}{5}$.	9. $5\frac{5}{6}$.	17. $16\frac{2}{3}$.	25. $62\frac{1}{2}$.
2. $4\frac{1}{7}$.	10. $6\frac{2}{3}$.	18. $5\frac{1}{2}$.	26. $87\frac{1}{2}$.
3. $3\frac{2}{9}$.	11. $12\frac{1}{2}$.	19. $13\frac{1}{3}$.	27. $6\frac{1}{4}$.
4. $1\frac{5}{11}$.	12. $10\frac{4}{5}$.	20. $15\frac{1}{4}$.	28. $18\frac{3}{4}$.
5. $5\frac{2}{3}$.	13. $7\frac{3}{10}$.	21. $11\frac{9}{10}$.	29. $30\frac{1}{3}$.
6. $3\frac{1}{9}$.	14. $2\frac{5}{11}$.	22. $2\frac{6}{25}$.	30. $31\frac{1}{4}$.
7. $8\frac{2}{3}$.	15. $9\frac{2}{5}$.	23. $8\frac{1}{3}$.	31. $1\frac{3}{100}$.
8. $6\frac{2}{7}$.	16. $8\frac{1}{2}$.	24. $37\frac{1}{2}$.	32. $2\frac{11}{100}$.

Exercise 66. — Written.

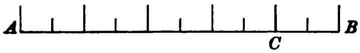
Reduce to an improper fraction:

1. $11\frac{5}{13}$.	9. $45\frac{10}{11}$.	17. $24\frac{1}{15}$.	25. $17\frac{11}{100}$.
2. $9\frac{13}{16}$.	10. $81\frac{5}{9}$.	18. $36\frac{3}{8}$.	26. $29\frac{53}{100}$.
3. $7\frac{11}{25}$.	11. $4\frac{17}{100}$.	19. $22\frac{7}{100}$.	27. $120\frac{5}{8}$.
4. $12\frac{3}{20}$.	12. $9\frac{9}{50}$.	20. $17\frac{8}{10}$.	28. $33\frac{5}{6}$.
5. $5\frac{16}{33}$.	13. $18\frac{7}{8}$.	21. $19\frac{3}{11}$.	29. $66\frac{1}{3}$.
6. $1\frac{87}{100}$.	14. $45\frac{3}{10}$.	22. $6\frac{41}{50}$.	30. $11\frac{29}{100}$.
7. $25\frac{1}{18}$.	15. $12\frac{5}{16}$.	23. $15\frac{6}{25}$.	31. $16\frac{4}{25}$.
8. $30\frac{12}{17}$.	16. $13\frac{7}{13}$.	24. $20\frac{6}{19}$.	32. $8\frac{5}{36}$.

33. To how many families can a man give $\frac{1}{8}$ of a ton of coal if he has $12\frac{1}{8}$ tons? $20\frac{3}{8}$ tons? $11\frac{7}{8}$ tons? $15\frac{1}{4}$ tons? $25\frac{5}{8}$ tons?

34. To how many boys can a man give $\frac{1}{10}$ of a dollar if he has $\$1\frac{1}{10}$? $\$2\frac{3}{10}$? $\$3\frac{7}{10}$? $\$5\frac{9}{10}$?

Reduction of Fractions to Lower Terms.



153. If AB is divided into 5 equal parts, how many of these parts are there in AC? What part of AB is AC?

If AB is divided into 10 equal parts, how many of these parts are there in AC? What part of AB is AC?

Which is the greater fraction, $\frac{8}{10}$ or $\frac{4}{5}$?

What must be done to $\frac{8}{10}$ to make $\frac{4}{5}$?

Which is greater, $\frac{1}{2}$ or $\frac{2}{4}$? $\frac{6}{10}$ or $\frac{3}{5}$? $\frac{15}{21}$ or $\frac{5}{7}$? $\frac{6}{20}$ or $\frac{3}{10}$?

Reduce to lowest terms, dividing the numerator and denominator of:

1. $\frac{5}{10}$ by 5.	4. $\frac{12}{30}$ by 6.	7. $\frac{18}{45}$ by 9.
2. $\frac{7}{21}$ by 7.	5. $\frac{15}{25}$ by 5.	8. $\frac{30}{100}$ by 10.
3. $\frac{11}{44}$ by 11.	6. $\frac{8}{32}$ by 8.	9. $\frac{36}{60}$ by 12.

154. To reduce a fraction to its lowest terms,

Cancel all the factors common to the numerator and denominator; or divide both terms by their G. C. M.

Reduce $\frac{70}{105}$ to its lowest terms.

$$\frac{\overset{5}{\cancel{70}}}{105} = \frac{\overset{7}{\cancel{14}}}{21} = \frac{2}{3}.$$

The common factors cancelled are 5 and 7.

Reduce $\frac{259}{333}$ to its lowest terms.

We first resolve 333 into its prime factors.

$$\begin{array}{r|l} 3 & 333 \\ \hline 3 & 111 \\ \hline & 37 \end{array}$$

Since the factor 3 of 333 will not exactly divide 259 we try 37, and find it is contained 7 times in 259.

Divide 259 and 333 each by 37, and we have $\frac{259}{333} = \frac{7}{9}$.

Exercise 67. — Oral.

Reduce to lowest terms:

1. $\frac{8}{12}$.	8. $\frac{5}{35}$.	15. $\frac{10}{15}$.	22. $\frac{24}{36}$.
2. $\frac{14}{16}$.	9. $\frac{11}{55}$.	16. $\frac{15}{20}$.	23. $\frac{8}{36}$.
3. $\frac{10}{50}$.	10. $\frac{16}{24}$.	17. $\frac{12}{16}$.	24. $\frac{25}{100}$.
4. $\frac{15}{18}$.	11. $\frac{21}{35}$.	18. $\frac{10}{12}$.	25. $\frac{50}{100}$.
5. $\frac{24}{32}$.	12. $\frac{13}{26}$.	19. $\frac{14}{28}$.	26. $\frac{75}{100}$.
6. $\frac{12}{36}$.	13. $\frac{4}{48}$.	20. $\frac{16}{40}$.	27. $\frac{30}{100}$.
7. $\frac{6}{42}$.	14. $\frac{9}{27}$.	21. $\frac{5}{25}$.	28. $\frac{45}{100}$.

Exercise 68. — Written.

Reduce to lowest terms:

1. $\frac{36}{144}$.	13. $\frac{135}{255}$.	25. $\frac{78}{416}$.	37. $\frac{216}{396}$.
2. $\frac{25}{125}$.	14. $\frac{47}{83}$.	26. $\frac{87}{290}$.	38. $\frac{225}{625}$.
3. $\frac{108}{180}$.	15. $\frac{63}{217}$.	27. $\frac{96}{480}$.	39. $\frac{289}{374}$.
4. $\frac{125}{375}$.	16. $\frac{65}{100}$.	28. $\frac{250}{1000}$.	40. $\frac{363}{660}$.
5. $\frac{120}{180}$.	17. $\frac{330}{660}$.	29. $\frac{465}{470}$.	41. $\frac{45}{540}$.
6. $\frac{144}{192}$.	18. $\frac{112}{128}$.	30. $\frac{550}{1000}$.	42. $\frac{196}{392}$.
7. $\frac{28}{154}$.	19. $\frac{128}{144}$.	31. $\frac{121}{253}$.	43. $\frac{625}{1000}$.
8. $\frac{48}{224}$.	20. $\frac{120}{600}$.	32. $\frac{132}{900}$.	44. $\frac{182}{455}$.
9. $\frac{84}{100}$.	21. $\frac{55}{121}$.	33. $\frac{150}{900}$.	45. $\frac{134}{230}$.
10. $\frac{28}{343}$.	22. $\frac{52}{143}$.	34. $\frac{105}{225}$.	46. $\frac{225}{1000}$.
11. $\frac{39}{156}$.	23. $\frac{66}{165}$.	35. $\frac{112}{480}$.	47. $\frac{825}{1000}$.
12. $\frac{42}{140}$.	24. $\frac{225}{450}$.	36. $\frac{187}{340}$.	48. $\frac{844}{1055}$.

Note. Pupils, unless otherwise directed, should reduce improper fractions to mixed numbers and proper fractions to their lowest terms in the answers.

Reduction of Fractions to Higher Terms.

155. How many quarters are there in $\$\frac{1}{2}$? Which is greater, $\frac{1}{2}$ of an apple or $\frac{2}{4}$ of an apple?

How many sixths of an orange can be made from $\frac{1}{2}$ of an orange? $\frac{1}{3}$? $\frac{2}{3}$?

Which is greater, $\frac{4}{5}$ or $\frac{8}{10}$? $\frac{3}{7}$ or $\frac{6}{14}$? $\frac{5}{8}$ or $\frac{15}{24}$? $\frac{7}{9}$ or $\frac{28}{36}$?

By what must you multiply both terms to change $\frac{5}{11}$ to $\frac{10}{22}$? $\frac{3}{5}$ to $\frac{15}{25}$? $\frac{5}{9}$ to $\frac{10}{18}$?

156. To reduce a fraction to higher terms,

Divide the required denominator by the denominator of the given fraction, and multiply both terms of the fraction by the quotient.

EXERCISE 69. — ORAL.

Reduce:

1. $\frac{3}{4}$ to 12ths.	5. $\frac{9}{13}$ to 26ths.	9. $\frac{3}{5}$ to 25ths.
2. $\frac{5}{6}$ to 24ths.	6. $\frac{4}{7}$ to 35ths.	10. $\frac{11}{8}$ to 32ds.
3. $\frac{5}{11}$ to 33ds.	7. $\frac{6}{25}$ to 100ths.	11. $\frac{3}{10}$ to 100ths.
4. $\frac{3}{20}$ to 100ths.	8. $\frac{9}{10}$ to 60ths.	12. $\frac{5}{9}$ to 36ths.

EXERCISE 70. — WRITTEN.

Reduce:

1. $\frac{5}{16}$ to 256ths.	10. $\frac{11}{19}$ to 399ths.
2. $\frac{3}{17}$ to 340ths.	11. $\frac{22}{23}$ to 460ths.
3. $\frac{11}{12}$ to 288ths.	12. $\frac{11}{14}$ to 168ths.
4. $\frac{9}{13}$ to 247ths.	13. $\frac{13}{15}$ to 315ths.
5. $\frac{7}{15}$ to 270ths.	14. $\frac{16}{17}$ to 187ths.
6. $\frac{17}{18}$ to 324ths.	15. $\frac{5}{9}$ to 279ths.
7. $\frac{7}{25}$ to 500ths.	16. $\frac{7}{11}$ to 220ths.
8. $\frac{9}{40}$ to 720ths.	17. $\frac{6}{7}$ to 196ths.
9. $\frac{8}{31}$ to 310ths.	18. $\frac{19}{20}$ to 500ths.

Multiplication of Fractions.

157. Compound Fractions. A fraction of a whole number, of a mixed number, or of a fraction, is called a *compound fraction.*

Thus, $\frac{3}{8}$ of 2, $\frac{3}{8}$ of $2\frac{1}{2}$, and $\frac{3}{8}$ of $\frac{1}{2}$ are compound fractions.

158. The expression $\frac{2}{3} \times \frac{1}{2}$ means the same as $\frac{2}{3}$ of $\frac{1}{2}$, and the sign $\times$ should be read *of* or *multiplied by* when it follows a fraction.

159. 7 times 3 horses are how many horses?
7 times 3 fifths ($\frac{3}{5}$) are how many fifths?
$\frac{3}{4}$ of 12 men are how many men?
$12 \times \$\frac{3}{4}$ are how many dollars?

160. To find the product of a whole number and a fraction,

Divide the product of the numerator and whole number by the denominator.

NOTE. A factor common to the whole number and denominator should be cancelled. Thus,

$$\frac{3}{\cancel{5}} \text{ of } \overset{8}{\cancel{40}} = 24. \qquad \frac{7}{\underset{3}{\cancel{9}}} \text{ of } \overset{4}{\cancel{12}} = \frac{28}{3} = 9\tfrac{1}{3}.$$

EXERCISE 71. — ORAL.

Find the product of:

1. $18 \times \frac{1}{3}$.
2. $25 \times \frac{1}{5}$.
3. $10 \times \frac{3}{5}$.
4. $24 \times \frac{2}{3}$.
5. $12 \times \frac{3}{4}$.
6. $14 \times \frac{2}{7}$.
7. $16 \times \frac{3}{8}$.
8. $30 \times \frac{1}{2}$.
9. $27 \times \frac{2}{9}$.
10. $20 \times \frac{3}{5}$.
11. $16 \times \frac{5}{4}$.
12. $10 \times \frac{9}{10}$.
13. $18 \times \frac{5}{6}$.
14. $12 \times \frac{3}{8}$.
15. $40 \times \frac{8}{5}$.
16. $32 \times \frac{1}{4}$.
17. $30 \times \frac{5}{6}$.
18. $45 \times \frac{3}{5}$.
19. $\frac{2}{9}$ of 45.
20. $\frac{5}{7}$ of 21.
21. $\frac{3}{10}$ of 100.
22. $\frac{5}{8}$ of 20.
23. $\frac{2}{13}$ of 26.
24. $\frac{3}{11}$ of 44.
25. $\frac{5}{12}$ of 36.
26. $\frac{9}{10}$ of 8.
27. $\frac{7}{5}$ of 15.
28. $\frac{7}{11} \times 22$.
29. $\frac{7}{8} \times 12$.
30. $\frac{5}{6} \times 15$.
31. $\frac{3}{5} \times 25$.
32. $\frac{11}{12} \times 24$.
33. $\frac{15}{16} \times 32$.
34. $\frac{11}{6} \times 9$.
35. $\frac{5}{11} \times 33$.
36. $\frac{4}{9} \times 24$.

EXERCISE 72. — WRITTEN.

Find the product of:

1. $48 \times \frac{11}{36}$.	12. $\frac{15}{16}$ of 512.	23. $\frac{8}{100} \times 1000$.
2. $72 \times \frac{17}{18}$.	13. $\frac{7}{8}$ of 144.	24. $\frac{9}{50} \times 210$.
3. $100 \times \frac{19}{40}$.	14. $\frac{17}{20}$ of 250.	25. $\frac{12}{25} \times 90$.
4. $132 \times \frac{11}{16}$.	15. $\frac{4}{25}$ of 210.	26. $\frac{5}{7} \times 434$.
5. $160 \times \frac{7}{100}$.	16. $\frac{3}{100}$ of 180.	27. $\frac{11}{9} \times 468$.
6. $256 \times \frac{13}{32}$.	17. $\frac{21}{25}$ of 90.	28. $\frac{12}{11} \times 30$.
7. $500 \times \frac{13}{40}$.	18. $\frac{5}{36}$ of 324.	29. $\frac{18}{15} \times 100$.
8. $125 \times \frac{17}{35}$.	19. $\frac{7}{13}$ of 273.	30. $\frac{25}{12} \times 54$.
9. $220 \times \frac{18}{55}$.	20. $\frac{10}{11}$ of 242.	31. $\frac{21}{32} \times 48$.
10. $147 \times \frac{8}{21}$.	21. $\frac{8}{17}$ of 340.	32. $\frac{19}{16} \times 72$.
11. $243 \times \frac{11}{12}$.	22. $\frac{7}{10}$ of 450.	33. $\frac{15}{32} \times 128$.

34. If an arithmetic costs $\$\frac{3}{4}$, what is the cost of 12 arithmetics? 14? 40?

35. If a loaf of bread weighs $\frac{5}{6}$ of a pound, what is the weight of 10 loaves? 15? 18? 21? 30?

36. A keg holds 24 quarts when full. How many quarts are there in it when it is $\frac{1}{2}$ full? $\frac{2}{3}$ full? $\frac{3}{4}$ full? $\frac{5}{6}$ full? $\frac{7}{8}$ full? $\frac{11}{12}$ full?

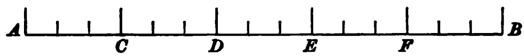
37. If a train travels 36 miles in an hour, how many miles will it travel in $\frac{5}{6}$ of an hour? $\frac{3}{4}$? $\frac{1}{8}$? $\frac{2}{3}$? $\frac{1}{7}$?

38. If a yard of cloth costs 48 cents, what is the cost of $\frac{1}{2}$ of a yard? $\frac{2}{3}$? $\frac{3}{8}$? $\frac{3}{4}$? $\frac{5}{6}$?

39. A company of soldiers contained 84 men. How many men were there in $\frac{7}{12}$ of the company? $\frac{11}{12}$? $\frac{9}{21}$? $\frac{5}{6}$? $\frac{3}{4}$? $\frac{23}{24}$? $\frac{9}{14}$? $\frac{11}{28}$? $\frac{6}{7}$?

161. To multiply a fraction by a fraction.

Multiply $\frac{4}{5}$ by $\frac{2}{3}$.



To multiply $\frac{4}{5}$ by $\frac{2}{3}$ means to find $\frac{2}{3}$ of $\frac{4}{5}$.

If the line AB is divided into 5 equal parts at the points C, D, E, and F, AF will be $\frac{4}{5}$ of AB.

If each part is subdivided into three equal parts, there will be 15 such parts in the whole line, and each part will be $\frac{1}{15}$ of the line.

That is, $\frac{1}{3}$ of $\frac{1}{5}$ is $\frac{1}{15}$ of the whole.

$\frac{1}{3}$ of $\frac{4}{5}$ is 4 times $\frac{1}{15}$, or $\frac{4}{15}$ of the whole. And $\frac{2}{3}$ of $\frac{4}{5}$ is twice $\frac{4}{15}$, or $\frac{8}{15}$ of the whole. Therefore,

162. To multiply a fraction by a fraction,

Find the product of the numerators for the required numerator, and of the denominators for the required denominator.

Mixed numbers and whole numbers are brought under the rule by first reducing them to improper fractions.

Any factor common to a numerator and a denominator should be cancelled before the multiplication.

Find the product of $\frac{13}{30} \times 2\frac{8}{13} \times \frac{10}{17}$.

Reducing the mixed number to an improper fraction, we have,

$$\frac{13}{30} \times \frac{34}{13} \times \frac{10}{17}.$$

By cancellation,

$$\frac{\cancel{13}}{\underset{3}{\cancel{30}}} \times \frac{\overset{2}{\cancel{34}}}{\cancel{13}} \times \frac{\overset{1}{\cancel{10}}}{\cancel{17}} = \frac{2}{3}.$$

EXERCISE 73. — ORAL.

Find the product of:

1. $\frac{1}{2} \times \frac{2}{3}$.	6. $\frac{3}{4}$ of $\frac{8}{9}$.	11. $\frac{3}{7} \times \frac{7}{9}$.	16. $\frac{1}{12} \times \frac{2}{5}$.
2. $\frac{1}{2} \times \frac{3}{5}$.	7. $\frac{1}{4} \times \frac{12}{15}$.	12. $\frac{5}{7}$ of $\frac{2}{5}$.	17. $\frac{5}{12} \times \frac{3}{10}$.
3. $\frac{1}{2} \times \frac{8}{11}$.	8. $\frac{2}{5}$ of $\frac{5}{8}$.	13. $\frac{1}{8}$ of $\frac{1}{2}$.	18. $\frac{7}{10} \times \frac{5}{8}$.
4. $\frac{1}{3} \times \frac{9}{10}$.	9. $\frac{3}{5} \times \frac{6}{7}$.	14. $\frac{3}{8} \times \frac{2}{8}$.	19. $\frac{9}{10} \times \frac{5}{3}$.
5. $\frac{2}{3} \times \frac{5}{7}$.	10. $\frac{4}{5} \times \frac{10}{11}$.	15. $\frac{6}{5} \times \frac{2}{3}$.	20. $\frac{3}{10} \times \frac{3}{8}$.

21. A man owning $\frac{3}{8}$ of a vessel sold $\frac{2}{3}$ of his share. What part of the vessel did he sell?

22. If a furnace burns $\frac{5}{8}$ of a ton of coal in a day, what part of a ton is consumed in $\frac{4}{5}$ of a day?

23. If a lot contains $\frac{7}{8}$ of an acre, what part of an acre is $\frac{3}{5}$ of the lot?

EXERCISE 74. — WRITTEN.

Find the product of:

1. $\frac{5}{7} \times \frac{14}{35}$.	11. $\frac{28}{100} \times \frac{75}{91}$.
2. $\frac{11}{12} \times \frac{20}{48}$.	12. $\frac{4}{25} \times 7\frac{1}{7} \times 6\frac{2}{3}$.
3. $2\frac{2}{3} \times \frac{3}{4} \times \frac{1}{5}$.	13. $12\frac{1}{2} \times \frac{8}{15} \times 16\frac{2}{3}$.
4. $3\frac{1}{5} \times \frac{7}{8} \times \frac{1}{12}$.	14. $37\frac{1}{2} \times \frac{12}{25} \times \frac{10}{11}$.
5. $\frac{17}{21} \times \frac{3}{85}$.	15. $\frac{5}{16} \times \frac{9}{20} \times \frac{4}{21} \times 2\frac{1}{3}$.
6. $\frac{9}{10} \times 8\frac{1}{3} \times \frac{1}{2}$.	16. $8\frac{1}{2} \times \frac{5}{12} \times 1\frac{1}{17}$.
7. $\frac{7}{8}$ of $\frac{8}{15}$ of $\frac{25}{2}$.	17. $62\frac{1}{2} \times \frac{9}{50} \times \frac{2}{3}$.
8. $\frac{5}{11}$ of $\frac{8}{9}$ of $4\frac{2}{7}$.	18. $\frac{8}{75} \times 87\frac{1}{2} \times \frac{3}{10}$.
9. $\frac{15}{13}$ of $5\frac{1}{5}$ of $\frac{3}{10}$.	19. $1\frac{1}{9} \times 1\frac{1}{10} \times \frac{36}{11}$.
10. $\frac{7}{100} \times \frac{25}{49}$.	20. $6\frac{3}{5} \times \frac{15}{22} \times \frac{8}{9}$.

163. To find the product of a mixed number and a whole number,

Find the product of the whole number and the fractional part of the mixed number, then the product of the whole number and the integral part, and add the products.

1. Multiply $7\frac{1}{8}$ by 9. 2. Multiply 8 by $2\frac{2}{3}$.

$$\begin{array}{r} (1) \\ 7\frac{1}{8} \\ \underline{9} \\ 64\frac{1}{8} \end{array} \qquad\qquad \begin{array}{r} (2) \\ 8 \\ \underline{2\frac{2}{3}} \\ 21\frac{1}{3} \end{array}$$

In (1), 9 times $\frac{1}{8}$ equals $\frac{9}{8}$, or $1\frac{1}{8}$, the $\frac{1}{8}$ is written, and the 1 is added to the product of 9×7, making 64.

In (2), $\frac{2}{3}$ of 8 equals $\frac{16}{3}$, or $5\frac{1}{3}$, the $\frac{1}{3}$ is written, and the 5 is added to the product of 2×8, making 21.

Exercise 75. — Written.

What is the cost of:

1. 14 cords of pine wood at $\$2\frac{1}{4}$ a cord?
2. 9 tons of coal at $\$5\frac{2}{3}$ a ton?
3. 5 yards of cloth at $12\frac{1}{2}$ cents a yard?
4. 6 pounds of beef at $16\frac{2}{3}$ cents a pound?
5. 24 pounds of sugar at $6\frac{3}{8}$ cents a pound?
6. 18 days' work at $\$1\frac{3}{4}$ per day?

Find the product of:

7.	$5 \times 8\frac{2}{7}$.	15.	$5\frac{3}{7} \times 9$.	23.	$36 \times 4\frac{1}{6}$.
8.	$8 \times 9\frac{2}{3}$.	16.	$8\frac{2}{11} \times 12$.	24.	$25 \times 3\frac{3}{8}$.
9.	$10 \times 7\frac{4}{5}$.	17.	$15\frac{3}{8} \times 7$.	25.	$12 \times 20\frac{2}{7}$.
10.	$12 \times 3\frac{1}{5}$.	18.	$20\frac{1}{4} \times 5$.	26.	$100 \times 3\frac{2}{9}$.
11.	$25 \times 2\frac{1}{6}$.	19.	$6\frac{2}{3} \times 18$.	27.	$100 \times 5\frac{3}{10}$.
12.	$100 \times 2\frac{1}{3}$.	20.	$8\frac{1}{2} \times 15$.	28.	$64 \times 8\frac{1}{7}$.
13.	$6 \times 5\frac{5}{12}$.	21.	$11\frac{1}{9} \times 11$.	29.	$22 \times 5\frac{8}{11}$.
14.	$14 \times 16\frac{2}{3}$.	22.	$12\frac{4}{5} \times 18$.	30.	$35 \times 8\frac{6}{7}$.

Division of Fractions.

164. Reciprocals. If the product of two numbers is 1, each of the numbers is called the *reciprocal* of the other.

Thus, $3 \times \frac{1}{3} = 1$; hence $\frac{1}{3}$ is the reciprocal of 3, and 3 is the reciprocal of $\frac{1}{3}$. The product of $\frac{2}{3} \times \frac{3}{2} = 1$; hence $\frac{3}{2}$ is the reciprocal of $\frac{2}{3}$, and $\frac{2}{3}$ is the reciprocal of $\frac{3}{2}$. The reciprocal of a whole number is 1 divided by the number, and the reciprocal of a fraction is the fraction inverted. In finding the reciprocal of a mixed number, first reduce the mixed number to an improper fraction.

EXERCISE 76. — ORAL.

Find the reciprocal of:

1. 2.	5. $\frac{1}{4}$.	9. $\frac{3}{5}$.	13. $2\frac{1}{3}$.
2. 5.	6. $\frac{1}{6}$.	10. $\frac{5}{7}$.	14. $3\frac{1}{5}$.
3. 7.	7. $\frac{1}{8}$.	11. $\frac{7}{9}$.	15. $5\frac{2}{7}$.
4. 9.	8. $\frac{1}{9}$.	12. $\frac{11}{12}$.	16. $6\frac{3}{4}$.

Reciprocal Relation of Division and Multiplication.

165. 1. Divide 12 by 3.

$12 \div 3 = 4.$

2. Multiply 12 by $\frac{1}{3}$.

$\frac{1}{3}$ of $12 = 4.$

3. Divide 12 by $\frac{2}{3}$.

$12 = \frac{36}{3}$

$\frac{36}{3} \div \frac{2}{3} = 36 \div 2 = 18.$

4. Multiply 12 by $\frac{3}{2}$.

$\frac{1}{2}$ of $12 = 6$

$\frac{3}{2}$ of $12 = 3 \times 6 = 18.$ Hence,

166. To divide by a whole number or by a fraction,
Multiply the dividend by the reciprocal of the divisor.

EXERCISE 77. — ORAL.

Find the quotient of:

1. $\frac{6}{7} \div 3$.	5. $\frac{12}{13} \div 6$.	9. $\frac{2}{3} \div 8$.	13. $\frac{11}{12} \div 9$.
2. $\frac{5}{9} \div 2$.	6. $\frac{15}{16} \div 3$.	10. $\frac{5}{6} \div 10$.	14. $\frac{3}{16} \div 4$.
3. $\frac{7}{10} \div 7$.	7. $\frac{5}{11} \div 6$.	11. $\frac{25}{27} \div 5$.	15. $\frac{5}{8} \div 12$.
4. $\frac{8}{11} \div 4$.	8. $\frac{7}{9} \div 5$.	12. $\frac{32}{39} \div 8$.	16. $\frac{3}{13} \div 6$.

17. A field containing $\frac{5}{8}$ of an acre is divided into 6 equal lots. What part of an acre is there in each lot?

18. If 5 men can build $\frac{10}{11}$ of a fence in a day, what part of the fence can 1 man build in a day?

19. A shipwrecked crew of 12 had $\frac{1}{2}$ of a barrel of biscuit. What part of a barrel will each man have after an equal division of the biscuit?

20. 8 bottles of the same size together contain $\frac{3}{4}$ of a gallon. What part of a gallon does each bottle hold?

21. 20 pencils cost $\$\frac{2}{5}$. What part of a dollar is the price of 1 pencil?

Exercise 78. — Written.

Find the quotient of:

1. $8 \div \frac{2}{3}$.	**11.** $6\frac{1}{2} \div 2\frac{1}{6}$.	**21.** $100 \div 62\frac{1}{2}$.
2. $12 \div \frac{3}{5}$.	**12.** $\frac{18}{25} \div \frac{5}{9}$.	**22.** $7\frac{1}{7} \div 6\frac{1}{4}$.
3. $\frac{5}{6} \div 1\frac{2}{3}$.	**13.** $100 \div 37\frac{1}{2}$.	**23.** $\frac{1}{11} \div \frac{1}{9}$.
4. $3\frac{1}{2} \div 2\frac{1}{3}$.	**14.** $4\frac{2}{3} \div \frac{7}{8}$.	**24.** $\frac{3}{7} \div \frac{7}{9}$.
5. $2\frac{1}{2} \div \frac{3}{4}$.	**15.** $3\frac{1}{6} \div 9\frac{1}{2}$.	**25.** $100 \div 11\frac{1}{9}$.
6. $\frac{5}{11} \div 2\frac{1}{2}$.	**16.** $8\frac{3}{5} \div 6\frac{1}{7}$.	**26.** $32 \div 6\frac{2}{5}$.
7. $\frac{8}{25} \div \frac{16}{5}$.	**17.** $8\frac{4}{5} \div 1\frac{1}{10}$.	**27.** $3\frac{3}{7} \div 3\frac{1}{7}$.
8. $100 \div 3\frac{1}{3}$.	**18.** $100 \div 6\frac{2}{3}$.	**28.** $\frac{11}{25} \div \frac{22}{15}$.
9. $100 \div 12\frac{1}{2}$.	**19.** $\frac{14}{15} \div \frac{12}{25}$.	**29.** $100 \div 33\frac{1}{3}$.
10. $100 \div 16\frac{2}{3}$.	**20.** $3\frac{1}{8} \div 5$.	**30.** $11\frac{1}{5} \div \frac{8}{9}$.

A Short Method of Dividing a Mixed Number by a Whole Number.

167. 1. Divide $16\frac{4}{5}$ by 4. 2. Divide $16\frac{2}{3}$ by 7.

$$\begin{array}{r} 4\underline{)16\frac{4}{5}} \\ 4\frac{1}{5} \end{array} \qquad\qquad \begin{array}{r} 7\underline{)16\frac{2}{3}} \\ 2\frac{8}{21} \end{array}$$

In the first problem we simply divide the whole number 16 by 4, and then the fraction $\frac{4}{5}$ by 4, and obtain the result at once, $4\frac{1}{5}$.

In the second problem we divide the 16 by 7, and obtain the quotient 2 and a remainder 2. The remainder 2 is joined with the $\frac{2}{3}$, making $2\frac{2}{3} = \frac{8}{3}$, and $\frac{8}{3} \div 7 = \frac{8}{21}$.

This method is the shortest method of dividing a mixed number by a whole number.

EXERCISE 79. — WRITTEN.

Find the quotient of:

1. $12\frac{1}{2} \div 2$.
2. $18\frac{2}{5} \div 3$.
3. $21\frac{1}{4} \div 7$.
4. $8\frac{1}{3} \div 5$.
5. $17\frac{1}{2} \div 4$.
6. $16\frac{2}{3} \div 10$.
7. $32\frac{4}{5} \div 8$.
8. $27\frac{6}{7} \div 9$.
9. $19\frac{5}{7} \div 6$.
10. $12\frac{3}{5} \div 9$.
11. $48\frac{24}{25} \div 12$.
12. $24\frac{4}{5} \div 11$.
13. $19\frac{3}{5} \div 7$.
14. $11\frac{1}{4} \div 8$.
15. $29\frac{1}{5} \div 5$.

168. Find the value of:

1. $(2\frac{1}{2} \div \frac{5}{6}) \times \frac{3}{8}$.

$$(2\frac{1}{2} \div \frac{5}{6}) \times \frac{3}{8}$$

$$= \frac{\cancel{5}}{\cancel{2}} \times \frac{\overset{3}{\cancel{6}}}{\cancel{5}} \times \frac{3}{8} = \frac{9}{8} = 1\frac{1}{8}.$$

2. $2\frac{1}{2} \div \frac{5}{6}$ of $\frac{3}{8}$.

$$2\frac{1}{2} \div \frac{5}{6} \text{ of } \frac{3}{8}$$

$$= \frac{\cancel{5}}{\cancel{2}} \times \frac{\overset{\cancel{2}}{\cancel{6}}}{\cancel{5}} \times \frac{8}{\cancel{3}} = 8.$$

In the first problem we have to divide $2\frac{1}{2}$ by $\frac{5}{6}$, and to multiply the result by $\frac{3}{8}$. But $2\frac{1}{2}$ is divided by $\frac{5}{6}$ by multiplying $2\frac{1}{2}$ by $\frac{6}{5}$. Hence we invert the divisor $\frac{5}{6}$ and find the product of the three fractions.

In the second problem we divide by $\frac{5}{6}$ of $\frac{3}{8}$; hence we may find the product of $\frac{5}{6}$ and $\frac{3}{8}$, and invert this product, or we may invert both factors of this product and multiply by the inverted factors $\frac{6}{5}$ and $\frac{8}{3}$.

EXERCISE 80. — WRITTEN.

Find the value of:

1. $\frac{2}{5}$ of $2\frac{1}{2} \div 5\frac{1}{3}$.
2. $\frac{3}{7} \div \frac{3}{10}$ of $\frac{5}{14}$.
3. $(\frac{3}{7} \div \frac{3}{10}) \times \frac{5}{14}$.
4. $(2\frac{5}{8} \div 2\frac{1}{3}) \times \frac{4}{5}$.
5. $\frac{2}{3}$ of $4\frac{1}{2} \div \frac{7}{9}$ of $3\frac{3}{5}$.
6. $2\frac{1}{5}$ of $2\frac{1}{2} \div \frac{3}{14}$ of $3\frac{2}{3}$.
7. $\frac{3}{8}$ of $\frac{5}{7}$ of $1\frac{4}{15} \div 6$.
8. $\frac{5}{9}$ of $3\frac{1}{3}$ of $\frac{6}{25} \div 5\frac{1}{2}$.
9. $\frac{3}{10} \div \frac{2}{5}$ of $2\frac{1}{4}$ of $1\frac{5}{7}$.
10. $\frac{3}{10} \div (\frac{2}{5} \times 2\frac{1}{4} \times 1\frac{5}{7})$.

169. To find the whole when a fractional part is given.

If $\frac{3}{4}$ of a barrel of flour costs \$3, what is the cost of $\frac{1}{4}$ of a barrel? of a barrel?

SOLUTION. If $\frac{3}{4}$ of a barrel of flour costs \$3, $\frac{1}{4}$ of a barrel will cost $\frac{1}{3}$ of \$3, or \$1. If $\frac{1}{4}$ of a barrel costs \$1, a barrel will cost $4 \times$ \$1, or \$4.

170. To find the whole when a fractional part is given,

Divide the given part by the numerator of the fraction, and multiply the quotient by the denominator.

EXERCISE 81. — ORAL.

1. If $\frac{5}{6}$ of a ton of hay costs \$15, what is the cost of one ton?

2. 15 is $\frac{5}{6}$ of what number? is $\frac{3}{7}$ of what number?

3. $8\frac{2}{5}$ is $\frac{7}{9}$ of what number? is $\frac{6}{7}$ of what number?

4. If $\frac{1}{12}$ of a dozen of eggs costs 22 cents, what is the cost of 1 dozen? of $3\frac{1}{2}$ dozen?

5. If $\frac{6}{13}$ of the scholars in a school are boys, and if there are 18 boys, how many scholars are there in the school? how many girls?

6. If $\frac{3}{5}$ of a yard of linen costs 60 cents, what is the cost of 1 yard? of $1\frac{1}{4}$ yards?

7. If $\frac{5}{8}$ of a bushel of corn costs 25 cents, what is the cost of $1\frac{1}{4}$ bushels?

8. If $\frac{4}{9}$ of an acre of land is worth \$32, find the cost of an acre.

9. If $\frac{7}{8}$ of a bushel of wheat is worth 56 cents, find the cost of $2\frac{1}{2}$ bushels.

10. If $\frac{3}{4}$ of a yard of silk is worth \$1, find the cost of 3 yards.

11. If $\frac{9}{10}$ of a ton of hay is worth \$18, find the cost of $3\frac{1}{2}$ tons.

Exercise 82. — Written.

1. $\frac{15}{16}$ of the goods in a store were sold for \$300. What was the value of the whole stock of goods?

2. $\frac{2}{11}$ of a ship's cargo was spoiled by water. If the loss was \$500, what was the value of the cargo?

3. If $\frac{5}{8}$ of an acre of land can be bought for \$40, how many acres can be bought for \$704?

4. If a man who owned $\frac{1}{8}$ of a schooner sold $\frac{3}{4}$ of his share for \$450, what was the value of the schooner?

5. If $\frac{7}{10}$ of a barrel of sugar costs $\$8\frac{1}{6}$, how many barrels can be bought for \$70?

6. $32\frac{1}{2}$ is $\frac{13}{17}$ of what number?

7. $11\frac{1}{5}$ is $\frac{14}{25}$ of what number?

8. $1\frac{22}{35}$ is $\frac{19}{23}$ of what number?

9. A man sold 63 acres of land, which was $\frac{7}{11}$ of his farm. How many acres were there in his farm?

10. If $\frac{5}{6}$ of a roll of carpeting is worth \$80.40, what is the whole roll worth?

11. A man sold $3\frac{1}{3}$ yards of cloth, which was $\frac{2}{15}$ of the whole piece. How many yards were there in the piece?

12. A farmer sold $\frac{3}{7}$ of his apples for \$141. At the same price, what are the rest of his apples worth?

13. \$145 is $\frac{1}{4}$ more than (that is, $\frac{5}{4}$ of) what sum of money?

14. \$144 is $\frac{1}{4}$ less than what sum of money?

15. 297 is $\frac{1}{8}$ more than what number?

16. 287 is $\frac{1}{8}$ less than what number?

17. 297 is $\frac{2}{13}$ less than what number?

18. 300 is $\frac{2}{13}$ more than what number?

19. If $\frac{5}{6}$ of a cord of wood is worth \$5, find the cost of $7\frac{1}{2}$ cords.

171. Complex Fractions. A fraction that has a fraction in one or both of its terms is called a *complex fraction.*

Thus, $\frac{\frac{3}{4}}{7}$, $\frac{\frac{3}{4}}{\frac{5}{6}}$, $\frac{3\frac{1}{2}}{4\frac{2}{3}}$, $\frac{20}{5\frac{2}{7}}$ are complex fractions.

A complex fraction simply indicates the division of its numerator by its denominator. Therefore,

172. To change a complex fraction to a simple fraction,

Multiply its numerator by the reciprocal of its denominator.

1. Change $\frac{\frac{3}{4}}{\frac{5}{6}}$ to a simple fraction.

$$\frac{3}{\underset{2}{\cancel{4}}} \times \frac{\overset{3}{\cancel{6}}}{5} = \frac{9}{10}.$$

2. Change $\frac{8\frac{3}{4}}{12\frac{5}{6}}$ to a simple fraction.

$$8\tfrac{3}{4} \times \frac{1}{12\frac{5}{6}} = \frac{\overset{5}{\cancel{35}}}{\underset{2}{\cancel{4}}} \times \frac{\overset{3}{\cancel{6}}}{\underset{11}{\cancel{77}}} = \frac{15}{22}.$$

EXERCISE 83. — WRITTEN.

Reduce to the simplest form:

1. $\frac{\frac{3}{4}}{5}$.	7. $\frac{8\frac{1}{3}}{10}$.	13. $\frac{11\frac{1}{9}}{100}$.	19. $\frac{66\frac{2}{3}}{100}$.
2. $\frac{\frac{7}{9}}{14}$.	8. $\frac{1\frac{5}{16}}{1\frac{7}{8}}$.	14. $\frac{16\frac{2}{3}}{100}$.	20. $\frac{37\frac{1}{2}}{100}$.
3. $\frac{\frac{10}{11}}{\frac{3}{5}}$.	9. $\frac{7\frac{1}{7}}{100}$.	15. $\frac{12\frac{1}{2}}{100}$.	21. $\frac{\frac{1}{8}}{2\frac{2}{3}}$.
4. $\frac{\frac{9}{13}}{6\frac{1}{2}}$.	10. $\frac{6\frac{2}{3}}{100}$.	16. $\frac{22\frac{2}{9}}{100}$.	22. $\frac{5\frac{1}{5}}{\frac{3}{7} \text{ of } 1\frac{3}{5}}$.
5. $\frac{\frac{14}{15}}{4\frac{2}{3}}$.	11. $\frac{33\frac{1}{3}}{100}$.	17. $\frac{6\frac{1}{4}}{100}$.	23. $\frac{2\frac{2}{9} \times \frac{4}{5}}{1\frac{1}{2} \text{ of } \frac{9}{22}}$.
6. $\frac{7\frac{1}{2}}{2\frac{3}{5}}$.	12. $\frac{62\frac{1}{2}}{100}$.	18. $\frac{87\frac{1}{2}}{100}$.	24. $\frac{3\frac{3}{7} \times 1\frac{7}{8}}{\frac{9}{11} \text{ of } 8\frac{1}{4}}$.

173. To find the fraction that one number is of another.

1. What fraction of 8 is 5?

Since $$1 = \frac{1}{8} \text{ of } 8,$$

$$5 = 5 \times \frac{1}{8} \text{ of } 8, \text{ or } \frac{5}{8} \text{ of } 8.$$

Here the number denoting the part is the numerator and the number denoting the whole is the denominator of the required fraction.

2. What fraction of $\frac{3}{2}$ is $\frac{3}{7}$?

Taking the number denoting the part for the numerator, and the number denoting the whole for the denominator, we have

$$\frac{\frac{3}{7}}{\frac{3}{2}}; \text{ and this becomes } \frac{\cancel{3}}{7} \times \frac{2}{\cancel{3}} = \frac{2}{7}.$$

174. To find the fraction that one number is of another,

Take the number denoting the part for the numerator, and the number denoting the whole for the denominator.

EXERCISE 84. — WRITTEN.

What fraction of:

1. 12 is 4?	7. $2\frac{1}{2}$ is $\frac{1}{2}$?	13. $11\frac{1}{4}$ is $\frac{9}{10}$?
2. 15 is 10?	8. $5\frac{1}{4}$ is 7?	14. $\frac{9}{10}$ is $11\frac{1}{4}$?
3. 10 is 15?	9. 12 is $3\frac{3}{7}$?	15. 100 is $12\frac{1}{2}$?
4. 36 is 16?	10. 16 is $2\frac{2}{3}$?	16. $5\frac{1}{11}$ is $1\frac{1}{7}$?
5. 50 is 4?	11. $6\frac{2}{5}$ is 24?	17. $3\frac{3}{8}$ is $6\frac{3}{4}$?
6. 15 is 25?	12. $6\frac{1}{4}$ is 5?	18. $13\frac{1}{3}$ is $8\frac{4}{7}$?

19. What part of a dollar is 25 cents? 50 cents? 84 cents? $14\frac{2}{7}$ cents? $28\frac{4}{7}$ cents? 75 cents? $66\frac{2}{3}$ cents? $87\frac{1}{2}$ cents? $33\frac{1}{3}$ cents? $16\frac{2}{3}$ cents? $12\frac{1}{2}$ cents? $6\frac{1}{4}$ cents? $8\frac{1}{3}$ cents? $83\frac{1}{3}$ cents? $37\frac{1}{2}$ cents? $62\frac{1}{2}$ cents? $22\frac{2}{9}$ cents? $44\frac{4}{9}$ cents? $18\frac{2}{11}$ cents? $7\frac{9}{13}$ cents?

EXERCISE 85. — WRITTEN.

What is the cost of:

1. $7\frac{1}{4}$ pounds of beef at $12\frac{1}{2}$ cents a pound?

2. $6\frac{2}{3}$ yards of silk at $\$1\frac{3}{4}$ a yard?

3. $15\frac{1}{2}$ yards of calico at $9\frac{1}{4}$ cents a yard?

4. 44 sheep at $\$4\frac{2}{3}$ apiece?

5. 68 pounds of copper at $11\frac{1}{4}$ cents a pound?

6. $12\frac{3}{8}$ tons of hay at \$18 a ton?

7. $20\frac{7}{8}$ pounds of halibut at $15\frac{1}{2}$ cents a pound?

8. If a yard of silk costs $\$2\frac{4}{5}$, how many yards can be bought for $\$18\frac{9}{10}$?

9. If $13\frac{1}{2}$ pounds of mutton cost $\$1\frac{1}{16}$, what is the cost of one pound?

10. A boy rides $21\frac{1}{4}$ miles on a bicycle in $2\frac{2}{3}$ hours. How many miles does he ride in an hour?

11. How many miles can a boy ride in $3\frac{2}{3}$ hours, if he rides $8\frac{4}{5}$ miles an hour?

12. If a painter paints $\frac{3}{8}$ of a fence in a day, what part can he paint in $\frac{2}{3}$ of a day?

13. 12 ducks were sold for $\$10\frac{1}{2}$. What was the price of 1 duck?

14. A man bought $\frac{5}{16}$ of a schooner and gave his purchase to 10 men in equal shares. What part of the schooner did each receive?

15. What is the cost of $8\frac{7}{10}$ thousand feet of gas at $\$1\frac{1}{4}$ a thousand?

16. To how many children can a man give $\frac{1}{4}$ of a pound of candy, if he has $13\frac{1}{2}$ pounds?

17. If a building is worth \$6000, what should a man receive for $\frac{2}{3}$ of his share if he owns $\frac{3}{4}$ of the building?

18. What will a ticket for a hundred miles cost at $1\frac{3}{5}$ cents a mile?

19. If it takes $2\frac{3}{4}$ yards of cloth to make a boy's suit, how many suits can be made from $24\frac{3}{4}$ yards?

20. If $5\frac{3}{4}$ pounds of sugar cost $34\frac{1}{2}$ cents, what is the cost of $\frac{1}{4}$ of a pound? of a pound?

21. If a man earns $\$1\frac{3}{8}$ a day, how many days will it take him to earn $\$12\frac{1}{2}$?

22. If a man earns $\$2\frac{1}{5}$ a day, how much will he earn in a month of 26 days?

23. At $\$4\frac{1}{2}$ a ton, how many tons of coal can be bought for $\$20\frac{1}{4}$?

24. A boy had $16\frac{1}{2}$ quarts of berries, and sold $\frac{7}{11}$ of them. How many quarts had he left?

25. How many bottles holding $1\frac{1}{6}$ quarts can be filled from a pail containing $9\frac{1}{3}$ quarts?

26. How many miles will a freight train travel in $3\frac{4}{5}$ hours, if it travels $22\frac{1}{2}$ miles an hour?

27. If 7 men shingle $\frac{7}{13}$ of a roof in an hour, what part of the roof will one man shingle in an hour?

28. If a man can dig a well in 7 days, what part can he dig in 1 day? $2\frac{1}{2}$ days? $3\frac{1}{2}$ days? $4\frac{2}{3}$ days?

29. If a man can mow a field in $2\frac{1}{5}$ days, what part can he mow in 1 day? $1\frac{2}{3}$ days?

30. If a man can dig a trench in $18\frac{1}{3}$ days, what part of the trench can 12 men dig in 1 day?

31. If a barrel of oil contains 42 gallons, how many gallons are there in $2\frac{5}{6}$ barrels? What is the cost at $9\frac{1}{4}$ cents a gallon?

32. Philip was given \$10 and spent \$$3\frac{3}{4}$. What part of his money did he spend?

33. From a barrel containing $22\frac{1}{2}$ gallons of oil $1\frac{7}{8}$ gallons leaked out. What part of the oil leaked out?

34. If a block of pine weighs $3\frac{1}{8}$ pounds, and a block of stone of the same size weighs $22\frac{1}{2}$ pounds, what part of the weight of the stone is the weight of the pine?

35. If a bottle holds $3\frac{3}{7}$ ounces of oil and $4\frac{1}{5}$ ounces of water, what part of the weight of the water is the weight of the oil?

36. If a suit of clothes costs \$$16\frac{1}{4}$ and the cost of the coat is \$$8\frac{3}{4}$, what part of the cost of the suit is the cost of the coat?

37. If a mass of lead and zinc weighing $16\frac{1}{5}$ pounds contains $9\frac{9}{10}$ pounds of lead, what part of the mass is lead?

38. If a piece of bronze weighing $7\frac{3}{5}$ pounds contains $6\frac{1}{2}$ pounds of copper, what part of the bronze is copper?

39. If $5\frac{1}{4}$ francs are equal to \$1, what part of a dollar would be returned if \$2 were given to pay a bill of 8 francs?

40. A man had \$$56\frac{1}{4}$ and spent \$9 for a railway ticket. What part of his money did he spend?

41. A man owned a field containing $19\frac{5}{8}$ acres. He sold $18\frac{1}{12}$ acres, reserving for himself enough for a house-lot and garden. What part of his land did he sell?

Similar Fractions.

175. Similar Fractions. Similar fractions are fractions that have a common denominator.

176. Change $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ to similar fractions.

The least common multiple of the denominators 2, 3, 4 is 12. Divide 12 by 2, the denominator of the given fraction $\frac{1}{2}$, and multiply both terms of $\frac{1}{2}$ by the quotient 6. Proceed in the same way with each of the other given fractions, and we obtain $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{12}$. Hence,

177. To change fractions to similar fractions,

Divide the least common multiple of the denominators by the denominator of the first fraction, and multiply both terms of this fraction by the quotient; and proceed in the same way with each of the other given fractions.

The least common multiple of the denominators is the *Least Common Denominator* (L. C. D.).

EXERCISE 86. — ORAL.

Change to similar fractions:

1. $\frac{1}{2}$, $\frac{1}{3}$.	6. $\frac{2}{3}$, $\frac{5}{6}$.	11. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$.	16. $\frac{7}{8}$, $\frac{5}{16}$, $\frac{3}{4}$.
2. $\frac{1}{3}$, $\frac{1}{4}$.	7. $\frac{1}{2}$, $\frac{2}{7}$.	12. $\frac{2}{3}$, $\frac{5}{6}$, $\frac{1}{12}$.	17. $\frac{1}{2}$, $\frac{3}{7}$, $\frac{9}{14}$.
3. $\frac{2}{3}$, $\frac{1}{5}$.	8. $\frac{4}{5}$, $\frac{7}{10}$.	13. $\frac{3}{4}$, $\frac{1}{6}$, $\frac{3}{8}$.	18. $\frac{2}{5}$, $\frac{1}{8}$, $\frac{1}{4}$.
4. $\frac{1}{2}$, $\frac{3}{5}$.	9. $\frac{3}{4}$, $\frac{3}{8}$.	14. $\frac{1}{3}$, $\frac{1}{6}$, $\frac{2}{9}$.	19. $\frac{1}{6}$, $\frac{5}{8}$, $\frac{11}{12}$.
5. $\frac{1}{4}$, $\frac{1}{5}$.	10. $\frac{2}{3}$, $\frac{5}{8}$.	15. $\frac{2}{3}$, $\frac{2}{5}$, $\frac{3}{10}$.	20. $\frac{2}{3}$, $\frac{5}{9}$, $\frac{11}{18}$.

EXERCISE 87. — WRITTEN.

Change to similar fractions:

1. $\frac{5}{11}$, $\frac{3}{4}$, $\frac{7}{22}$.	11. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$.
2. $\frac{7}{12}$, $\frac{9}{16}$, $\frac{11}{24}$.	12. $\frac{2}{5}$, $\frac{13}{100}$, $\frac{19}{20}$, $\frac{2}{25}$.
3. $\frac{5}{7}$, $\frac{7}{8}$, $\frac{9}{28}$.	13. $\frac{4}{5}$, $\frac{7}{8}$, $\frac{17}{20}$, $\frac{9}{10}$.
4. $\frac{2}{3}$, $\frac{11}{14}$, $\frac{25}{42}$.	14. $\frac{8}{9}$, $\frac{11}{12}$, $\frac{1}{2}$, $\frac{1}{4}$.
5. $\frac{3}{4}$, $\frac{2}{5}$, $\frac{13}{15}$.	15. $\frac{3}{4}$, $\frac{5}{13}$, $\frac{25}{26}$, $\frac{1}{8}$.
6. $\frac{7}{9}$, $\frac{5}{6}$, $\frac{23}{27}$.	16. $\frac{4}{7}$, $\frac{3}{8}$, $\frac{2}{5}$, $\frac{41}{42}$.
7. $\frac{9}{10}$, $\frac{7}{8}$, $\frac{4}{25}$.	17. $\frac{7}{11}$, $\frac{2}{9}$, $\frac{1}{2}$, $\frac{5}{6}$.
8. $\frac{7}{15}$, $\frac{1}{3}$, $\frac{5}{12}$.	18. $\frac{8}{15}$, $\frac{7}{10}$, $\frac{5}{8}$, $\frac{6}{25}$.
9. $\frac{8}{17}$, $\frac{1}{4}$, $\frac{31}{34}$.	19. $\frac{3}{20}$, $\frac{9}{50}$, $\frac{11}{60}$, $\frac{19}{100}$.
10. $\frac{5}{13}$, $\frac{1}{6}$, $\frac{11}{26}$.	20. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$.

Addition of Fractions.

178. Add $\frac{7}{8}$, $\frac{5}{12}$, $\frac{8}{15}$.

$$\text{Denominators} \ldots\ldots \begin{cases} 8 = 2 \times 2 \times 2. \\ 12 = 2 \times 2 \times 3. \\ 15 = 3 \times 5. \end{cases}$$

Hence, L. C. D. $= 2^3 \times 3 \times 5 = 120$.

$$\text{Numerators} \ldots\ldots \begin{cases} 105 \\ 50 \\ \underline{64} \end{cases}$$

Sum of numerators $= 219$

Therefore, sum of fractions $= \frac{219}{120} = \frac{73}{40} = 1\frac{33}{40}$. *Ans.*

179. To add fractions,

Change the fractions to similar fractions, if they are not similar, and write the sum of their numerators over the common denominator.

Find the sum of $2\frac{3}{20}$, $1\frac{7}{15}$, $5\frac{11}{12}$.

L. C. D. of the fractions $= 2^2 \times 3 \times 5 = 60$.

$$\text{Numerators} \ldots\ldots \begin{cases} 9 \\ 28 \\ \underline{55} \end{cases}$$

Sum of numerators $= 92$

Sum of fractions $= \frac{92}{60} = \frac{23}{15} = 1\frac{8}{15}$

Sum of integers $= 2 + 1 + 5 = \underline{8}$

$9\frac{8}{15}$. *Ans.*

If any of the expressions are integers or mixed numbers,

Add together separately the fractions and the integers, and find the sum of the results.

Exercise 88. — Oral.

Find the sum of:

1. $\frac{4}{5}$, $\frac{3}{5}$.
2. $\frac{5}{8}$, $\frac{6}{8}$, $\frac{1}{8}$.
3. $\frac{4}{9}$, $\frac{5}{9}$, $\frac{8}{9}$.
4. $\frac{2}{7}$, $\frac{6}{7}$, $\frac{3}{7}$.
5. $\frac{9}{10}$, $\frac{5}{10}$, $\frac{7}{10}$.
6. $2\frac{3}{4}$, $5\frac{1}{4}$.
7. $8\frac{1}{6}$, $7\frac{5}{6}$.
8. $10\frac{3}{5}$, $6\frac{4}{5}$.
9. $12\frac{6}{8}$, $9\frac{5}{8}$.
10. $2\frac{5}{11}$, $7\frac{4}{11}$.
11. $\frac{1}{2}$, $\frac{2}{3}$.
12. $\frac{1}{2}$, $\frac{3}{4}$.
13. $\frac{2}{3}$, $\frac{3}{4}$.
14. $\frac{5}{6}$, $\frac{3}{4}$.
15. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{7}{8}$.
16. $8\frac{2}{5}$, $5\frac{7}{10}$.
17. $6\frac{5}{12}$, $9\frac{1}{6}$.
18. $11\frac{4}{9}$, $4\frac{2}{3}$.
19. $5\frac{2}{7}$, $3\frac{3}{14}$.
20. $1\frac{1}{10}$, $9\frac{2}{5}$.

Exercise 89. — Written.

Find the sum of :

1. $\frac{3}{4}$, $\frac{1}{2}$, $\frac{5}{8}$.	17. $\frac{7}{10}$, $\frac{3}{5}$, $\frac{13}{20}$, $2\frac{1}{4}$.
2. $\frac{5}{6}$, $\frac{11}{12}$, $\frac{1}{4}$.	18. $\frac{3}{8}$, $1\frac{1}{6}$, $\frac{17}{24}$, $2\frac{1}{12}$.
3. $\frac{7}{8}$, $\frac{4}{5}$, $\frac{3}{10}$.	19. $3\frac{7}{10}$, $1\frac{7}{15}$, $\frac{3}{20}$, $2\frac{1}{2}$.
4. $\frac{5}{6}$, $\frac{8}{9}$, $\frac{19}{27}$.	20. $\frac{15}{16}$, $\frac{11}{18}$, $1\frac{1}{8}$, $\frac{2}{9}$.
5. $\frac{13}{20}$, $\frac{2}{5}$, $\frac{5}{6}$.	21. $1\frac{1}{11}$, $2\frac{5}{6}$, $3\frac{1}{4}$, $\frac{15}{22}$.
6. $\frac{2}{11}$, $\frac{5}{6}$, $\frac{9}{22}$.	22. $2\frac{11}{14}$, $5\frac{6}{7}$, $\frac{7}{8}$, $1\frac{1}{4}$.
7. $\frac{8}{13}$, $\frac{1}{2}$, $\frac{1}{4}$.	23. $5\frac{3}{8}$, $\frac{8}{15}$, $\frac{9}{10}$, $12\frac{1}{4}$.
8. $\frac{14}{15}$, $\frac{7}{12}$, $\frac{4}{5}$.	24. $\frac{35}{36}$, $2\frac{1}{9}$, $7\frac{3}{4}$, $10\frac{1}{8}$.
9. $\frac{2}{9}$, $\frac{3}{16}$, $\frac{5}{12}$.	25. $\frac{17}{18}$, $11\frac{1}{16}$, $\frac{11}{12}$, $\frac{1}{4}$.
10. $\frac{11}{25}$, $\frac{9}{20}$, $\frac{17}{100}$.	26. $\frac{11}{14}$, $\frac{19}{21}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{6}{7}$.
11. $5\frac{8}{9}$, $11\frac{5}{6}$, $13\frac{1}{3}$.	27. $\frac{7}{15}$, $\frac{2}{35}$, $\frac{3}{7}$, $\frac{1}{2}$, $\frac{9}{14}$.
12. $3\frac{4}{15}$, $4\frac{13}{18}$, $8\frac{1}{6}$.	28. $28\frac{7}{9}$, $41\frac{1}{6}$, $53\frac{3}{8}$.
13. $9\frac{1}{6}$, $4\frac{3}{8}$, $8\frac{2}{9}$.	29. $47\frac{13}{15}$, $76\frac{9}{10}$, $29\frac{1}{6}$, $18\frac{3}{5}$.
14. $10\frac{2}{3}$, $18\frac{8}{13}$, $7\frac{1}{2}$.	30. $55\frac{5}{12}$, $27\frac{19}{36}$, $92\frac{8}{9}$, $45\frac{7}{8}$.
15. $24\frac{1}{8}$, $16\frac{7}{12}$, $13\frac{5}{16}$.	31. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{3}{8}$, $\frac{2}{9}$, $\frac{11}{12}$.
16. $33\frac{1}{3}$, $28\frac{2}{11}$, $15\frac{7}{9}$.	32. $\frac{4}{5}$, $\frac{7}{10}$, $\frac{13}{20}$, $\frac{18}{25}$, $\frac{49}{50}$, $\frac{53}{100}$.

Exercise 90. — Written.

1. A grain dealer put in his wagon $11\frac{7}{8}$ bushels of oats, $9\frac{1}{2}$ bushels of corn, and $14\frac{3}{4}$ bushels of barley. How many bushels of grain were in the wagon ?

2. Henry earned $\$1\frac{4}{5}$, John $\$5\frac{3}{10}$, Joseph $\$11\frac{1}{2}$, Thomas $\$8\frac{1}{4}$, and Charles $\$7\frac{9}{10}$. How much did they all earn ?

3. A gardener raised $28\frac{1}{3}$ bushels of onions, $40\frac{5}{12}$ bushels of carrots, and $85\frac{3}{8}$ bushels of potatoes. How many bushels of vegetables did he raise ?

4. Philip rode 4 hours on a bicycle, going $9\frac{2}{7}$ miles the first hour, $8\frac{9}{10}$ miles the second, $10\frac{2}{5}$ the third, and $7\frac{3}{4}$ the fourth. How many miles did he ride ?

Subtraction of Fractions.

180. From $\frac{11}{16}$ take $\frac{7}{12}$.

The L. C. D. of the fractions is 48.

$$\tfrac{11}{16} = \tfrac{33}{48};\quad \tfrac{7}{12} = \tfrac{28}{48};\quad \text{and } \tfrac{33}{48} - \tfrac{28}{48} = \tfrac{5}{48}.$$

181. To subtract one fraction from another,

Change the fractions to similar fractions, if they are not similar, and write the difference of their numerators over the common denominator.

182. 1. Subtract $5\frac{7}{8}$ from $15\frac{3}{4}$.

$$\begin{aligned} 15\tfrac{3}{4} &= 15\tfrac{6}{8} \\ 5\tfrac{7}{8} &= \underline{\ 5\tfrac{7}{8}} \\ & \quad\ 9\tfrac{7}{8} \end{aligned}$$

SOLUTION. Since we cannot take $\frac{7}{8}$ from $\frac{6}{8}$, we take 1 from 15, change it to $\frac{8}{8}$, and add it to $\frac{6}{8}$, making $\frac{14}{8}$. Then $\frac{7}{8}$ from $\frac{14}{8}$ leaves $\frac{7}{8}$, and 5 from 14 leaves 9.

2. From 9 take $2\frac{19}{24}$.

$$9 = 8\tfrac{24}{24},\ \text{and } 8\tfrac{24}{24} - 2\tfrac{19}{24} = 6\tfrac{5}{24}.$$

Another method. *Adding the same number to both the minuend and the subtrahend does not alter their difference.* Hence, we may add to the subtrahend such a fraction as will make it a whole number, provided we add the same fraction to the minuend. Thus,

$$1.\ \begin{aligned} 15\tfrac{3}{4} + \tfrac{1}{8} &= 15\tfrac{7}{8} \\ 5\tfrac{7}{8} + \tfrac{1}{8} &= \underline{\ 6\ \ } \\ & \quad 9\tfrac{7}{8} \end{aligned} \qquad\qquad 2.\ \begin{aligned} 9 \quad + \tfrac{5}{24} &= 9\tfrac{5}{24} \\ 2\tfrac{19}{24} + \tfrac{5}{24} &= \underline{3\ \ \ } \\ & \quad 6\tfrac{5}{24} \end{aligned}$$

EXERCISE 91. — ORAL.

Find the value of:

1. $\frac{5}{6} - \frac{1}{6}$.	9. $\frac{17}{18} - \frac{5}{9}$.	17. $\frac{7}{9} - \frac{7}{12}$.
2. $\frac{7}{8} - \frac{3}{8}$.	10. $\frac{27}{50} - \frac{9}{20}$.	18. $11 - \frac{4}{5}$.
3. $\frac{7}{10} - \frac{4}{10}$.	11. $\frac{33}{100} - \frac{4}{25}$.	19. $15 - \frac{5}{8}$.
4. $\frac{11}{12} - \frac{3}{4}$.	12. $\frac{47}{100} - \frac{16}{50}$.	20. $23 - \frac{11}{16}$.
5. $\frac{7}{12} - \frac{5}{24}$.	13. $\frac{79}{100} - \frac{3}{10}$.	21. $37 - \frac{21}{25}$.
6. $\frac{11}{12} - \frac{5}{16}$.	14. $\frac{63}{100} - \frac{1}{6}$.	22. $19 - \frac{13}{30}$.
7. $\frac{14}{15} - \frac{6}{7}$.	15. $\frac{15}{24} - \frac{5}{16}$.	23. $28 - \frac{46}{51}$.
8. $\frac{13}{16} - \frac{7}{12}$.	16. $\frac{19}{36} - \frac{1}{4}$.	24. $42 - \frac{29}{100}$.

Exercise 92. — Written.

Find the value of:

1. $41 - 10\frac{5}{11}$.
2. $65 - 26\frac{3}{13}$.
3. $53 - 48\frac{2}{7}$.
4. $27 - 26\frac{7}{10}$.
5. $48 - 21\frac{13}{20}$.
6. $100 - 20\frac{15}{16}$.
7. $25\frac{5}{8} - 16\frac{1}{2}$.
8. $37\frac{2}{15} - 11\frac{1}{5}$.
9. $24\frac{7}{9} - 18\frac{5}{6}$.
10. $33\frac{1}{3} - 16\frac{2}{3}$.
11. $37\frac{1}{2} - 8\frac{1}{3}$.
12. $62\frac{1}{2} - 16\frac{2}{3}$.
13. $11\frac{1}{9} - 6\frac{1}{4}$.
14. $77\frac{3}{10} - 42\frac{9}{20}$.
15. $65\frac{29}{100} - 57\frac{4}{25}$.
16. $13\frac{11}{18} - 8\frac{5}{24}$.
17. $32\frac{11}{12} - 17\frac{5}{16}$.
18. $9\frac{17}{30} - 8\frac{19}{20}$.
19. $17\frac{23}{36} - 16\frac{7}{8}$.
20. $45\frac{9}{11} - 18\frac{3}{7}$.
21. $54\frac{11}{13} - 29\frac{1}{6}$.
22. $2\frac{47}{100} - 1\frac{1}{50}$.
23. $15\frac{6}{25} - 7\frac{71}{100}$.
24. $62\frac{3}{8} - 16\frac{3}{5}$.
25. $\frac{5}{9} + \frac{2}{3} - \frac{1}{6}$.
26. $\frac{7}{8} + \frac{15}{16} - 1\frac{1}{2}$.
27. $1\frac{11}{12} - 1\frac{1}{8} + 2\frac{3}{4}$.
28. $4\frac{3}{10} - 3\frac{2}{15} + 2\frac{4}{5}$.
29. $12\frac{1}{11} + 8\frac{3}{4} - 15\frac{1}{2}$.
30. $8\frac{5}{7} - 2\frac{16}{21} + 3\frac{5}{14}$.
31. $\frac{49}{100} - \frac{17}{50} + \frac{23}{25}$.
32. $\frac{17}{20} - \frac{43}{100} + 1\frac{1}{2}$.
33. $\frac{29}{30} + 1\frac{1}{15} - \frac{9}{25}$.
34. $\frac{1}{2} + \frac{1}{11} + \frac{7}{22} - \frac{1}{4}$.
35. $\frac{5}{13} - \frac{3}{26} + 2\frac{1}{2} + \frac{3}{4}$.
36. $2\frac{4}{5} - 1\frac{1}{8} + 9\frac{9}{10} - 9\frac{7}{20}$.
37. $\frac{16}{25} - \frac{11}{20} + \frac{67}{100} - \frac{3}{8}$.
38. $6\frac{2}{3} - 6\frac{1}{2} + 11\frac{3}{12} - 10\frac{7}{8}$.
39. $\frac{11}{13} - \frac{1}{26} - \frac{1}{2} + \frac{1}{4}$.
40. $\frac{8}{17} + \frac{5}{34} + \frac{3}{4} - 1\frac{1}{8}$.
41. $\frac{18}{19} + \frac{2}{3} + \frac{1}{2} - 1\frac{1}{6}$.
42. $3\frac{4}{7} + 9\frac{1}{14} + 6\frac{2}{3} - 18$.
43. $25 - 1\frac{1}{8} - 3\frac{2}{7} - 5\frac{1}{2}$.
44. $19 - 7\frac{1}{8} - 5\frac{1}{10} - \frac{4}{5}$.
45. $100 - 20\frac{3}{5} - 16\frac{1}{12} - 25\frac{17}{20}$.
46. $29 - \frac{7}{100} - \frac{11}{20} - \frac{9}{50}$.
47. $42 - \frac{7}{15} - 1\frac{2}{3} - 8\frac{2}{5}$.
48. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} - \frac{1}{6} - \frac{1}{8} - \frac{1}{9}$.

Exercise 93. — Written.

1. Henry bought some collars for $\$\frac{1}{2}$, a pair of gloves for $\$1\frac{3}{4}$, and 4 shirts at $\$\frac{7}{8}$ apiece. What change should he receive for a \$10 bill?

2. What change should be returned when a \$2 bill is given in payment for 8 pounds of steak at \$0.15 a pound?

3. From a piece of cloth containing $42\frac{1}{2}$ yards a clerk sold $9\frac{1}{4}$ yards to A, $15\frac{3}{8}$ yards to B, and $11\frac{1}{2}$ yards to C. How many yards were left?

4. Mr. French has a farm containing $12\frac{5}{8}$ acres. How many acres will he have left after selling 14 lots, each containing $\frac{2}{3}$ of an acre?

5. A man spent $\frac{1}{3}$ of his income for rent, $\frac{1}{4}$ for food, and $\frac{1}{8}$ for other expenses. What part of his income remained?

6. A train from Boston reached Portland at quarter past 11 after traveling $3\frac{3}{4}$ hours. At what hour did the train leave Boston?

7. $\frac{1}{2}$ the length of a pole was broken off and $\frac{1}{8}$ of the length of the pole was then cut off. What part of the pole was left?

8. $\frac{1}{2}$ of a pole was broken off and $\frac{1}{8}$ of the *remainder* of the pole was cut off. What part of the pole was left?

9. Mr. A spent $\frac{3}{11}$ of his wages for food and $\frac{1}{2}$ the remainder for other expenses. What part of his wages did he save?

10. Mr. B started to walk $15\frac{7}{8}$ miles. After walking 3 hours at the rate of $3\frac{3}{8}$ miles per hour, how many miles of his journey remained?

11. A farmer sold $6\frac{1}{2}$ dozen eggs at $\$\frac{1}{8}$ a dozen. He then bought at the same store 10 yards of cloth at $\$\frac{1}{2}$

a yard. What change should he receive for a $10 bill?

12. A man invested $\frac{2}{9}$ of his capital in bonds, $\frac{3}{7}$ of the remainder in bank stock, and the rest, $8000, in real estate. Find his capital.

13. A man owning a vessel gave $\frac{3}{8}$ of it to his son and sold $\frac{1}{4}$ of the remainder for $2000. What was the value of the vessel?

14. How much would be gained if $6\frac{1}{4}$ acres of land were bought for $\$15\frac{1}{5}$ an acre and sold for $\$21\frac{3}{4}$ an acre?

15. How much would be gained or lost if a carriage bought for $\$80\frac{1}{2}$ were exchanged for 2 cows worth $\$26\frac{7}{8}$ apiece?

16. To what number must $8\frac{5}{11}$ be added to make $16\frac{2}{9}$?

17. From what number must $25\frac{3}{7}$ be subtracted to give a remainder of $9\frac{7}{10}$?

18. If $\frac{7}{12}$ of the weight of an ore is lost in smelting and $\frac{1}{5}$ of the remainder in refining, how many tons of ore will be required to make $7\frac{1}{2}$ tons of pure metal?

19. Two men are $45\frac{5}{16}$ miles apart. If they travel toward each other, one at the rate of $2\frac{3}{4}$ miles an hour, the other at the rate of $3\frac{1}{5}$ miles, how far apart will they be at the end of 7 hours?

20. Name the largest and the smallest of the fractions, $\frac{13}{15}$, $\frac{25}{27}$, $\frac{9}{10}$, and $\frac{2}{3}$.

21. If $4\frac{1}{5}$ marks can be bought for $1, what part of a dollar would a man receive in exchange for a $5 bill after paying a bill of $18\frac{9}{10}$ marks?

22. A man gave his eldest son $\frac{1}{3}$ of his property, $\frac{1}{3}$ of the remainder to a second son, and the rest to his daughter. What part of his property did the daughter receive?

Conversion of Fractions.

183. A decimal fraction may be changed to a common fraction.

Change 0.15 to a common fraction.

$$0.15 = \frac{15}{100} = \frac{3}{20}. \textit{ Ans.}$$

184. To change a decimal to a common fraction,

Write the figures of the decimal for the numerator; and 1 with as many zeros after it as there are figures after the decimal point for the denominator.

EXERCISE 94. — WRITTEN.

Change to a common fraction:

1. 0.5.	**6.** 0.0225.	**11.** 5.005.	**16.** 0.00375.
2. 0.05.	**7.** 0.0032.	**12.** 0.00625.	**17.** 54.85.
3. 0.25.	**8.** 2.128.	**13.** 0.1004.	**18.** 0.015625.
4. 0.325.	**9.** 11.06.	**14.** 24.163.	**19.** 242.88.
5. 0.008.	**10.** 6.125.	**15.** 20.0125.	**20.** 8.9625.

185. A common fraction may be changed to a decimal.

Change $\frac{3}{8}$ to a decimal.

$$8\overline{)3.000}$$
$$0.375$$

186. To change a common fraction to a decimal,

Divide the numerator by the denominator.

NOTE. If a fraction, when reduced to its lowest terms, contains in the denominator any other factor than 2 or 5 (the prime factors of 10), the division of the numerator by the denominator will not terminate. In general, it will be sufficient to obtain five decimal places in the quotient. But the number in the fifth place of the quotient should be increased by 1 if the number in the next place of the quotient would be five, or greater than five.

Exercise 95. — Written.

Change the following fractions to decimals:

1. $\frac{7}{16}$.	5. $16\frac{7}{64}$.	9. $13\frac{11}{32}$.	13. $192\frac{27}{50}$.
2. $\frac{12}{25}$.	6. $\frac{13}{5}$.	10. $7\frac{13}{250}$.	14. $10\frac{3}{25}$.
3. $\frac{19}{200}$.	7. $\frac{144}{125}$.	11. $\frac{499}{500}$.	15. $\frac{125}{800}$.
4. $\frac{17}{40}$.	8. $\frac{335}{8}$.	12. $\frac{37}{80}$.	16. $\frac{165}{125}$.

Express the following as decimals to five places:

17. $\frac{5}{6}$.	20. $\frac{5}{11}$.	23. $\frac{23}{30}$.	26. $\frac{109}{21}$.
18. $\frac{3}{14}$.	21. $1\frac{4}{9}$.	24. $12\frac{13}{75}$.	27. $55\frac{9}{26}$.
19. $\frac{2}{9}$.	22. $3\frac{7}{13}$.	25. $25\frac{10}{11}$.	28. $101\frac{1}{18}$.

Exercise 96. — Oral.

Review Problems.

1. $\frac{1}{4}$ of a stock of goods was spoiled by fire and $\frac{1}{8}$ by water. What part remained uninjured?

2. What part would have been uninjured if $\frac{1}{4}$ had been spoiled by fire and $\frac{1}{3}$ of the remainder by water?

3. How do you find $\frac{2}{5}$ of a number?

4. What is meant by $\frac{2}{5}$ of a unit?

5. Change to mixed numbers $\frac{22}{5}$, $\frac{17}{3}$, $\frac{29}{7}$, $\frac{35}{12}$, $\frac{45}{8}$, $\frac{59}{11}$.

6. What is the cost of 8 pounds of mutton at $12\frac{1}{2}$ cents a pound?

7. Change to improper fractions $8\frac{3}{4}$, $9\frac{4}{5}$, $8\frac{4}{9}$, $5\frac{6}{11}$.

8. From a farm of $19\frac{3}{8}$ acres $11\frac{1}{2}$ acres have been sold. How many acres are left?

9. At $\$\frac{2}{3}$ a bushel, how many bushels of salt can be bought for $8?

10. Reduce to lowest terms $\frac{15}{18}$, $\frac{28}{35}$, $\frac{22}{55}$, $\frac{17}{51}$, $\frac{26}{52}$, $\frac{18}{54}$, $\frac{12}{42}$.

11. A bottle holds 2 pounds of water, or 3 pounds of acid. What part of the weight of the acid is the weight of the water? How many quarts of water will weigh as much as 4 quarts of the acid?

12. If $\frac{3}{4}$ of a ton of coal costs $\$4\frac{1}{2}$, what is the cost of a ton? of $2\frac{1}{5}$ tons? of $3\frac{1}{2}$ tons? of $4\frac{1}{4}$ tons?

13. What part of $\frac{3}{8}$ is $\frac{2}{3}$? of $\frac{4}{5}$ is $\frac{3}{10}$? of $\frac{5}{6}$ is $\frac{7}{12}$?

14. 8 pints make a gallon. How many bottles containing $1\frac{1}{2}$ pints each can be filled from a keg of 3 gallons?

15. $2\frac{1}{3}$ days make what part of a week?

16. 9 is $\frac{3}{7}$ of what number?

17. What is the cost of board for a week at $\$1\frac{1}{4}$ a day?

18. A farmer gets each day $\frac{2}{5}$ as many eggs as he has hens. How many dozen will he get each day if he has 90 hens? 60 hens? 120 hens?

19. If $2\frac{3}{4}$ bushels of apples make a barrel, how many bushels will it take to fill 12 barrels?

20. How many barrels can be filled from a heap of apples containing $8\frac{1}{4}$ bushels?

21. If 6 yards of cloth cost $\$3\frac{3}{4}$, what is the cost of 1 yard? $\frac{1}{2}$ of a yard? 5 yards?

22. Four quarts make a gallon. If a grocer has $5\frac{1}{4}$ gallons of vinegar, how many quarts has he? How many will he have after selling $2\frac{1}{2}$ gallons?

23. Henry spilled $\frac{1}{5}$ of his berries and had 8 quarts left. How many had he at first?

24. A carpet dealer cut a roll of carpet containing 45 yards into 3 equal pieces. How many yards were there in each piece? How much should he charge for each piece at $\$1\frac{1}{5}$ a yard?

25. A farmer bought 12 sheep at $\$3\frac{1}{2}$ apiece. He sold 10 of them for enough to pay the cost of all. What did he receive for each sheep?

26. How many celery plants will be needed to plant a row $18\frac{1}{2}$ feet long, if the plants are half a foot apart?

27. How many bushels of potatoes can be dug from 9 rows, if the rows average $\frac{5}{12}$ of a bushel?

28. How many pints of peas will be needed to plant 10 rows, if it takes $\frac{3}{4}$ of a pint to plant 1 row?

29. How many cords of pine can be bought for $24 at $\$2\frac{1}{2}$ a cord?

30. If it takes $1\frac{1}{4}$ quarts of alcohol to weigh as much as 1 quart of water, how many quarts of alcohol will weigh as much as 6 quarts of water?

31. How many quarts of water will equal the weight of 10 quarts of alcohol?

32. If 1 boy can do $\frac{2}{3}$ as much as a man, how many boys will be needed to do the work of 14 men?

33. How many men will be needed to do the work of 15 boys?

34. How much milk will a farmer get from 12 cows in a day, if their average is $7\frac{3}{4}$ quarts a day?

35. How many quarter-pound packages of soda can be made from $11\frac{1}{2}$ pounds?

36. How many bushels of potatoes at $\$\frac{3}{8}$ a bushel can be bought for $18?

37. A pulp mill runs 9 hours a day. How many cords of spruce will it use in a day, if it uses $1\frac{1}{8}$ cords an hour?

38. If a packer received $1\frac{1}{2}$ cents for each dozen articles packed, how many dozen must he pack to earn 90 cents?

39. What is the car fare for 48 miles at $2\frac{3}{8}$ cents a mile?

40. A house destroyed by fire was insured for $\frac{7}{8}$ of its value. If the insurance company paid the owner $1400, what was its value?

41. A man having walked $\frac{3}{7}$ of the distance between two towns found that he had walked 15 miles. What was the distance between the towns?

42. James had $$\frac{3}{5}$, and his uncle gave him enough more to make $$1\frac{1}{2}$. How much did his uncle give him?

43. What number must be added to $1\frac{2}{3}$ to make $2\frac{1}{4}$?

44. Henry had $2\frac{1}{6}$ pounds of candy. After eating a part of it he had $1\frac{3}{4}$ pounds left. How much did he eat?

45. What number must be taken from $5\frac{3}{10}$ to leave $4\frac{4}{5}$?

46. If Philip picks $2\frac{2}{7}$ quarts of berries in an hour, how many hours will he take to pick 16 quarts?

47. By what number must $2\frac{3}{8}$ be multiplied to give $9\frac{1}{2}$?

48. If a boy rides $21\frac{6}{11}$ miles in 3 hours, how far does he ride in one hour?

49. By what number must $15\frac{10}{13}$ be divided to give 5 as a quotient?

50. If coal costs $5 a ton, what part of a ton can be bought for $$1\frac{1}{4}$? $2? $$2\frac{1}{2}$? $$4\frac{1}{2}$?

51. If $\frac{3}{8}$ of a ton of hay can be bought for $$7\frac{1}{2}$, what part of a ton can be bought for $15? $18? $$2\frac{1}{2}$?

52. If $\frac{3}{7}$ of the timber on a wood-lot was destroyed by fire, and $\frac{1}{2}$ the remainder sold, what part remained?

53. A can hoe a field in 6 hours and B in 5 hours. What part can each hoe in an hour? What part can both hoe in an hour?

54. A can cut a cord of wood in $7\frac{1}{2}$ hours. What part of a cord can he cut in an hour? in $2\frac{1}{2}$ hours?

55. C can pick a quart of berries in $\frac{2}{3}$ of an hour. How many quarts can he pick in an hour? $1\frac{1}{2}$ hours?

56. One pipe can fill a tank in 2 hours; a second can empty it in 5 hours. If both pipes are open, what part of the tank will be filled in 1 hour?

57. How many minutes will it take to empty a tank containing 36 gallons by a pipe carrying $2\frac{2}{5}$ gallons a minute?

58. A is 20 yards ahead of B. If A runs $6\frac{1}{3}$ yards a second and B 7 yards a second, in how many seconds will B overtake A?

59. How many seconds will it take a boy to run 40 yards if he runs $6\frac{2}{3}$ yards a second?

60. If a man runs 50 yards in $5\frac{1}{2}$ seconds, how many yards does he run in 1 second?

61. A field contains 8 acres. If $3\frac{1}{3}$ acres of the field have been plowed, what part of the field remains unplowed?

62. How much is gained by buying 2 dozen oranges at $1\frac{1}{2}$ cents apiece and selling the lot for 55 cents?

63. Mr. D sold a sleigh for $\frac{4}{5}$ of its cost and lost $9. What was the cost of the sleigh?

64. A house was sold for $\frac{7}{8}$ of its cost. If the selling price was $2100, what was the cost?

65. How much was lost by selling a house-lot costing $450 for $\frac{2}{5}$ of its cost?

66. A farmer gave $5\frac{3}{4}$ bushels of potatoes, worth 40 cents a bushel, for 23 yards of cotton cloth. What did the cloth cost him a yard?

67. How much greater than $\frac{2}{3}$ of $4\frac{1}{2}$ is $\frac{1}{6}$ of 24?

68. Mr. B sold 20 bushels of corn for $12. What was the price per bushel? If he had sold the corn for $\frac{5}{6}$ of its cost, what would have been the cost per bushel?

69. If grain costing 48 cents a bushel is sold for 60 cents a bushel, what part of the cost is gained?

Exercise 97. — Written.

Review Problems.

1. If a cup holds $\frac{1}{4}$ of a quart, how many cupfuls are there in $11\frac{7}{9}$ quarts?

2. How many house lots, each containing $\frac{1}{6}$ of an acre, can be made from $17\frac{2}{3}$ acres?

3. A real-estate broker bought 25 acres of land. He kept $\frac{3}{8}$ of it, used $\frac{1}{10}$ to make a street, and divided the rest into lots, each containing $\frac{5}{8}$ of an acre. How many lots were there?

4. A and B are painters working for the same wages. If A worked $4\frac{1}{9}$ days and B $5\frac{1}{8}$ days painting a house, what part of the money paid should each man receive?

5. If B's share was $\$14\frac{2}{3}$, what did he receive for a day's work?

6. A and B bought a boat, A paying 5 times as much as B; if the boat cost \$96, what sum did each man pay?

7. If A paid $2\frac{1}{2}$ times as much for the boat as B, and they agreed to use it in proportion to their payments, what part of the time is each to have the boat?

8. \$150 was paid for a horse and saddle. If the cost of the saddle was $\frac{3}{7}$ of the cost of the horse, what was the cost of each?

9. 150 is $\frac{10}{7}$ of what number?

10. 64 is $\frac{8}{5}$ of what number?

11. 64 is $\frac{2}{5}$ of what number?

12. A cattle dealer reduced the price of a pair of oxen $\frac{2}{9}$ and sold them for \$140. What was the original price?

13. What number diminished by $\frac{2}{9}$ of itself equals 63?

14. What number increased by $\frac{3}{11}$ of itself equals 104?

15. Mr. Smith bought a house, paying $\frac{1}{2}$ of the price in cash. One month later he paid $\frac{1}{5}$ of the price and then owed $3000. What was the price of the house?

16. What was the price if after paying $\frac{1}{2}$ of the price and $\frac{1}{5}$ of the *remainder* he still owed $3000?

17. If a pound of coffee worth $37\frac{1}{2}$ cents was mixed with a pound worth $44\frac{3}{8}$ cents, what was the value of a pound of the mixture?

18. By working $8\frac{1}{2}$ hours a day a carpenter built a fence in $5\frac{1}{5}$ days. How many hours a day must he work to build the fence in $4\frac{2}{3}$ days?

19. A box contains 345 eggs. What is their value at $16\frac{2}{3}$ cents a dozen?

20. A man bought $2\frac{1}{4}$ thousand feet of lumber at $30 a thousand, and gave in payment 40 barrels of apples worth $\$1\frac{3}{8}$ a barrel. How much did he then owe the lumber dealer?

21. A man bought 25 pounds of sugar at the rate of 16 pounds for $1, $3\frac{1}{2}$ pounds of crackers at 8 pounds for $1, 2 pounds of tea at $\$\frac{3}{4}$ a pound, and 18 cans of peas at 15 cents each. What change should he receive for a $10 bill?

22. If the circumference of a bicycle wheel is $7\frac{1}{3}$ feet, and the circumference of the wheel of another bicycle is $7\frac{6}{7}$ feet, how many more times will the smaller wheel turn than the larger in going 1 mile, or 5280 feet?

23. 9 men reap $\frac{3}{5}$ of a field of wheat in $1\frac{1}{5}$ days. How many days will it take one man to reap the whole field?

24. A coal dealer bought a cargo of coal for $\$3\frac{3}{4}$ a ton and paid $\$\frac{1}{2}$ a ton for freight charges. If there were 300 tons in the cargo, for what price per ton must he sell it to gain $175?

25. A milk dealer had in his wagon 26 cans, each containing $8\frac{1}{2}$ quarts. How much should he receive for it at 5 cents a quart?

26. How many preserve jars, each holding $\frac{7}{8}$ of a quart, can be filled from a keg of syrup containing $15\frac{3}{4}$ quarts?

27. A man can plaster a house in $11\frac{3}{8}$ days. What part of it can he plaster in $7\frac{9}{10}$ days?

28. A piece of oak weighs $5\frac{5}{12}$ pounds; a piece of iron of the same size weighs 45 pounds. Iron is how many times as heavy as oak?

29. A man paid $8.75 for the pasturage for 5 cows for $2\frac{1}{2}$ weeks. What was the charge per day for each cow?

30. A man lost $\$13\frac{1}{2}$, which was $\frac{3}{8}$ of what he had left after the loss. How many dollars had he before the loss?

31. Change $11\frac{3}{8}$, $19\frac{7}{20}$, $45\frac{11}{16}$ to decimals and find their sum.

32. A and B hire a pasture and pay in proportion to the number of horses each puts in. If A pays $\frac{5}{11}$ of the cost and puts in 15 horses, what part of the cost does B pay and how many horses does he put in?

33. From 2 fields 240 bushels of potatoes were dug. If one field yielded $\frac{2}{3}$ as many potatoes as the other, how many bushels were dug from each field?

34. Three tubs of butter were sold for $18.70. If the first tub contained $37\frac{1}{2}$ pounds, the second $24\frac{1}{4}$ pounds, and the third $31\frac{3}{4}$ pounds, what was the price per pound?

35. If the slope of a hill rises 5 feet in every 29, and if the top of the hill is 81 feet high, how many feet is it up the slope from the bottom to the top of the hill?

36. If a cubic foot of water weighs $62\frac{1}{2}$ pounds, how many pounds will $\frac{2}{5}$ of a cubic foot weigh?

37. If iron is $7\frac{9}{10}$ times as heavy as water, what is the weight of $\frac{2}{5}$ of a cubic foot of iron?

38. If copper is $8\frac{4}{5}$ times as heavy as water, what is the weight of a cubic foot of copper?

39. A can build a wall in $7\frac{2}{3}$ days and B in $8\frac{1}{5}$ days. What part can both together build in 1 day? In how many days can both together build the wall?

40. A can dig a ditch in 10 days, B in 12 days, and C in 15 days. What part of the ditch will be dug if A works 3 days, B 4 days, and C 5 days?

41. A can dig a well in $12\frac{1}{2}$ days, and A and B together can dig $\frac{1}{7}$ of the well in one day. What part does A dig in one day? What part does B dig in one day?

42. What is the cost of bricks per thousand, if a single brick costs $\frac{9}{10}$ of a cent?

43. If bricks cost $8.50 a thousand, what is the cost of one brick?

44. Counting 2000 pounds as a ton, what is the cost of a ton of steel at $1\frac{1}{8}$ cents a pound?

45. If steel costs $1\frac{1}{5}$ cents a pound, what part of a ton can be bought for $\$14\frac{1}{2}$?

46. At $10\frac{3}{5}$ cents a pound, how many pounds of copper can be bought for $100?

47. If aluminum is $2\frac{1}{2}$, and copper $8\frac{4}{5}$ times as heavy as water, copper is how many times as heavy as aluminum?

48. What part of a cubic foot of copper would weigh as much as a cubic foot of aluminum?

49. A bankrupt's property is worth \$4500, and with it he can pay $33\frac{1}{3}$ cents for every dollar he owes. How much does he owe?

50. If a bankrupt pays $37\frac{1}{2}$ cents on a dollar, how much will a creditor receive whose claim is \$112?

51. If a man's debts amount to \$3500, and his property is worth \$1550, how many cents on a dollar can he pay?

52. If a man's property is worth \$7500, and he owes \$5100, what part of his property would pay his debts?

53. A ship has a crew of 16 men, and has 400 pounds of ship-biscuit. What part of a pound should be given daily to each man to make the biscuit last 45 days?

54. How many pounds of ship-biscuit will be needed for a voyage of 60 days, if there are 18 men in the crew, and each man's daily allowance is $\frac{9}{16}$ of a pound?

55. What should be the price for a 1000-mile book of tickets at $1\frac{7}{8}$ cents a mile?

56. If a 1000-mile book is sold for \$16.75, what is the price per mile?

57. A man owns $87\frac{5}{16}$ acres of land, his wife owns $42\frac{5}{8}$ acres, and his son $29\frac{5}{8}$ acres. How many acres do they own together?

58. A merchant was taxed \$144. $\frac{3}{8}$ of the tax was on his store, $\frac{2}{5}$ on his stock of goods, and the rest upon his house and lot. What was the tax on each?

59. A man wishes to buy a bicycle which is $\frac{1}{7}$ as heavy as himself. What should the weight of the bicycle be if the man weighs 156 pounds?

60. If a man can row $4\frac{3}{10}$ miles an hour in still water, how many miles can he row in an hour up a river that runs at the rate of $1\frac{5}{8}$ miles an hour?

61. How many miles can he row in an hour down the same river?

62. How many hours will it take him to row 12 miles down the river? 12 miles up the river?

63. There are 32 quarts in a bushel. If a miller takes $\frac{1}{16}$ as toll, how many bushels must he grind to earn a bushel for himself?

64. How many quarts should he take for grinding $2\frac{5}{16}$ bushels?

65. If a bushel of wheat makes 40 pounds of flour when ground, what part of a quart will make a pound of flour?

66. If a man's heart weighs $\frac{3}{4}$ of a pound, and the man weighs 150 pounds, what part of his weight is the weight of his heart?

67. If a man's brain is $\frac{1}{50}$ of his weight, and weighs $3\frac{3}{32}$ pounds, what is his weight?

68. A, B, and C can plaster a house in $13\frac{3}{4}$ days. If A and B can plaster $\frac{1}{20}$ of it in a day, what part can C plaster in a day? In how many days can C do the whole, working alone?

69. If an express train travels $42\frac{1}{2}$ miles in an hour, what part of a mile does it travel in a minute?

70. If a freight train travels $\frac{3}{11}$ of a mile in a minute, how many miles does it travel in an hour?

71. A freight train travels $23\frac{7}{10}$ miles an hour, and an express $47\frac{1}{2}$ miles an hour. If the freight train is $129\frac{2}{7}$ miles ahead of the express, how many miles apart will the trains be 3 hours later?

72. Which is greater, $\frac{40}{103}$ or $\frac{5}{13}$? Prove your answer by changing them to similar fractions and also to decimal fractions.

73. If a bale of cotton sells for $33.84 at the rate of $9\frac{2}{5}$ cents a pound, how many pounds are there in the bale?

74. What is the freight charge for carrying $85\frac{4}{5}$ tons of grain 45 miles, if the charge for carrying 1 ton 1 mile is $1\frac{3}{10}$ cents?

75. If 5 is added to both terms of $\frac{8}{15}$, is the fraction increased or diminished? How much is it increased or diminished?

76. If 7 is added to both terms of $\frac{11}{6}$, how much is the fraction increased or diminished?

77. Which is greater, 1.567 or $1\frac{21}{40}$? How much greater?

78. To make a bell, $180\frac{7}{10}$ pounds of copper were melted with $45\frac{1}{8}$ pounds of tin. What was the weight of the bell? What part of the bell was tin?

79. The selling price of a horse was $\$100\frac{3}{4}$, and the gain $\$22\frac{4}{5}$. Find the cost price.

80. A man raised $93\frac{3}{4}$ bushels of barley on $3\frac{7}{8}$ acres of land. How many bushels an acre did he raise?

81. If $\$2\frac{5}{8}$ will pay a woman's wages for $2\frac{1}{3}$ days, how much will pay her for $5\frac{1}{3}$ days?

82. A man bought $7\frac{3}{4}$ thousand feet of boards for $155. Find the cost of $19\frac{5}{8}$ thousand feet at the same rate.

83. Of a certain farm $\frac{1}{6}$ is in pasture, $\frac{5}{8}$ is under cultivation, and the remainder is woodland. If the woodland is 50 acres, how many acres are there in the whole farm?

Review Questions.

What are the factors of a number? What are prime numbers? Composite numbers? Even numbers? Odd numbers? Prime factors? Exact divisors? What are exponents? Powers? Measures of a number? What is the greatest common measure of two or more numbers? What factors does the greatest common measure of two or more numbers contain? What are multiples of a number? What is the least common multiple of two or more numbers? What factors does the least common multiple of two or more numbers contain? What is cancellation? What is the advantage of cancellation? On what principle does cancellation depend? What is a fractional unit? What are common fractions? What is a simple fraction? A compound fraction? A complex fraction? What does the denominator of a simple fraction show? What does the numerator show? What are the terms of a fraction? How does multiplying or dividing both terms of a fraction by the same number affect the value of the fraction? What is a proper fraction? An improper fraction? A mixed number? How is an improper fraction reduced to a whole or mixed number? How is a whole or mixed number reduced to an improper fraction? How is a fraction reduced to its lowest terms? How is a fraction reduced to higher terms? How do we find the product of a whole number and a fraction? The product of two fractions? The product of a mixed number and a whole number? What is the reciprocal of a whole number? What is the reciprocal of a fraction? By what must we multiply the dividend to produce the same result as to divide the dividend by a whole number? By what must we multiply the dividend to produce the same result as to divide the dividend by a fraction? What is the shortest method of dividing a mixed number by a whole number? How do we find a number when a fractional part of it is given? How do we change a complex fraction to a simple fraction? How do we find the fraction that one number is of another? What are similar fractions? How are fractions changed to similar fractions? How do we add fractions? How do we add mixed numbers? How do we subtract one fraction from another? What effect does it have on the difference if the same number is added to the minuend and to the subtrahend? How can we make the subtrahend in every case a whole number? Can we by this method bring the subtraction of fractions under addition of fractions? How do we change a decimal to a common fraction? A common fraction to a decimal?

CHAPTER X.

COMPOUND QUANTITIES.

187. A quantity expressed in units of *one* denomination is a *simple quantity;* expressed in units of *two or more* denominations is a *compound quantity,* or a *compound denominate number.*

Thus, $20\frac{1}{4}$ pounds is a simple quantity, but 20 pounds 4 ounces is a compound quantity.

188. The process of changing the *denomination* in which a quantity is expressed, without changing the *value* of the quantity, is called *reduction.*

189. If the change is from a higher denomination to a lower, it is called *reduction descending;* if from a lower to a higher, it is called *reduction ascending.*

Thus, 1 yard = 36 inches is an example of reduction descending; and 24 inches = 2 feet is an example of reduction ascending.

Liquid Measure.

190. Liquid Measure is used in measuring liquids, as water, milk, etc.

TABLE.

4 gills (gi.) = 1 pint (pt.).
2 pints = 1 quart (qt.).
4 quarts = 1 gallon (gal.).

1 gal. = 4 qt. = 8 pt. = 32 gi.   1 gal. contains 231 cubic inches.

$31\frac{1}{2}$ gal. = 1 barrel (bbl.).
63 gal. = 1 hogshead (hhd.).

NOTE. Casks holding from 28 gal. to 43 gal. are called barrels, and casks holding from 54 gal. to 63 gal. are called hogsheads. Whenever barrels or hogsheads are used as *measures*, a barrel means $31\frac{1}{2}$ gallons, and a hogshead means 63 gallons.

191. Reduction of compound quantities.

Reduce 12 gallons 2 quarts 1 pint to pints.

gal.	qt.	pt.
12	2	1
4		
50		
2		
101		

SOLUTION. 12 gal. = 12 × 4 qt. = 48 qt., and 48 qt. with the 2 qt. added are 50 qt.

50 qt. = 50 × 2 pt. = 100 pt., and 100 pt. with the 1 pt. added are 101 pt. 101 pt. *Ans.*

Hence, **in reduction descending,**

Multiply the given number of units of the highest denomination by the number of units of the next lower denomination required to make one of this higher; and add to the product the given number of units of this lower denomination.

Proceed in this way with each successive result until the required denomination is reached.

EXERCISE 98. — WRITTEN.

Reduce:

1. 6 qt. 1 pt. to pints.
2. 4 gal. 1 pt. to pints.
3. 4 qt. to gills.
4. 7 qt. 1 pt. to pints.
5. 9 gal. 3 qt. 1 pt. to pints.
6. 12 gal. 1 pt. to pints.
7. 2 hhd. to quarts.
8. 1 hhd. 1 gal. 1 qt. 1 pt. to pints.

Reduce 221 pints to higher units.

2 | 221 pt.
4 | 110 qt. . . . 1 pt.
27 gal. . . . 2 qt.

SOLUTION. 221 pt. = 110 qt. and 1 pt. over. 110 qt. = 27 gal. and 2 qt. over. 27 gal. 2 qt. 1 pt. *Ans.*

Hence, **in reduction ascending,**

Divide by the number of units required to make one of the next higher denomination.

Divide this quotient and each successive quotient in like manner until the required denomination is reached.

The last quotient with the several remainders, arranged in order, is the answer sought.

EXERCISE 99. — WRITTEN.

Reduce to higher units:

1. 333 pints.
2. 461 gills.
3. 513 pints.
4. 718 quarts.
5. 1041 gills.
6. 606 pints.

192. Addition and subtraction of compound quantities.

Add 3 gal. 2 qt. 1 pt.; 5 gal. 3 qt.; 13 gal. 1 qt. 1 pt.; 9 gal. 1 pt.

gal.	qt.	pt.
3	2	1
5	3	0
13	1	1
9	0	1
31	3	1

SOLUTION. We write units of the same name in the same column; and add the columns, beginning with the pints. 3 pt. = 1 qt. and 1 pt. over. We write the 1 pt. under the pints, and add the 1 qt. to the quarts. 7 qt. = 1 gal. and 3 qt. over. We write the 3 qt. under the quarts, and add the 1 gal. to the gallons.

From 6 gal. 2 qt. 1 pt. take 3 gal. 3 qt. 1 pt.

gal.	qt.	pt.
6	2	1
3	3	1
2	3	0

SOLUTION. 1 pt. − 1 pt. = 0 pt. We write 0 under the pints. Since we cannot take 3 qt. from 2 qt., we take 1 gal. (4 qt.) from the 6 gal. and add it to the 2 qt. 4 qt. + 2 qt. = 6 qt. 6 qt. − 3 qt. = 3 qt. We write 3 qt. under the quarts. Then 5 gal. − 3 gal. = 2 gal.

EXERCISE 100. — WRITTEN.

Add:

1. gal.	qt.	pt.	2. gal.	qt.	pt.	3. gal.	qt.	pt.
3	3	1	18	3	1	25	3	0
7	1	0	6	2	1	16	3	1
5	2	1	30	0	0	22	3	1
6	3	1	8	0	1	5	3	1

Find the difference between:

4. gal.	qt.	pt.	5. gal.	qt.	pt.	6. gal.	qt.	pt.
10	2	0	13	1	1	25	1	0
4	2	1	5	3	1	16	2	1

7. After 25 gal. 3 qt. have been drawn from a hogshead of molasses, how much will remain?

8. What must be added to 21 gal. 2 qt. to fill a barrel holding 40 gal.?

193. Multiplication and division of compound quantities.

Multiply 15 gal. 3 qt. 1 pt. by 5.

gal.	qt.	pt.
15	3	1
		5
79	1	1

SOLUTION. 5×1 pt. $= 5$ pt. $= 2$ qt. 1 pt. We write the 1 pt. under the pints, and reserve the 2 qt. to be added to 5×3 qt. 5×3 qt. $= 15$ qt., and this with the 2 qt. added $= 17$ qt. 17 qt. $= 4$ gal. 1 qt. We write the 1 qt. under the quarts, and add the 4 gal. to 5×15 gal., and we have 79 gal.

Divide 139 gal. 1 qt. 1 pt. by 5.

	gal.	qt.	pt.
5)	139	1	1
	27	3	1

SOLUTION. 139 gal. $\div 5 = 27$ gal. and 4 gal. over. 4 gal. $= 16$ qt., and this added to the 1 qt. $= 17$ qt. 17 qt. $\div 5 = 3$ qt. and 2 qt. over. 2 qt. $= 4$ pt., and this added to the 1 pt. $= 5$ pt. 5 pt. $\div 5 = 1$ pt.; and we have 27 gal. 3 qt. 1 pt.

Divide 11 gal. 2 qt. by 5 gal. 3 qt.

11 gal. 2 qt. $= 46$ qt. 5 gal. 3 qt. $= 23$ qt.
46 qt. $\div$ 23 qt. $= 2$. *Ans.*

EXERCISE 101. — WRITTEN.

Multiply:

1. 8 gal. 2 qt. 1 pt. by 8.
2. 12 gal. 3 qt. 1 pt. by 10.
3. 15 gal. 1 qt. 1 pt. by 7.
4. 21 gal. 3 qt. 1 pt. 2 gi. by 6.

Divide:

5. 116 gal. 2 qt. by 4.
6. 162 gal. 3 qt. by 6.
7. 226 gal. 1 qt. 1 pt. 1 gi. by 3.
8. 70 gal. 3 qt. 1 pt. by 9.
9. 70 gal. 3 qt. 1 pt. by 7 gal.

194. *All* compound quantities are reduced, added, subtracted, multiplied, divided, by the methods given for liquid measure.

Dry Measure.

195. Dry Measure is used in measuring dry articles, as grain, seeds, fruit, vegetables.

TABLE.

2 pints (pt.)	=	1 quart (qt.).
8 quarts	=	1 peck (pk.).
4 pecks	=	1 bushel (bu.).

1 bu. = 4 pk. = 32 qt. **1 bu. contains 2150.42 cubic inches.**

NOTE. In measuring grain, seeds, and small fruits, the measure must be *even* full. In measuring apples, potatoes, and other large articles, the measure must be *heaping* full.

EXERCISE 102. — WRITTEN.

1. Reduce 3 bu. 3 pk. 3 qt. to quarts.
2. Reduce 1012 quarts to higher denominations.
3. Multiply 3 bu. 2 pk. 6 qt. by 7.
4. Divide 69 bu. 1 pk. 7 qt. by 9.
5. How many peck measures will 5 bushels fill?
6. Multiply 2 bu. 2 pk. 2 qt. by 8.

Add:

7. bu.	pk.	qt.	8. bu.	pk.	qt.	9. bu.	pk.	qt.
6	1	1	11	3	4	7	2	7
5	1	7	18	2	4	9	3	7
3	0	2	7	1	1	8	1	2

Subtract:

10. bu.	pk.	qt.	11. bu.	pk.	qt.	12. bu.	pk.	qt.
17	2	4	18	3	4	156	1	6
11	3	5	8	4	6	121	3	7

Avoirdupois Weight.

196. Avoirdupois Weight is used in weighing all articles except gold, silver, and precious stones.

TABLE.

16 ounces (oz.)	= 1 pound (lb.).
100 pounds	= 1 hundredweight (cwt.).
2000 pounds	= 1 ton (t.).

The long ton is used in the United States Custom Houses and in wholesale transactions in iron and coal.

112 pounds Avoirdupois = 1 long hundredweight.
2240 pounds Avoirdupois = 1 long ton.

The pound Avoirdupois contains 7000 grains.

NOTE. Many articles are sold by weight, as follows:

1 bu. of wheat, peas, or beans	= 60 lb.
1 bu. of corn or rye	= 56 lb.
1 bu. of corn or rye meal, or cracked corn	= 50 lb.
1 bu. of oats	= 32 lb.
1 bu. of barley	= 48 lb.
1 bu. of timothy seed	= 45 lb.
1 bu. of potatoes, carrots, beets, turnips, or onions	= 60 lb.
1 barrel of flour	= 196 lb.
1 barrel of beef or pork	= 200 lb.
1 cask of lime	= 240 lb.
1 quintal of fish	= 100 lb.

EXERCISE 103. — WRITTEN.

1. Reduce 5 t. 12 cwt. 95 lb. to pounds.
2. Reduce 8174 lb. to higher denominations.
3. Reduce 39 long tons 596 lb. of coal to pounds.
4. Multiply 1 t. 500 lb. by 7.
5. Reduce 2 cwt. 55 lb. 5 oz. to ounces.
6. Reduce 6242 oz. to higher denominations.
7. How many bushels of corn meal are there in 2 t.?

8. Divide 15 t. 17 cwt. 29 lb. 7 oz. by 4.

9. Add 1 t. 1236 lb., 2 t. 1650 lb., 3 t. 1278 lb.

10. A cubic foot of water weighs 1000 oz. How many tons, etc., will 144 cubic feet of water weigh?

11. From 2 t. 3 cwt. 97 lb. 6 oz. take the sum of 1 t. 17 cwt. 8 lb., 5 cwt. 29 lb. 8 oz., 1 cwt. 42 lb. 6 oz.

12. Is it cheaper to buy coal at $6 a short ton or at $7 a long ton?

13. How much must I pay for a ton of hay at 8 mills a pound?

14. I put into my cellar 8 loads of coal averaging 1 t. 1200 lb. each. How many tons do I put in?

15. If 4 horses weigh 1 t. 1600 lb., what is their average weight?

16. If a dealer has 27 long tons of coal and sells 6480 lb., how many short tons has he left?

17. If butter is 25 cents a pound, how much hay at $16 a ton can be bought with 48 lb. of butter?

18. How many 4-oz. packages can be put up from 8 lb. 12 oz. of ginger?

19. Divide 102 t. 2 cwt. 20 lb. 8 oz. by 8.

20. Multiply 19 t. 13 cwt. 66 lb. by 19.

21. How many pounds will a car-load of 600 bu. of oats weigh?

22. How many tons will 100 bu. of potatoes weigh?

23. Find the difference in weight between 75 bu. of wheat and 75 bu. of barley.

24. Find the cost of 1512 lb. of rye at 56 cents a bushel.

25. Find the cost of 2 t. of carrots at 30 cents a bushel.

Troy Weight.

197. Troy Weight is used in weighing gold, silver, and precious stones.

TABLE.

24 grains (gr.)	=	1 pennyweight (dwt.).
20 pennyweights	=	1 ounce (oz.).
12 ounces	=	1 pound (lb.).

The pound Troy contains 5760 grains.

EXERCISE 104. — WRITTEN.

1. How many more grains does a pound Avoirdupois contain than a pound Troy?

2. How many more grains does an ounce Troy contain than an ounce Avoirdupois?

3. Find to the nearest cent the value of one pennyweight of gold, an ounce of gold being worth $20.67.

4. When silver is worth $68\frac{3}{4}$ cents an ounce, find the value of 160,000 ounces.

5. How many spoons weighing 2 oz. 5 dwt. can be made from 27 oz. of silver?

6. Divide 401 oz. 2 dwt. by 7.

7. Multiply 1 oz. 19 dwt. by 16.

8. Add 10 oz. 10 dwt. 10 gr., 7 oz. 19 dwt. 22 gr., 18 oz. 17 dwt. 13 gr., 4 oz. 18 dwt. 12 gr., 3 oz. 4 dwt. 5 gr.

9. From 7 oz. 2 dwt. 6 gr. take 2 oz. 9 dwt. 13 gr.

10. Reduce 1760 gr. to higher denominations.

NOTE. Apothecaries, in compounding medicines, divide the Troy ounce (480 gr.) as follows: 20 gr. = 1 scruple; 3 scruples = 1 dram; 8 drams = 1 ounce; and use the following

APOTHECARIES' LIQUID MEASURE.

60 minims (♏)	=	1 dram (♏ lx.).
8 fluid drams	=	1 ounce (fl. drm. viij.).
16 ounces	=	1 pint (fl. oz. xvj.).

Long Measure.

198. Long Measure is used in measuring lines or distances.

TABLE.

12 inches (in.)	= 1 foot (ft.).
3 feet	= 1 yard (yd.).
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet	= 1 rod (rd.).
320 rods	= 1 mile (mi.).

1 mi. = 320 rd. = 1760 yd. = 5280 ft.

NOTE 1. A hand (used in measuring the height of horses) = 4 in.; a knot (used in navigation) = 6086 ft.; a league = 3 knots; a fathom (used in measuring depths at sea) = 6 ft.; a cable length = 120 fathoms.

NOTE 2. Lengths measured by yards are generally expressed in yards and fractions of a yard; and distances of 160 rd. and 80 rd. are called *half-miles* and *quarter-miles*, respectively.

Reduce 320 yards to rods.

$$\begin{array}{r|l} 5\frac{1}{2} & 320 \\ & \underline{\quad 2} \\ 11 & 640 \ \textit{half-yards.} \\ & 58 \ldots 2 \ \textit{half-yards.} \end{array}$$

SOLUTION. Since it takes $5\frac{1}{2}$ yards, or 11 *half-yards*, to make a rod, reduce the 320 yards to *half-yards* and divide by 11. The quotient is 58 rods, and the remainder is 2 *half-yards*. The 2 half-yards are equal to 1 yard. 58 rd. 1 yd. *Ans.*

Add:

mi.	rd.	yd.	ft.	in.
6	80	3	2	1
4	75	1	2	7
5	170	2	1	8
16	6	$2\frac{1}{2}$	0	4
			1	6
16	6	2	1	10

16 mi. 6 rd. 2 yd. 1 ft. 10 in. *Ans.*

We have for the sum 16 mi. 6 rd. $2\frac{1}{2}$ yd. 0 ft. 4 in. We reduce the $\frac{1}{2}$ yd. of this sum and add its value 1 ft. 6 in. to the 0 ft. 4 in. We thus have for the answer 16 mi. 6 rd. 2 yd. 1 ft. 10 in.

Exercise 105. — Written.

1. Reduce 7 yd. 1 ft. 9 in. to inches.
2. Reduce 4 yd. 2 ft. 7 in. to inches.
3. Reduce 3 rd. 5 yd. 1 ft. to feet.
4. Reduce 5 mi. 125 rd. to rods.
5. Reduce 7043 yd. to units of higher denominations.
6. Reduce 31,415 ft. to units of higher denominations.
7. Reduce 33,768 in. to units of higher denominations.
8. Change 5301 rd. to units of higher denominations.

Add:

9.			10.			11.		
yd.	ft.	in.	yd.	ft.	in.	yd.	ft.	in.
4	2	3	13	2	11	15	1	7
5	1	9	15	2	9	3	1	6
3	2	8	18	2	10	12	2	5

12.			13.			14.		
mi.	rd.	yd.	rd.	yd.	ft.	mi.	rd.	ft.
12	25	5	114	5	2	7	230	9
25	12	4	79	3	2	92	164	14
37	13	3	65	4	1	46	189	15
16	31	4	87	5	2	23	207	8

Find the difference between:

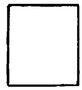
15.			16.			17.		
yd.	ft.	in.	rd.	ft.	in.	rd.	ft.	in.
28	1	5	27	3	1	173	4	1
13	2	7	14	4	2	37	5	2

18.			19.			20.		
mi.	rd.	yd.	mi.	rd.	ft.	mi.	rd.	ft.
7	140	3	13	35	11	5	230	5
5	230	5	11	57	15	3	175	11

21. Multiply 15 yd. 1 ft. 9 in. by 13.
22. Divide 171 yd. 1 ft. 3 in. by 11.

Square Measure.

199. A *square* is a flat surface that has four equal sides and four square corners.



200. The *unit* of surface is a square, each side of which is a unit of length, as a square inch.

201. The *area* of a surface is the number of square units it contains.

TABLE.

202.

144 square inches (sq. in.)	= 1 square foot (sq. ft.).
9 square feet	= 1 square yard (sq. yd.).
$30\frac{1}{4}$ square yards, or $272\frac{1}{4}$ square feet	= 1 square rod (sq. rd.).
160 square rods or 43,560 square feet	= 1 acre (A.).

EXERCISE 106. — WRITTEN.

1. Reduce 9 A. 110 sq. rd. to square rods.
2. Reduce 3 sq. yd. 7 sq. ft. to square feet.
3. Reduce 17 sq. yd. 8 sq. ft. 76 sq. in. to square inches.
4. Reduce 67,592 sq. in. to higher denominations.
5. Reduce 22,471 sq. yd. to higher denominations.
6. Reduce 47,916 sq. ft. to higher denominations.
7. A farm has 87 A. 137 sq. rd. 17 sq. yd. in tillage land, 13 A. 117 sq. rd. 19 sq. yd. in pasture, 1 A. 96 sq. rd. 29 sq. yd. in orchard, and 27 A. 115 sq. rd. 3 sq. yd. in woodland. Find the area of the farm.
8. A farm containing 160 acres has 47 A. 13 sq. rd. 17 sq. yd. wild land, and the rest is under cultivation. Find the area of the land under cultivation.
9. Multiply 80 A. 120 sq. rd. 20 sq. yd. 8 sq. ft. 136 sq. in. by 25.
10. Divide 182 A. 89 sq. rd. 23 sq. yd. by 9.
11. How many lots each containing 48 sq. rd. can be made from 9 A.?

Surveyors' Measure.

203. Surveyors use a chain, called Gunter's chain which, is 4 rods, or 66 feet, long. The chain has 100 links, and therefore links are written as *hundredths* of a chain.

SURVEYORS' TABLE OF

LONG MEASURE.	SQUARE MEASURE.
7.92 in. = 1 link (l.).	16 sq. rd. = 1 sq. ch.
100 links = 1 chain (ch.).	10 sq. ch. = 1 A.
80 chains = 1 mile (mi.).	640 A. = 1 sq. mi.
	1 sq. mi. = 1 section (sec.).
	36 sec. = 1 township.

EXERCISE 107. — WRITTEN.

1. A square piece of land is just 80 chains on a side. How many miles must a man walk to go round it?

2. A side of Russell square in London is 660 feet. How many chains on a side is it? What part of a mile must a man walk to go round the square?

3. Reduce 3168 inches to chains.

4. A chain is 66 ft. long and has 100 links. How many inches long is one link?

5. Find the cost of making a road 5 mi. 60 ch. long at $1600 a mile.

6. Reduce 1440 sq. rd. to square chains.

7. Reduce 980 sq. ch. to acres.

8. Reduce 5 sq. ch. to square rods.

9. Reduce 3375 sq. ch. to acres.

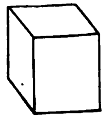
10. At $27.60 an acre, find the cost of 200.5 square chains.

11. At $3.50 a square chain, find the cost of 35 acres 60 square rods of land.

Cubic Measure.

204. A *cube* is a solid bounded by 6 equal squares.

205. The *unit* of volume is a cube each edge of which is a unit of length.



206. The *volume* of a body is the number of cubic units it contains.

TABLE FOR CUBIC MEASURE.

207.

1728 cubic inches (cu. in.)	=	1 cubic foot (cu. ft.).
27 cubic feet	=	1 cubic yard (cu. yd.).
16 cubic feet	=	1 cord foot (cd. ft.).
8 cord feet, or 128 cubic feet	=	1 cord (cd.).

EXERCISE 108. — WRITTEN.

1. Reduce 64 cu. ft. 620 cu. in. to cubic inches.
2. Reduce 12 cd. 64 cu. ft. to cubic feet.
3. Reduce 10 cu. yd. 18 cu. ft. to cubic feet.
4. Reduce 7680 cu. ft. to cords.
5. Reduce 1080 cu. ft. to cubic yards.
6. Reduce 51,840 cu. in. to cubic feet.
7. How much wood is there in 8 piles each containing 15 cords and 32 cubic feet?
8. How much wood is there in 3 piles containing respectively 12 cords 18 cubic feet, 10 cords 90 cubic feet, 11 cords 100 cubic feet?
9. A wood dealer had 70 cords of wood and sold 37 cords 96 cubic feet. How much had he left?
10. Divide 136 cu. yd. 14 cu. ft. by 19.
11. Divide 5 cu. yd. 1 cu. ft. 84 cu. in. by 429 cu. in.
12. Take 23 cu. yd. 13 cu. ft. 1600 cu. in. from 34 cu. yd. 7 cu. ft. 1400 cu. in.
13. Multiply 7 cu. yd. 11 cu. ft. 370 cu. in. by 16.

Time Measure.

208.

60 seconds (sec.)	= 1 minute (min.).
60 minutes	= 1 hour (hr.).
24 hours	= 1 day (dy.).
7 days	= 1 week (wk.).
365 days	= 1 common year (yr.).
366 days	= 1 leap-year.
100 years	= 1 century.

The names of the months (mo.), called **calendar months**, and the number of days in each are:

	dy.		dy.
1. January (Jan.)	31	7. July	31
2. February (Feb.)	28 or 29	8. August (Aug.)	31
3. March (Mar.)	31	9. September (Sept.)	30
4. April (Apr.)	30	10. October (Oct.)	31
5. May	31	11. November (Nov.)	30
6. June	30	12. December (Dec.)	31

NOTE. The number of days in each month may be easily remembered by committing to memory the following lines:

"Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Except the second month alone,
Which has but twenty-eight, in fine,
Till leap-year gives it twenty-nine."

A *year* is the time in which the earth performs one revolution in its orbit round the sun, and consists of 365 dys. 5 hrs. 48 min. 48 sec.; that is, nearly $365\frac{1}{4}$ days. As there are 365 days in a common year, a common year lacks nearly $\frac{1}{4}$ of a day of being a solar year, and this defect is made up by reckoning for some years (leap-years) 366 days.

The leap-years are the years whose dates are exactly divisible by 4; except in the case of complete hundreds, and these are leap-years if exactly divisible by 400. Thus, 1600 and 1884 were leap-years; 1800 and 1885 were not; 1900 will not be a leap-year.

The present, or Gregorian Calendar (Pope Gregory XIII, 1582), leaves a slight error equal to one day in about 3300 years.

Exercise 109. — Written.

1. Reduce 15 hr. 32 min. 17 sec. to seconds.

2. Reduce 2 dy. 7 hr. 29 min. to minutes.

3. Reduce 986,753 min. to units of higher denominations.

4. A lunar month contains 2,551,443 seconds. Express this number of seconds in units of higher denominations.

5. Find the sum of 2 wk. 3 dy. 12 hr. 18 min., 1 wk. 5 dy. 7 hr. 25 min., 2 wk. 4 dy. 11 hr. 30 min., 5 dy. 13 hr. 40 min.

6. A boy walks from 3 hr. 30 min. 30 sec. past noon until 5 hr. 10 min. 10 sec. past noon. How long does he walk?

7. Multiply 1 dy. 14 hr. 15 min. 20 sec. by 13.

8. Divide 6 dy. 23 hr. 40 min. 50 sec. by 20.

9. Divide 31 dy. 12 hr. by 18 hr.

10. Divide 26 wk. 2 dy. by 7 hr. 40 min.

11. Divide 58 dy. 15 hr. 14 min. by 14.

12. Multiply 5 dy. 12 min. 14 sec. by 50.

13. How long is it from 35 min. 25 sec. past 8 o'clock in the morning to 12 min. 15 sec. past 3 in the afternoon?

14. A man receives $2.25 a day, omitting Sundays. Find the amount of his wages for February, 1896. (Feb., 1896, began on Saturday.)

15. A solar year is 365 dy. 5 hr. 48 min. 48 sec. How much time is there in 4 solar years?

16. If a solar year is reckoned 365 dy. 6 hr., instead of 365 dy. 5 hr. 48 min. 48 sec., how much will the difference amount to in 100 years?

17. How many days of 8 hr. each will it take to count a million silver dollars at the rate of 80 a minute?

18. Find the number of seconds the month of February, 1900, will contain.

Difference between Two Dates.

209. Find the difference between April 3, 1885, and May 7, 1837.

yr.	mo.	dy.
1885	4	3
1837	5	7
47	10	26

In finding the difference between *long* dates, 30 days are considered a month. As April is the fourth, and May the fifth, month, we write 4 and 5 instead of the names of the months.

Find the number of days from March 7 to June 15.

The number of days in March = 24
The number of days in April = 30
The number of days in June = 15
Total number of days = 69

In finding the difference between *short* dates, the exact number of days is generally counted.

EXERCISE 110. — WRITTEN.

1. On the 1st day of May, 1896, how much time had passed since the discovery of the Island of San Salvador by Columbus, Oct. 12, 1492?

2. Find the difference in time between July 4, 1776, and Oct. 12, 1492.

3. If a note is dated March 7, 1896, and has 93 days to run, when is the note due?

4. If a note is dated June 17 and is due in 63 days, find the date when it is due.

5. Find the number of days from May 3 to Aug. 4.

6. Find the number of days from Oct. 1 to Dec. 31.

7. Find the number of days from Sept. 25 to Dec. 15.

8. If a note is dated Aug. 4 and is due in 90 days, find the date when it is due.

9. Find the date of 60 days after April 13.

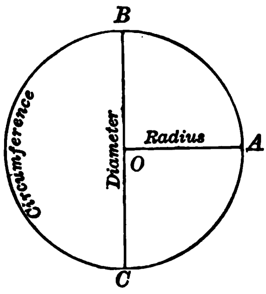
10. George Washington was born in Wakefield, Virginia, Feb. 22, 1732. Abraham Lincoln was born Feb. 12, 1809. Find the difference between these two dates.

Angular Measure.

210. A circle is a plane figure bounded by a curved line called the *circumference*, all points of which are equally distant from a point within called the *centre*. Any part of the circumference is called an *arc*.

211. A straight line drawn through the centre, having its ends in the circumference, is called a *diameter;* and half the diameter is called the *radius*.

If a straight line fixed at one end is revolved, the other end makes the circumference of a circle; and the straight line moving from its position at the start to any other given position makes an angle. Thus, if OA revolve on a fixed point O, the end A makes the circumference ABC. When OA has reached the position OB, the part of the circumference between A and B has been made by A, and the part of the angular magnitude about the point O, between OA and OB, has been made by OA. The angle AOB is such a part of the angular magnitude about O as AB is of the circumference.



The circumference of every circle is divided into 360 equal parts, called *degrees* (arc-degrees), and corresponding to every one of these equal parts is an angle at the centre of the circle. Hence, the whole angular magnitude about any point in a plane is divided into 360 equal parts called *degrees* (angle-degrees), and the number of degrees in the angle formed by two lines drawn from the centre of a circle is the same as the number of degrees in the arc which is intercepted between these two lines.

212. An angle described by a line making one fourth of a revolution contains 90°, and is called a *right angle*, as AOB; and OA and OB are said to be *perpendicular* to each other. An angle less than a right angle is called an *acute angle;* an angle greater than a right angle and less than two right angles is called an *obtuse angle*.

Units of Angular Measure.

213.

60 seconds (″)	=	1 minute (′).
60 minutes	=	1 degree (°).
360 degrees	=	1 revolution.

NOTE. A degree of the circumference of the earth at the equator contains 60 geographical miles, or 69.16 statute miles.

EXERCISE 111. — WRITTEN.

1. Reduce 37° 18′ 39″ to seconds.
2. Reduce 9783″ to units of higher denominations.
3. Add 7° 35′ 14″, 8° 46′ 56″, 12° 37′ 39″, 26° 8′ 22″.
4. From 17° 0′ 26″ take 13° 21′ 49″.
5. From 42° 31′ 20″ take 40° 42′ 47″.
6. Multiply 5° 39′ 28″ by 7.
7. Divide 39° 36′ 42″ by 6.

MISCELLANEOUS.

214.

Numbers.

12 units	=	1 dozen.
12 dozen	=	1 gross.
12 gross	=	1 great gross.
20 units	=	1 score.

Paper.

24 sheets	=	1 quire.
20 quires	=	1 ream.
2 reams	=	1 bundle.
5 bundles	=	1 bale.

FOREIGN CURRENCIES.

215.

French:	100 centimes	= 1 franc (fr.)	= $0.193.
Italian:	100 centisimi	= 1 lira	= $0.193.
German:	100 pfennigs	= 1 mark	= $0.238.
Russian:	100 kopecks	= 1 rouble	= $0.734.
Austrian:	100 kreuzers	= 1 florin (fl.)	= $0.453.

English:

4 farthings (far.)	=	1 penny (*d.*).
12 pence	=	1 shilling (*s.*).
20 shillings	=	1 pound (£).

A guinea	=	21 *s.*
A sovereign	=	20 *s.*
A crown	=	5 *s.*
A florin	=	2 *s.*

£1 = $4.8665.

216. To find the cost of goods sold by the hundred, by the thousand, by the ton.

1. Find the cost of 1863 laths at $0.38 per hundred.

1863 = 18.63 hundreds.
18.63 × $0.38 = the cost.

$$\begin{array}{r} \$18.63 \\ \underline{0.38} \\ 14904 \\ \underline{5589} \\ \$7.0794 \\ \$7.08.\ \textit{Ans.} \end{array}$$

SOLUTION. Divide the number denoting the *quantity* by 100, and multiply the *price* of a hundred by this quotient.

We interchange multiplicand and multiplier (§ 47).

2. Find the cost of 3924 feet of boards at $9 per thousand.

3924 = 3.924 thousands.
3.924 × $9 = the cost.

$$\begin{array}{r} \$3.924 \\ \underline{9} \\ \$35.316 \\ \$35.32.\ \textit{Ans.} \end{array}$$

SOLUTION. Divide the number denoting the *quantity* by 1000, and multiply the *price* of a thousand by this quotient.

3. What is the cost of 8712 pounds of coal at $8 per ton?

$8 ÷ 2 = $4.
8712 = 8.712 thousands.
8.712 × $4 = the cost.

$$\begin{array}{r} \$8.712 \\ \underline{4} \\ \$34.848 \\ \$34.85.\ \textit{Ans.} \end{array}$$

SOLUTION. Divide the price of one ton by 2, to get the price of 1000 pounds. Then divide the number denoting the *quantity* by 1000, and multiply the *price* of a thousand pounds by this quotient.

NOTE. C is often used for hundred and M for thousand.

EXERCISE 112. — WRITTEN.

1. What will 8725 feet of boards cost at $10.50 per M?
2. Find the cost of 12,700 feet of boards at $11.25 per M.
3. What must be paid for 3726 paving stones at $8.75 per C?
4. Find the cost of 12,760 pounds of coal at $6.20 per ton.

5. 1,626,400 bricks were required for a dormitory. Find the cost at $7.50 per M.

6. If 8760 pounds of plaster cost $32.85, what is the price per ton?

7. What is the cost of 18,800 laths at $0.35 per C?

8. A coal dealer retails coal at $8.20 per ton. If a ton costs $3.75 at the mine, $1.40 for freight, and $0.75 for handling, what will he make on 16,760 pounds of coal?

9. Find the cost of 12,645 feet of boards at $12.25 per M.

10. A lumber dealer paid $2.75 per M for shingles, and sold them for $3.50. What did he gain on 48 M?

11. What must be paid for 16,820 pounds of coal at $6.50 per ton?

12. At $11 per M, how many thousand feet of boards can be bought for $107.80?

13. Find the cost of 7250 shingles at $3.25 per M.

14. Required the cost of 1872 laths at $0.33 per C.

15. At $0.18 per C, what is the freight on a car-load of 30,000 pounds of flour?

16. 9500 tiles are to be used for the roof of a building. What will they cost at $8.25 per M?

17. Find the cost of 18,620 pounds of coal at $6.70 per ton.

18. What is the retail price of coal per ton when 14,600 pounds are sold for $46.72?

19. When 27,000 pounds of plaster are sold for $74.25, what is the price per ton?

20. Find the cost of 19,720 feet of boards at $11.50 per M.

21. Find the cost of 27,000 feet of boards at $10.25 per M.

22. Find the cost of 28,760 pounds of coal at $6.20 per ton.

Denominate Fractions.

217. **1.** Express $\frac{2}{3}$ of a mile as a compound quantity.

$$\tfrac{2}{3}\text{ mi.} = \tfrac{2}{3}\text{ of }320\text{ rd.} = 213\tfrac{1}{3}\text{ rd.}$$

We reduce to feet the $\frac{1}{3}$ rd. of the $213\frac{1}{3}$ rd.

$$\tfrac{1}{3}\text{ rd.} = \tfrac{1}{3}\text{ of }16\tfrac{1}{2}\text{ ft.} = 5\tfrac{1}{2}\text{ ft.}$$

We reduce to inches the $\frac{1}{2}$ ft. of the $5\frac{1}{2}$ ft.

$$\tfrac{1}{2}\text{ ft.} = \tfrac{1}{2}\text{ of }12\text{ in.} = 6\text{ in.}$$

Therefore the answer is 213 rd. 5 ft. 6 in.

2. Find the value of $\frac{7}{9}$ of 4 rd. 12 ft. 6 in.

	rd.	ft.	in.
9	4	12	6
	0	8	$8\frac{2}{3}$
			7
	3	$11\frac{1}{2}$	$0\frac{2}{3}$
			6
	3	11	$6\frac{2}{3}$

Solution. Here we divide by 9 to get $\frac{1}{9}$ of 4 rd. 12 ft. 6 in., and multiply the quotient by 7 to get $\frac{7}{9}$ of 4 rd. 12 ft. 6 in. The $\frac{1}{2}$ ft. = 6 in. We therefore cancel $\frac{1}{2}$ ft. and add 6 in. to the inches.

3. Express 0.31375 of a mile as a compound quantity.

```
  0.31375
      320
  -------
   627500
   94125
  -------
 100.40000
      16½
  -------
      6.6
       12
  -------
      7.2
```

Solution. 0.31375 mi. = 0.31375 of 320 rd. = 100.4 rd.
0.4 rd. = 0.4 of $16\frac{1}{2}$ ft. = 6.6 ft.
0.6 ft. = 0.6 of 12 in. = 7.2 in.

The answer is 100 rd. 6 ft. 7.2 in.

Exercise 113. — Written.

Express in units of lower denominations:

1. $\frac{7}{16}$ bu.	4. $\frac{9}{16}$ t.	7. 0.7575 bu.	10. 0.94375 A.
2. $\frac{4}{9}$ mi.	5. $\frac{5}{9}$ A.	8. 0.3755 dy.	11. 0.96875 t.
3. $\frac{7}{8}$ rd.	6. $\frac{9}{8}$ dy.	9. 0.2556°.	12. 0.77625 mi.

Find:

13. $\frac{2}{9}$ of 5 bu. 6 qt. 1 pt.	16. $\frac{7}{11}$ of 13 mi. 3 rd.
14. $\frac{4}{11}$ of 1 mi. 1 rd.	17. $\frac{3}{5}$ of 3 gal. $1\frac{3}{4}$ qt.
15. $\frac{5}{6}$ of 1 t. 240 lb.	18. $\frac{4}{9}$ of 1 A. 620 sq. yd.

218. 1. Express 203 rd. 3 yd. 1 ft. 6 in. as the fraction of a mile.

$$6\text{ in.} = \tfrac{6}{12}\text{ ft.} \quad = \tfrac{1}{2}\text{ ft.}$$
$$1\tfrac{1}{2}\text{ ft.} = \frac{1\frac{1}{2}}{3}\text{ yd.} \quad = \tfrac{1}{2}\text{ yd.}$$
$$3\tfrac{1}{2}\text{ yd.} = \frac{3\frac{1}{2}}{5\frac{1}{2}}\text{ rd.} \quad = \tfrac{7}{11}\text{ rd.}$$
$$203\tfrac{7}{11}\text{ rd.} = \frac{203\frac{7}{11}}{320}\text{ mi.} = \tfrac{7}{11}\text{ mile.}$$

SOLUTION. Here the 6 in. = $\frac{1}{2}$ ft., and this $\frac{1}{2}$ ft. put with the 1 ft. makes $1\frac{1}{2}$ ft. The $1\frac{1}{2}$ ft. = $\frac{1}{2}$ yd., and this $\frac{1}{2}$ yd. put with the 3 yd. makes $3\frac{1}{2}$ yd. The $3\frac{1}{2}$ yd. = $\frac{7}{11}$ rd., and this $\frac{7}{11}$ rd. put with the 203 rd. makes $203\frac{7}{11}$ rd. = $\frac{7}{11}$ mi.

2. Express 127 rd. 0 ft. 7.92 in. as the decimal of a mile.

SOLUTION.
$7.92 \div 12 = 0.66.$
$0.66 \div 16\frac{1}{2} = 0.04.$
$127.04 \div 320 = 0.397.$

12	7.92	in.
$16\frac{1}{2}$	0.66	ft.
320	127.04	rd.
	0.397	mi.

3. Express 5 lb. 4 oz. as the fraction of 6 lb. 12 oz.

5 lb. 4 oz. = 84 oz. 6 lb. 12 oz. = 108 oz.

$\frac{84}{108} = \frac{7}{9}$. *Ans.*

EXERCISE 114. — WRITTEN.

1. What fraction of a yard is 2 ft. 6 in.?
2. What fraction of a mile is 3 rd. 1 yd.?
3. What fraction of an acre is 13 sq. rd. 22 sq. yd.?
4. What fraction of $3\frac{3}{4}$ lb. is $4\frac{1}{2}$ oz.?
5. What fraction of 2 mi. is 4 rd. 2 yd. 1 ft. 4 in.?
6. Reduce 5 cwt. 64 lb. to the decimal of a ton.
7. Reduce 11 hr. 55 min. 40.8 sec. to the decimal of a day.
8. Reduce 3267 sq. ft. to the decimal of an acre.
9. Reduce 146 days to the decimal of a common year.
10. Reduce 14 ounces to the decimal of a pound.
11. What decimal of 72 mi. 80 rd. is 9 mi. 10 rd.?
12. What decimal of 118 bu. 2 pk. is 7 bu. 1 pk. 5 qt.?

Longitude and Time.

219. A *meridian* is a line drawn straight around the earth, passing through both poles.

220. Longitude is reckoned in degrees, minutes, and seconds east or west from a **standard meridian,** as the meridian of Greenwich, near London. The longitude of a place is never greater than 180°, half the distance round the earth.

221. When two places are both east or both west of the standard meridian, the difference of their longitudes is found by subtracting the one from the other.

When one place is east and the other west of the standard meridian, the difference of their longitudes is found by adding the two longitudes.

If the sum of two longitudes is greater than 180°, this sum must be subtracted from 360° to obtain the correct difference of longitude.

222. As the earth turns upon its axis once in twenty-four hours, a point on the earth's surface will describe a circumference (360°) in twenty-four hours. Therefore longitude may be reckoned in *time* as well as in degrees.

In one hour a point on the earth's surface describes $\frac{1}{24}$ of $360° = 15°$; in one minute, $\frac{1}{60}$ of $15° = 15'$; and in one second, $\frac{1}{60}$ of $15' = 15''$.

Again, since it requires one hour (60 min.) for a point to pass over 15°, to pass over 1° it requires $\frac{1}{15}$ of 60 min. = 4 min.; and to pass over 1′ it requires $\frac{1}{60}$ of 4 min. = 4 sec.

223. Express 20° 36′ 15″ of longitude in time.

SOLUTION. Since 15° longitude give 1 hr. in time, 15′ longitude 1 min., and 15″ longitude 1 sec., divide 20° 36′ 15″ by 15, as in compound division, and the quotient will be the time required.

$$15\overline{)20°\ 36'\ 15''}$$
1 hr. 22 min. 25 sec.

224. **Express 1 hr. 4 min. 4 sec. in degrees.**

1 hr. 4 min. 4 sec.	
15	
16° 1′ 0″	

Solution. Since 1 hr. of time equals 15° of longitude, 1 min. of time 15′, and 1 sec. of time 15″, multiply 1 hr. 4 min. 4 sec. by 15, as in compound multiplication, and the product will be the longitude required.

225. Hence, if longitude is expressed in **degree-measures, divide** by 15; the quotient gives the longitude in **time-measures.**

226. If longitude is expressed in **time-measures, multiply** by 15; the product gives the longitude in **degree-measures.**

EXERCISE 115. — WRITTEN.

Find the difference in longitude between two cities if the difference in time is:

1. 4 hr. 12 min. 30 sec.
2. 5 hr. 15 min. 55 sec.
3. 1 hr. 48 min. 20 sec.
4. 7 hr. 55 min. 10 sec.

Find the difference in time between two cities if the difference in longitude is:

5. 55° 25′.
6. 44° 36′ 55″.
7. 18° 12′ 15″.
8. 104° 58′ 40″.

9. The difference in longitude between Boston and Chicago is 16° 30′ 38″. Find the difference in time.

10. The difference in time between St. Louis and Paris is 6 hr. 10 min. 22 sec. Find the difference in longitude.

11. Find the difference in time between Halifax, longitude 63° 36′ West, and San Francisco, longitude 122° 26′ 15″ West.

12. The difference in time between Constantinople and New York is 6 hr. 51 min. 56 sec. What is the difference in longitude?

13. The difference in time between Berlin and Boston is 5 hr. 37 min. 49 sec. What is the difference in longitude?

14. The difference in longitude between St. Petersburg and New Orleans is 120° 19′. What is the difference in time?

227. Since the sun *appears* to move from east to west, sunrise will occur earlier at all points east, and later at all points west, of a given place. Hence, clock-time will be later in all places east, and earlier in all places west of a given meridian.

Therefore, if the time of a place is given,

To find the time of a place **east, add** to the given time the difference of time between the two places.

To find the time of a place **west, subtract** from the given time the difference of time between the two places.

228. To find the difference in clock-time when the difference in longitude is known.

When it is noon at Boston (long. 71° 3′ 30″ West), what is the time at Paris (long. 2° 20′ 22″ East)?

$$\begin{array}{rrrl} 71^\circ & 3' & 30'' & \text{W.} \\ 2^\circ & 20' & 22'' & \text{E.} \\ \hline 73^\circ & 23' & 52'' & \ldots \text{ difference in longitude.} \end{array}$$

$$15\overline{)73^\circ \; 23' \; 52''}$$
$$4 \text{ hr. } 53 \text{ min. } 35\tfrac{7}{15} \text{ sec.}$$

53 min. $35\frac{7}{15}$ sec. past 4 P.M. *Ans.*

Since Boston is west and Paris is east of the meridian of Greenwich, the difference between their longitudes is found by taking the sum of their longitudes.

Their difference in longitude, 73° 23′ 52″, is equivalent to 4 hr. 53 min. $35\frac{7}{15}$ sec., and as Paris is *east* of Boston, the time at Paris is found by *adding* the 4 hr. 53 min. $35\frac{7}{15}$ sec. to the time at Boston.

Exercise 116. — Written.

1. When it is noon at Greenwich, what is the clock-time at Calcutta, longitude 88° 19′ 2″ E.?

2. When it is half-past four P.M. at Boston, longitude 71° 3′ 30″ W., what is the clock-time at San Francisco, longitude 122° 26′ 15″ W.?

3. When it is ten o'clock A.M. at Constantinople, longitude 28° 59′ E., what is the clock-time at Paris, longitude 2° 20′ 22″ E.?

4. When it is ten o'clock A.M. at Berlin, longitude 13° 23′ 43″ E., what is the clock-time at Boston, longitude 71° 3′ 30″ W.?

5. When it is seven o'clock A.M. at Pekin, longitude 116° 23′ 45″ E., what is the clock-time at Constantinople, longitude 28° 59′ E.?

6. When it is six o'clock P.M. at Rome, longitude 12° 27′ 14″ E., what is the clock-time at Pekin, longitude 116° 23′ 45″ E.?

7. When it is four o'clock P.M. at San Francisco, longitude 122° 26′ 15″ W., what is the clock-time at St. Louis, longitude 90° 15′ 15″ W.?

8. Find the longitude of a steamer when the time is found to be 2 hr. 45 min. 15 sec. later than Greenwich time.

Note. **Standard time** is the clock-time of some selected meridian. **Eastern standard time** is the clock-time of the meridian 75° west of Greenwich, and is five hours slower than Greenwich time. **Central standard time** is the clock-time of 90° west of Greenwich, and is just one hour slower than Eastern standard time. **Mountain standard time** is the clock-time of the meridian of 105°, and is one hour slower than that of 90°. **Western standard time** is the clock-time of the meridian of 120°, and is one hour slower than that of 105°. The railroads and many cities and towns of the United States have adopted standard time.

Places not more than 7½° east or west of the meridians of 75°, 90°, 105°, 120° are reckoned to have the same time respectively as places on these meridians.

Exercise 117. — Oral.

Review Problems.

1. How many pints are there in 2 gallons?

2. How many gallons are there in 32 pints?

3. How many pecks are there in 8 bushels?

4. How many bushels are there in 40 pecks?

5. How many bushels are there in 64 quarts?

6. How many quarts are there in 3 bushels?

7. How many quarts are there in 7 pecks?

8. How many pecks are there in 64 quarts?

9. How many pints are there in 3 quarts and 2 pints?

10. How many quarts are there in 4 gallons and 3 quarts? 5 gallons and 1 quart?

11. How many ounces are there in 4 pounds?

12. How many pounds are there in 32 ounces?

13. How many tons are there in 10,000 pounds?

14. How many pounds of hay are there in 3 tons 500 pounds? in 2 tons 750 pounds?

15. How many pounds in $\frac{1}{2}$ a ton? $\frac{1}{4}$? $\frac{1}{8}$? $\frac{1}{10}$? $\frac{1}{20}$?

16. How many feet are there in 72 inches?

17. How many rods are there in a mile? $\frac{1}{2}$ mi.? $\frac{1}{4}$ mi.? $\frac{1}{8}$ mi.? $\frac{1}{16}$ mi.? $\frac{1}{32}$ mi.? $\frac{1}{40}$ mi.? $\frac{1}{80}$ mi.?

18. How many yards are there in a mile? $\frac{1}{2}$ mi.? $\frac{1}{4}$ mi.? $\frac{1}{8}$ mi.? $\frac{1}{16}$ mi.? $\frac{1}{32}$ mi.? $\frac{1}{40}$ mi.? $\frac{1}{80}$ mi.?

19. How many feet are there in a mile? $\frac{1}{2}$ mi.? $\frac{1}{4}$ mi.? $\frac{1}{8}$ mi.? $\frac{1}{16}$ mi.? $\frac{1}{32}$ mi.? $\frac{1}{40}$ mi.? $\frac{1}{80}$ mi.?

20. How many square rods in an acre? $\frac{1}{2}$ A.? $\frac{1}{4}$ A.? $\frac{1}{8}$ A.? $\frac{1}{16}$ A.? $\frac{1}{32}$ A.? $\frac{1}{40}$ A.? $\frac{1}{80}$ A.?

21. How many square yards in an acre? $\frac{1}{2}$ A.? $\frac{1}{4}$ A.? $\frac{1}{8}$ A.? $\frac{1}{16}$ A.? $\frac{1}{40}$ A.? $\frac{1}{80}$ A.?

22. How many acres in a square mile? $\frac{1}{2}$ sq. mi.? $\frac{1}{4}$ sq. mi.? $\frac{1}{8}$ sq. mi.? $\frac{1}{16}$ sq. mi? $\frac{1}{40}$ sq. mi.? $\frac{1}{80}$ sq. mi.?

23. Which is heavier, a pound of lead or a pound of gold? an ounce of lead or an ounce of gold?

24. How many cubic feet are there in 1 cubic yard? in $\frac{1}{3}$ cu. yd.? in $\frac{1}{9}$ cu. yd.? in $\frac{1}{2}$ cu. yd.?

25. How many cubic feet are there in $1\frac{1}{3}$ cubic yards?

26. How many cubic feet are there in $\frac{1}{2}$ a cord? in $\frac{1}{4}$? in $\frac{1}{8}$? in $\frac{1}{16}$? in $\frac{1}{32}$?

27. How many cubic feet are there in a cord foot? in 2 cord feet? in 3 cord feet? in 4 cord feet?

28. How many inches are there in $\frac{1}{4}$ yd.? $\frac{1}{3}$ yd.? $\frac{1}{2}$ yd.? $\frac{1}{6}$ yd.? $\frac{1}{9}$ yd.? $\frac{1}{12}$ yd.?

29. How many square yards are there in 54 sq. ft.? in 108 sq. ft.? 45 sq. ft.? 63 sq. ft.?

30. At 4 cents a quart, what is the cost of a can of milk if the can holds 2 gallons and 1 pint?

31. What part of a peck is 1 qt.? 2 qt.? 4 qt.? 6 qt.?

32. What part of a bushel is 1 qt.? 8 qt.? 16 qt.? 24 qt.? 12 qt.? 20 qt.?

33. What part of a gallon is 1 qt.? 1 pt.? $\frac{1}{2}$ pt.?

34. What part of an acre is 10 sq. rd.? 60 sq. rd.? 80 sq. rd.? 120 sq. rd.? 40 sq. rd.?

35. What part of a square yard is 1 sq. ft.? 3 sq. ft.? 6 sq. ft.? 5 sq. ft.? 7 sq. ft.?

36. What part of a square foot is 1 sq. in.? 72 sq. in.? 36 sq. in.? 48 sq. in.? 60 sq. in.?

37. What part of a mile is 1 rd.? 160 rd.? 80 rd.? 40 rd.? 32 rd.? 64 rd.? 120 rd.?

38. What part of a yard is 1 in.? 9 in.? 18 in.? 24 in.? 27 in.? 6 in.? 3 in.? 8 in.?

39. What part of a cubic yard is 1 cu. ft.? 9 cu. ft.? 18 cu. ft.? 3 cu. ft.? 6 cu. ft.?

40. What part of a cord is 1 cu. ft.? 16 cu. ft.? 32 cu. ft.? 64 cu. ft.?

41. What part of a day is 1 hr.? 4 hr.? 6 hr.? 8 hr.? 10 hr.? 12 hr.? 18 hr.?

42. What part of an hour is 1 min.? 30 min.? 45 min.? 15 min.? 20 min.? 36 min.? 48 min.?

43. What part of a minute is 1 sec.? 20 sec.? 25 sec.? 50 sec.? 40 sec.? 45 sec.? 24 sec.?

44. What part of a circumference is 1°? 90°? 45°? 60°? 120°? 180°? 270°? 240°?

45. What part of a right angle is 1°? 30°? 60°? 45°? $22\frac{1}{2}$°? $11\frac{1}{4}$°?

46. What part of a mile is 1 yd.? 440 yd.? 880 yd.?

47. What part of a mile is 1 ch.? 20 ch.? 40 ch.? 60 ch.?

48. What part of an acre is 1 sq. ch.? 2 sq. ch.? 4 sq. ch.? 5 sq. ch.? 8 sq. ch.?

49. What part of a bushel of corn is 1 lb.? 14 lb.? 28 lb.? 40 lb.? 48 lb.?

50. What part of a bushel of wheat is 1 lb.? 15 lb.? 30 lb.? 40 lb.? 48 lb.?

51. A surveyors' chain has 100 links, and is 66 ft. long. What decimal of a foot is the length of each link?

Exercise 118. — Written.

Review Problems.

1. London covers 689 square miles and Greater New York 359 square miles. How many more acres does London cover than Greater New York?

2. If 100 hills of corn yield a bushel, and each hill occupies 12 square feet, how many bushels of corn will an acre produce?

3. How many seconds are there in 365 days 5 hours 48 minutes 48 seconds?

4. What will 13 hogsheads of molasses cost at 11 cents a quart, if each hogshead contains 63 gallons?

5. What is the cost of 7 bushels 3 pecks 2 quarts of blueberries at 12 cents a quart?

6. How many quart boxes will be needed to hold 12 bu. 2 pk. 7 qt. of blueberries?

7. How many pint bottles will be needed to hold 17 gal. 3 qt. 1 pt. of vinegar?

8. How many steps $\frac{3}{4}$ yd. long will a man take in walking 1 mile and 370 yards?

9. A grocer bought 4 barrels of cranberries, each containing 2 bu. 3 pk. 7 qt., for $8 a barrel, and sold them at 10 cents a quart. How much did he gain?

10. What is the cost of 11 feet of lead pipe weighing 6 pounds to the foot, at 12 cents a pound?

11. How many bushels of carrots will a ten-acre field produce, if each square rod produces 4 bushels?

12. How many bushels of wheat are there in 1260 pounds?

13. How many bushels of oats are there in 1600 pounds?

14. How many bushels of corn are there in 1680 pounds?

15. How many bushels of beans are there in 1740 pounds?

16. How many bushels of potatoes are there in 2240 pounds?

17. In quick marching, soldiers take 120 steps, 30 inches each, per minute. At this rate, how far will they march in an hour?

18. In double-quick marching, soldiers take 180 steps, 36 inches each, per minute. At this rate, how far will they march in 20 minutes?

19. A certain miller bought 18 tons 1700 pounds of wheat one week, 19 tons 1500 pounds the next week, 21 tons 1345 pounds the next week, and 20 tons 1835 pounds the fourth week. At 56 cents a bushel, what was the total amount paid for the wheat?

20. In four piles of wood there are 13 cords 76 cu. ft., 37 cords 26 cu. ft., 25 cords 55 cu. ft., 33 cords 102 cu. ft. Find the value of the four piles at $4 a cord.

21. A firkin of butter weighed 37 lb. 7 oz., and the firkin weighed 5 lb. 12 oz. Find the weight of the butter.

22. If two boys play tennis from 2 hr. 47 min. 45 sec. past noon until 5 o'clock, how long do they play?

23. St. Paul's Cathedral in London is in latitude 51° 30′ 48″, and St. Peter's in Rome is in latitude 41° 53′ 54″. Find the difference in their latitudes.

24. The longitude of Boston is 71° 3′ 30″ W., the longitude of San Francisco is 122° 26′ 15″ W. When it is noon at San Francisco what is the time at Boston?

25. A coal dealer sold $22\frac{1}{2}$ tons of coal. After he delivered 13 tons 1750 pounds, how much more had he to deliver?

26. A bicycle wheel 7 ft. 4 in. in circumference is making 3 revolutions a second. How many miles an hour is the rider going?

27. The fore quarters of a lamb weighed 7 lb. 3 oz. each, and the hind quarters weighed 9 lb. 5 oz. each. What was the weight of the lamb?

28. A farm of 100 acres is divided into house lots. The streets require one-eighth of the whole farm, and the lots number 140. How many square rods are there in each lot?

29. How many sleepers laid 2 ft. 6 in. from centre to centre will be required for a mile of railway?

30. How many turns will a bicycle wheel 7 ft. 4 in. round make in going $5\frac{5}{8}$ miles?

31. A man bought 17 long tons of soft coal at $5 a ton, and sold it at 20 cents a bushel. If a bushel of soft coal weighs 70 pounds, how much did he gain?

32. A farmer sold a load of hay weighing 1876 pounds at $18.50 a ton. What did he get for his load?

33. If 7 horses eat 17 bushels 2 quarts of oats in a week, what does 1 horse eat?

34. A cistern holds 3780 gallons of water. How many barrels does it hold?

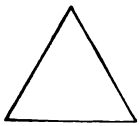
35. How many iron bars each weighing 13 lb. 9 oz. will make a load of 2 tons 1425 pounds?

36. If the average speed of a railway train is 27 miles 110 rods an hour, how many hours will it be in going 409 miles 220 rods?

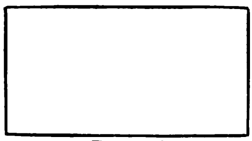
CHAPTER XI.

MEASUREMENTS.

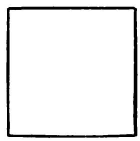
229. A **Triangle** is a plane figure with three straight sides. The side on which the triangle rests is the *base*, and the opposite corner the *vertex*. The shortest distance from the vertex to the base, or base produced, is the *height* or *altitude* of the triangle.



Triangle.



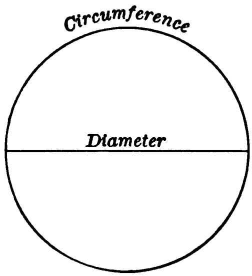
Rectangle.



Square.

230. A **Rectangle** is a plane figure with four straight sides and four square corners.

231. A **Square** is a plane figure with four equal straight sides and four square corners.



232. The **Perimeter** of a plane figure is the distance round it.

233. A **Circle** is a plane figure bounded by a curved line called the *circumference*, all points of which are equally distant from a point within called the *centre*. A straight line drawn through the centre, having its ends in the circumference, is called a *diameter;* and half the diameter is called the *radius*.

To find the circumference when the diameter is given,

Multiply the diameter by 3.1416.

To find the diameter when the circumference is given,

Divide the circumference by 3.1416.

NOTE. In oral exercises use $3\frac{1}{7}$ for 3.1416.

EXERCISE 119. — ORAL.

1. What is the perimeter of a triangle whose sides are $3\frac{1}{2}$ ft., $4\frac{3}{4}$ ft., and 5 ft.?

2. What is the perimeter of an equilateral triangle, if each side is $5\frac{3}{8}$ in.?

3. If the perimeter of an equilateral triangle is 10 yd., what is the length of each side in feet?

4. A rectangular field is 23 rd. long and 15 rd. wide. How many rods of fence will enclose the field?

5. It requires 72 rd. of fence to enclose a square field. How long is a side of the field?

6. One side of a triangle is $10\frac{1}{2}$ ft. and a second side is $8\frac{1}{4}$ ft. What is the length of the third side if the perimeter is 25 ft.?

7. The perimeter of a triangle is 18 in. One side is 7 in. and the other two sides are equal. How long is one of the equal sides?

8. What is the circumference of a circle whose diameter is 14 feet?

9. How many posts 1 rd. apart will be needed for the fence of a circular field whose diameter is 21 rods?

10. What is the diameter of a circle whose circumference is 22 ft.? What is its radius?

11. A hexagon (a figure of six sides) has each side 5 in. long. What is the length of its perimeter?

12. The perimeter of a hexagon having equal sides is 24 in. What is the length of each side?

Exercise 120. — Written.

1. What is the cost for fencing a rectangular field $185\frac{1}{2}$ ft. long, and $123\frac{1}{4}$ ft. wide, if the fence costs \$0.50 a yard?

2. How many feet of molding will be needed for the four walls of a room 18 ft. 4 in. long and 15 ft. 10 in. wide?

3. The ceiling of a room is a hexagon having equal sides, each side of which is $9\frac{1}{2}$ ft. How many yards of border will be needed to go round the room?

4. The sides of a triangular field are 197 ft., $256\frac{11}{12}$ ft., and $301\frac{3}{4}$ ft. What is the cost of the wire for fencing the field, if the fence contains 4 wires and wire costs $\frac{3}{8}$ of a cent a yard?

5. What is the circumference of a circle whose diameter is 23 rd.? Whose radius is 15 ft.?

6. What is the diameter of a circle whose circumference is 84 in.?

7. What is the diameter of a circle whose circumference is 1 ft.? (Carry the answer to five decimal places.)

8. Multiply 84 by 0.31831, and compare your answer with the answer of the sixth problem. In what two ways can you find the diameter of a circle knowing the circumference? Which is easier?

9. If the circumference of a wheel is 88 in., how many turns will the wheel make in going 770 ft.?

10. The diameter of a wheel is $3\frac{1}{2}$ ft. How many feet will the wheel go in turning 24 times?

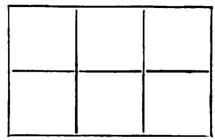
11. A bicycle geared at 60 goes as far in one complete turn of the pedals as a wheel whose diameter is 60 in. How many feet will such a bicycle go at each turn of the pedals?

234. A **surface** has two **dimensions,** *length* and *breadth.*

235. The **unit of surface measure** is a **square,** each side of which is a unit of length.

236. The **area of a surface** is the **number of square units** it contains.

Suppose the rectangle in the margin is 3 in. long and 2 in. wide. If lines are drawn as represented in the figure, the surface will be divided into *square inches.* There will be 2 horizontal rows of 3 square inches each; that is, in all, 2 × 3 square inches. Hence,



Express the length and breadth of a rectangle in the same linear unit; the product of these two numbers will express its area in square units of the same name as the linear unit.

The number of square units in a rectangle divided by the number of linear units in one dimension gives the number of linear units in the other dimension.

237. The **area of a triangle** is half the product of its base by its height, or altitude.

238. The **area of a circle** is found by taking half the product of the circumference by the radius; the product of the square of the radius by 3.1416; or, the product of the square of the diameter by 0.7854 ($\frac{1}{4}$ of 3.1416).

NOTE. In writing the dimensions of surfaces and solids, the sign × is used for the word *by*, an accent (′) for the word *feet*, and two accents (″) for the word *inches*. Thus, the dimensions of the floor of a room 15 ft. 6 in. by 13 ft. 8 in. are denoted by 15′ 6″ × 13′ 8″.

EXERCISE 121. — ORAL.

1. How many square rods are there in a rectangular field 16 rd. by $4\frac{1}{2}$ rd.? What is the unit of surface?

2. A square board is 9 in. long. How many square inches does it contain? What is the unit of surface?

3. What is the area of a triangle if its base is 10 ft. and its altitude $2\frac{2}{5}$ ft.?

4. The area of a rectangle is 26 sq. ft. If the length of the rectangle is 6 ft., what is its breadth?

5. A square is 8 in. long; a rectangle having the same perimeter is 4 in. wide. What is the length of the rectangle? How many square inches are there in the rectangle? In the square?

6. What is the area of a circle whose radius is 7 yd.? What is the unit of surface?

7. How many tiles 6 in. square will cover a floor $12' \times 10'$?

8. A square is 7 in. long; a rectangle 12 in. long has the same area. How wide is the rectangle?

EXERCISE 122. — WRITTEN.

1. How many acres are there in a rectangular field 145 rd. long and 86 rd. wide?

2. A rectangular field contains $2\frac{3}{4}$ A. and is $12\frac{2}{3}$ rods wide. How many rods long is the field?

3. A city lot in the shape of a triangle has a base of 82 ft. and an altitude of $45\frac{1}{2}$ ft. What is the value of the lot at \$5.50 a square foot?

4. How many paving blocks $8'' \times 2''$ will be needed to pave a street 250 ft. long and 40 ft. wide, making no allowance for spaces between the blocks?

5. What is the cost of concreting a walk $82\frac{1}{2}' \times 6'$ at \$0.70 a square yard?

6. How many square feet of glass will be needed for 6 windows, if each window contains 8 panes each $14'' \times 10''$?

7. A rectangle and a triangle have equal areas. The rectangle is $18\frac{1}{8}' \times 14'$; the altitude of the triangle is 22 ft. What is the base of the triangle?

8. What is the area of a circle whose radius is 28 in.?

9. The diameter of a barrel head is 18 in. What is its radius? How many square inches are there in the surface of the barrel head?

10. The radii of two circles are $3\frac{1}{2}$ in. and 7 in. What is the area of each circle? If the circles are placed so that their centres coincide, what is the area of the ring between the two circumferences?

11. A circular grass plot has a radius of 42 ft. What is the area of a walk 4 ft. wide surrounding the grass plot?

12. The radius of a circle is 1 ft. How does its area compare with the area of a circle whose radius is 2 ft.? 3 ft.? 4 ft.? 5 ft.?

13. The radius of the rotunda of the Pantheon of Rome is 71 ft. 6 in. Find the area of the floor in square feet.

14. The two dials of the clock of St. Paul's, London, are each $18\frac{1}{2}$ ft. in diameter. Find the area of each in square feet.

15. A side of Russell Square in London is 660 ft. How many acres does it contain?

16. What part of a circle is a semicircle? What is the area of a semicircle whose radius is $9\frac{2}{11}$ yd.?

17. Make a circle in a square touching the sides of the square. If the side of the square is 10 in. long, what is the diameter of the circle? What is the area of the square? What is the area of the circle?

Carpeting Rooms.

239. In determining the number of yards of carpeting required for a room, we first decide whether the strips shall run lengthwise or across the room, and then find the number of strips needed. The number of yards in a strip, including the waste in matching the pattern, multiplied by the number of strips will give the required number of yards.

How many yards of carpet $2\frac{1}{4}$ ft. wide will cover a floor 17 ft. by 15 ft., if the strips run across the room?

$17 \div 2\frac{1}{4} = 7\frac{5}{9}$. Hence, 8 strips are required and $\frac{4}{9}$ of the width of a strip will be *turned under*. 15 ft. = 5 yd., and 8 × 5 yd. = 40 yd., the number of yards required, if there is no waste.

Exercise 123. — Written.

1. How many yards of carpet 27 in. wide will it take to carpet a room $16\frac{1}{2}'$ by $12'$, if the strips run lengthwise? What is the cost at $0.90 a yard?

2. A man wishes to carpet a room $18' \times 16'$, and does not wish to cut a strip or turn any carpet under. Which way must the strips run if the carpet is 27 in. wide?

3. Find the cost of carpeting a room $14\frac{1}{2}' \times 13'$ with carpet 1 yd. wide, worth $0.85 a yard, if the strips run lengthwise and $\frac{1}{8}$ of a yard is allowed for each strip for matching the pattern.

4. How many yards of carpet $\frac{3}{4}$ yd. wide will be needed to carpet a room $19' \times 14\frac{1}{2}'$, if the strips run across the room? How many if the strips run lengthwise?

5. Find the cost of linoleum, which is sold by the square yard, for a floor $12\frac{3}{4}' \times 11'$ at $0.85 a square yard.

Papering Rooms.

240. Wall paper is 18 in. wide, and is sold in single rolls 8 yd. long, or in double rolls 16 yd. long.

In estimating the number of rolls of paper required for a room of ordinary height, find the number of feet in the perimeter of the room, leaving out the widths of the doors and windows, and allow a double roll or two single rolls for every 7 ft.

NOTE. A room is considered of ordinary height when the distance from the base board to the border is not more than 8 ft.

How many double rolls of paper will be required for a room of ordinary height, 18′ × 16′, with one door and three windows, each 4 ft. wide?

Perimeter of room $= 2(18' + 16') = 68'$
Width of door and windows $= 4 \times 4' = \underline{16'}$
Deducting door and windows $= 52'$
$52 \div 7 = 7\frac{3}{7}$. 8 double rolls. *Ans.*

EXERCISE 124. — WRITTEN.

1. How many double rolls of paper will be required for a room of ordinary height, 14′ 8″ × 13′ 6″, with 1 door and 2 windows, each 3′ 6″ wide?

2. What is the cost of paper, at 35 cents a single roll, for a room of ordinary height, 16′ 4″ × 15′, with 2 doors and 3 windows, each 3 ft. 6 in. wide?

3. What is the cost of border for the same room at 45 cents a yard, making no allowance for doors and windows?

4. What is the cost of the paper at 90 cents a double roll for a room 18′ 10″ × 16′ 5″, with 2 doors, each 3 ft. 8 in. wide, 2 windows, each 4 ft. wide, and 1 window 3 ft. wide?

Plastering, Painting, and Paving.

241. The unit of plastering, painting, paving, and ceiling is the square yard.

The rule for estimating such work is:

Measure the total area; deduct from this total area half the area of doors, windows, and other openings, and express the result to the nearest square yard.

At 18 cents a square yard, find the cost of plastering a room $18' \times 16' \times 10'$, if the room has a base board 10 in. high, two doors $7'\ 4'' \times 4'$, and three windows $6'\ 6'' \times 4'$.

Perimeter of room $= 2\,(18' + 16')$	$= 68'$	
Height above base board $= 10' - 10''$	$= 9\frac{1}{6}'$	
Total wall area is $9\frac{1}{6}' \times 68'$		$= 623\frac{1}{3}$ sq. ft.
Area of ceiling is $18' \times 16'$		$= 288$ sq. ft.
Total area		$= 911\frac{1}{3}$ sq. ft.
Height of doors above base board is $7'\ 4'' - 10'' = 6'\ 6''$		
Area of 2 doors $= 2\,(4' \times 6'\ 6'')$	$= 52$ sq. ft.	
Area of 3 windows $= 3\,(4' \times 6'\ 6'')$	$= 78$ sq. ft.	
Total area of openings	$= 130$ sq. ft.	
Half of area of openings		$= 65$ sq. ft.
Net area		$= 846\frac{1}{3}$ sq. ft.
		$= 94\frac{1}{27}$ sq. yd.

At 18 cents a square yard, 94 sq. yd. will cost

94×18 cts. $= \$16.92$. *Ans.*

Exercise 125. — Written.

Find at 20 cents a square yard the cost of plastering:

1. A room (walls and ceiling) $22' \times 18' \times 10'$, with two doors $7'\ 6'' \times 4'$, three windows $6'\ 6'' \times 4'$, and a base board of 10 inches.

2. A room (walls and ceiling) $16' \times 15' \times 10'$, with two doors $7'\ 6'' \times 4'$, three windows $6' \times 4'$, and a base board of 10 inches.

3. A room (walls and ceiling) $16' \times 15'\ 6'' \times 9'\ 6''$, with one door $7'\ 4'' \times 4'$, three windows $6' \times 4'$, and a base board of 9 inches.

Find the cost at 15 cents a square yard of painting:

4. The outside of the walls of a cottage-roofed house $36' \times 32' \times 13'$.

5. The outside of the walls of a house $42' \times 34' \times 19'$, with gables extending 10 ft. above the end walls, the two gables to be reckoned as one full wall.

6. The walls of a room $24' \times 20' \times 12'$, allowing for two doors $7'\ 6'' \times 4'$ and four windows $6'\ 6'' \times 4'$.

7. How many bricks $8'' \times 4'' \times 2\frac{1}{2}''$, laid flatwise, will be needed to pave a rectangular yard $60' \times 30'$?

8. How many bricks of the same size will be needed to pave the yard of Example 7, if the bricks are laid on edge?

9. How much will it cost to pave a road 50 feet wide and half a mile long at $1.75 a square yard?

10. How many panes of glass $8'' \times 10''$ will there be in a box containing 50 sq. ft.?

11. A stone-cutter dressed the tops of four stone steps, each $6' \times 18''$, at 40 cents a sq. ft. How much was his bill?

Laths and Clapboards.

242. Laths. Laths are put up in bundles of 100 pieces, each 4 ft. long, and a bundle is estimated to cover 5 sq. yd.

In estimating the number of bundles, deduct the area of all openings.

How many bundles of laths will be required for the ceiling of a room $17' \times 14'$?

$$17 \times 14 = 238 \text{ sq. ft.} = 26\tfrac{4}{9} \text{ sq. yd.}$$

$$26\tfrac{4}{9} \div 5 = 5\tfrac{13}{45}. \qquad \textbf{6 bundles. } \textit{Ans.}$$

243. Clapboards. Clapboards are put up in bundles of 25, each four ft. long, and are laid $3\frac{1}{2}$ inches to the weather. Therefore each clapboard covers $1\frac{1}{6}$ sq. ft.

Find the number of clapboards required for the front of a house 42 ft. long and 22 ft. high, allowing 200 sq. ft. for doors and windows.

$$42 \times 22 = 924 \text{ sq. ft.}$$
$$924 \text{ sq. ft.} - 200 \text{ sq. ft.} = 724 \text{ sq. ft.}$$
$$\frac{724}{1\frac{1}{6}} = \frac{6}{7} \text{ of } 724 = 620\tfrac{4}{7}. \qquad \textbf{621 clapboards. } \textit{Ans.}$$

Exercise 126. — Written.

1. How many bundles of laths will be required for the walls and ceiling of a room 15′ × 13′ and 9′ high, allowing 19 sq. yd. for doors, windows, and base board?

2. How many bundles of laths will be required for a room 21′ × 18′ 6″ and 9′ 8″ high, allowing for a base board 8″ high, 2 doors each 7′ × 4′, and 4 windows each 6′ × 3′ 10″?

3. How many clapboards will be required for the front of a house 26 ft. long and 19 ft. high, allowing 126 sq. ft. for doors and windows?

4. How many clapboards will be required for a house 42′ × 34′ × 21′ with gables extending 10 ft. above the end walls, the two gables to be reckoned as one full wall, and 540 sq. ft. to be allowed for doors and windows?

5. How many clapboards will be required for a barn 40 ft. square and 16 ft. to the eaves, with gables extending 12 ft. above the walls, allowing 200 sq. ft. for doors and windows? What will the clapboards cost at $40 a thousand?

Roofing and Flooring.

244. **The unit of measure for roofing and flooring is a square containing 100 sq. ft.**

Shingles are 16 in. long, and are estimated to average 4 in. wide, so that a shingle laid $4\frac{1}{2}$ in. to the weather will cover 18 sq. in., and 8 shingles will cover 1 sq. ft. At this rate, 800 shingles would cover a square, but to allow for waste it is usual to reckon 1000 shingles to the square.

It is found, however, in practice, that 1000 shingles of the best quality, laid $4\frac{1}{2}$ in. to the weather, will cover about 120 sq. ft.

Shingles are put up in bunches of 250, and therefore it takes 4 bunches for a thousand.

EXERCISE 127. — WRITTEN.

1. Allowing 1000 shingles for 110 sq. ft., how many thousand will be required to cover the pitched roof of a house 36 ft. long, if the width of each side of the roof is 21 ft.?

2. Allowing 1000 shingles for 120 sq. ft., how many thousand would be required to cover the four walls of a flat-roofed house $42' \times 36'$ and $22'$ high, allowing 500 sq. ft. for doors and windows?

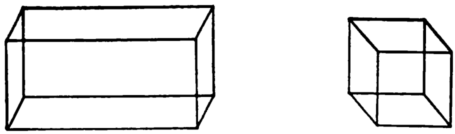
3. How many slates at 3 to the square foot will be required to cover 13 squares of roof?

4. Find the cost of a floor for a room $18' \times 16'$, at $22 per square.

5. Find the cost of a birch floor for a room $16' \times 15'$, if birch floor-boards are worth $40 a thousand square feet, and if the waste is reckoned at $\frac{1}{5}$ of the area of the floor.

6. Find the cost of laying an oak floor for a dining-room $16' \times 17'$, reckoning the nails and labor at $12, the oak floor-boards at $80 a thousand, and the waste at 34 sq. ft.

Rectangular Solids.



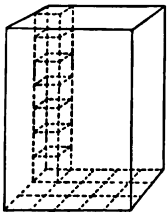
245. A **Rectangular Solid** is a solid bounded by six rectangles.

If the rectangles are all squares, the solid is called a **cube**.

246. A rectangular solid has three **dimensions**: *length, breadth, and thickness.*

How many *edges* has a rectangular solid? Are the edges of a cube equal or unequal?

247. Find the volume of a rectangular solid whose length is 5 in., breadth 3 in., and height 7 in.



The face on which the solid rests may be divided into square inches; there will be three rows of 5 sq. in. each; in all 15 sq. in. Upon each square inch may be placed a pile of 7 cu. in., so that the solid will contain 15×7 cu. in.; that is, $3 \times 5 \times 7$ cu. in.

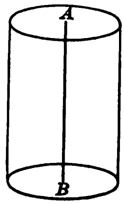
248. To find the volume of a rectangular solid,

Express its length, breadth, and height in the same linear unit; the product of these numbers will express its volume in cubic units of the same name as the linear unit.

If the number of cubic units in the volume is divided by the product of the numbers of linear units in any two dimensions, the quotient is the number of linear units in the third dimension.

The Cylinder.

249. A **Cylinder** is a solid bounded by two equal and parallel circles called its *bases,* and a uniformly curved surface called its *lateral surface.*



NOTE. Two circles are parallel if all points of one are equally distant from the other.

250. Find the volume of a cylinder if its length is 20 in. and the diameter of its base is 20 inches.

	Area of base	= 10 × 10 × 3.1416
		= 314.16 sq. in.
	Length	= 20 in.
Therefore,	Volume	= 6283.20 cu. in. Hence,

251. To find the volume of a cylinder,

Multiply the number of square units in its base by the number of linear units of the same name in its length.

252. To find the area of the lateral surface of a cylinder,

Multiply the length of the circumference of the base by the height of the cylinder.

EXERCISE 128. — ORAL.

What is the volume of:

1. A rectangular solid 8″ × 6″ × 2″?
2. A rectangular solid 5′ × 4′ × $2\frac{1}{2}$′?
3. A rectangular solid 10′ × 6′ × $1\frac{1}{2}$′?
4. A rectangular solid 2 yd. × 2′ × 6″?
5. A cube whose edge is 3′? 5″? 6″?
6. A rectangular cistern 8′ × 5′ × 3′?

EXERCISE 129. — WRITTEN.

1. How many cubic inches are there in a rectangular cistern $8' \times 5' \times 3'$?

2. How many cubic feet of air are there in a rectangular room 36 ft. long, 32 ft. wide, and 14 ft. high?

3. A rectangular pile of bricks contains 160 cubic yards, and is 20 ft. wide and 8 ft. high. How many feet long is it?

4. How many cubic feet in a cylinder 3 ft. in diameter and 10 ft. long?

5. How many cubic feet in a well 3 ft. in diameter and 16 ft. deep?

6. What is the cost of digging a cellar 42 ft. long, 36 ft. wide, and 7 ft. deep at 41 cents a cubic yard?

7. What is the cost of the digging for a round cistern 8 ft. deep and 8 ft. in diameter at 50 cents a cubic yard?

Brick Work and Stone Work.

253. The number of bricks required for brick walls is estimated by the thousand, and 22 bricks of common size laid in mortar are reckoned for each cubic foot of wall.

254. Stone work is reckoned by the cubic foot or cubic yard, and sometimes by the perch of 25 cubic feet.

NOTE. A perch is $16\frac{1}{2}' \times 1\frac{1}{2}' \times 1'$, or $24\frac{3}{4}$ cu. ft.; but in practice a perch is understood to be 25 cu. ft.

In making estimates for brick and stone work, masons measure *the outside of the walls*, thus reckoning the corners twice.

1. Find the number of bricks required for a wall $42' \times 25'$ and 1 ft. thick.

$$42 \times 25 \times 1 \times 22 \text{ bricks} = 23{,}100 \text{ bricks.}$$

2. Find the number of cubic feet, masons' measure, in a cellar wall $40' \times 30'$, 6 ft. high, and 2 ft. thick.

Outside circuit is $2(40 + 30) = 140$ ft.
Therefore, the number is $6 \times 2 \times 140 = 1680$ cu. ft.

Exercise 130. — Written.

Solve by masons' measure, and by actual measure:

1. How many bricks will be required for the walls of a flat-roofed house $42' \times 36' \times 21'$, if the walls are 1 ft. thick and if 300 cu. ft. are deducted for doors and windows?

2. How many bricks will be required for a house with a pitched roof $40' \times 36' \times 20'$ with gable peaks 12 ft. above the end walls, the walls to be 1 ft. thick, and 350 cu. ft. to be allowed for doors and windows? (Reckon the two gables as one full wall $36' \times 12'$.)

3. How many bricks will be required for the four walls of a building $100' \times 60' \times 22'$, if the walls are $1\frac{1}{2}$ ft. thick and if 600 cu. ft. are allowed for doors and windows?

4. How many cubic feet of stone will be required for the cellar walls of a house $28' \times 36'$, if the wall is $1\frac{1}{2}$ ft. thick and 6 ft. high?

5. How many cubic yards of wall will there be in the foundation of a building $142' \times 58'$, if the wall is 6 ft. high and 3 ft. thick?

6. How many perches of split stone will be required for the foundation walls of a building $100' \times 80'$, if the walls are 6 ft. high and average 3 ft. thick?

Coal, Sand, and Gravel.

255. A short ton of anthracite coal measures about 37 cu. ft. A long ton of soft coal measures about 48 cu. ft., a short ton about 42 cu. ft. A bushel of hard coal weighs 80 lb. A bushel of soft coal weighs 70 lb.

A cubic yard of earth, sand, or gravel is called a load.

Exercise 131. — Written.

1. How many short tons of hard coal will a rectangular bin hold, if it is 10 ft. long, 8 ft. wide, and 6 ft. high?

2. How many short tons of hard coal can be put into a rectangular bin 10 ft. long, 7 ft. wide, and 5 ft. high?

3. How many long tons of soft coal can be put into a rectangular bin 12 ft. long, 10 ft. wide, and 6 ft. high?

4. How many loads are there in a rectangular cut 300 ft. long, 15 ft. wide, and 10 ft. deep?

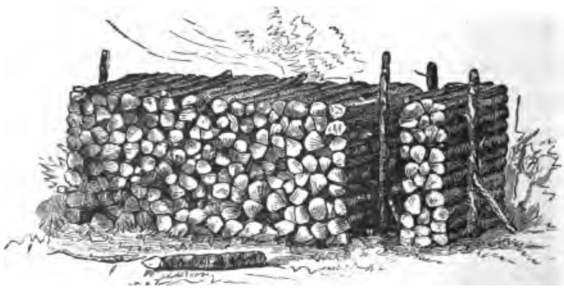
5. How many loads in an embankment 200 ft. long, 20 ft. wide, and 4 ft. high?

6. How many cubic yards are there in a semicircular tunnel 8 ft. radius and 16 rd. long?

Wood Measure.

256. A Cord of wood or stone is a pile 8 ft. long, 4 ft. wide, and 4 ft. high, making 128 cu. ft.

257. A Cord Foot is a pile 1 ft. long, 4 ft. wide, and 4 ft. high, and is therefore *one eighth* of a cord, or 16 cu. ft.



Find the number of cords in a pile of wood 30 ft. long, 4 ft. wide, and 6 ft. high.

$$\frac{\overset{15}{\cancel{30}} \times \cancel{4} \times \overset{3}{\cancel{6}}}{\underset{4}{\cancel{8}} \times \cancel{4} \times \underset{2}{\cancel{4}}} = \frac{45}{8} \text{ cords} = 5\tfrac{5}{8} \text{ cords.} \quad \text{Hence,}$$

258. To find the number of cords in a pile of wood,

Divide the product of the length, width, and height, expressed in feet, by $8 \times 4 \times 4$.

EXERCISE 132. — WRITTEN.

1. How many cords of wood in a pile 40 ft. long, 8 ft. wide, and 6 ft. high?

2. How many cords of wood in a pile 32 ft. long, 4 ft. wide, and 5 ft. high?

3. What part of a cord is there in a load of wood 8 ft. long, 4 ft. wide, and $3\frac{1}{2}$ ft. high? What does the teamster mean by calling his load 7 ft. of wood?

4. Find the cost of the wood that can be piled in a shed 20 ft. long, 16 ft. wide, 8 ft. high at $4.50 a cord.

5. What is the height of a pile of wood containing 25 cords, if it is 100 ft. long and 8 ft. wide?

6. How long must a pile of cord wood be to contain 21 cords, if it is piled 6 ft. high?

Board Measure.

259. Boards *one inch or less* in thickness are sold by the square foot.

Boards *more than one inch* in thickness, and all squared lumber, are sold by the number of square feet of boards one inch in thickness to which they are equal.

Thus, a board 16 ft. long, 1 ft. wide, and 1 in. thick contains 16 ft. *board measure.* If only $\frac{7}{8}$, $\frac{3}{4}$, or $\frac{1}{2}$ of an inch thick, it still contains 16 ft.; but if $1\frac{1}{4}$ in. thick, it contains $1\frac{1}{4} \times 16 = 20$ ft. board measure.

260. For boards more than an inch thick and squared lumber,

Express the length and width in feet, and the thickness in inches. The product of these three numbers will be the number of feet board measure.

In practice the width of a board, unless sawed to order, is reckoned only to the next smaller half-inch. Thus, a width of $11\frac{3}{8}$ inches is reckoned 11 inches; of $13\frac{5}{8}$ or $13\frac{3}{4}$ inches is reckoned $13\frac{1}{2}$ inches.

How many feet in a 2-inch plank 18 ft. long and 14 in. wide?

$$14 \text{ in.} = 1\tfrac{1}{6} \text{ ft.}$$
$$2 \times 1\tfrac{1}{6} \times 18 = 42 \text{ ft. board measure. } \textit{Ans.}$$

Exercise 133. — Written.

How many feet board measure:

1. In 12 planks 3 in. thick, 12 ft. long, 12 in. wide?
2. In 18 planks 4 in. thick, 16 ft. long, 10 in. wide?
3. In 14 planks 2 in. thick, 18 ft. long, 10 in. wide?
4. In 2 sticks of timber 18 ft. long and 1 ft. square?
5. In 30 joists 10″ by 2″ and 18 ft. long?
6. In 40 joists 8″ by 2″ and 16 ft. long?
7. In 60 joists 3″ by 4″ and 12 ft. long?
8. In 4 beams 14″ by 12″ and 30 ft. long?
9. In 40 boards $\frac{3}{4}$ in. thick, 8 in. wide, and 14 ft. long?
10. In 50 inch-boards that are 5 in. wide and 16 ft. long?
11. In 60 boards $\frac{7}{8}$ in. thick, 3 in. wide, and 12 ft. long?
12. In 100 planks 2 in. thick, 12 in. wide, and 18 ft. long?
13. In 24 sticks 7″ by 8″ and 20 ft. long?
14. In 20 beams 10″ by 12″ and 30 ft. long?
15. In 36 joists 3″ by 9″ and 16 ft. long?

Round Logs.

261. Round logs are sold by the number of board feet that can be cut from them.

If logs are not more than 16 feet long, the length of the log and the diameter of the *small end* are measured, and a table stamped on calipers, used in measuring logs, gives the number of feet board measure in the log.

262. The table stamped on calipers is calculated as follows:

Express the diameter in inches, subtract twice the diameter from the square of the diameter, and $\frac{21}{40}$ *of the remainder will be the number of feet board measure in a log 10 ft. long.*

The formula is $\frac{21}{40}(d^2-2d)$, in which d stands for the diameter of the log in inches.

Find the number of feet board measure in a log 16 feet long and 20 inches in diameter.

$$20^2 - 2 \times 20 = 400 - 40 = 360.$$
$$\tfrac{21}{40} \text{ of } 360 = 189.$$
$$\tfrac{16}{10} \text{ of } 189 = 302.4 \text{ ft. board measure. } \textit{Ans.}$$

Exercise 134. — Written.

By this rule find the number of feet board measure:

1. In a log 12 ft. long, having diameter of end 15 in.
2. In a log 16 ft. long, having diameter of end 16 in.
3. In a log 14 ft. long, having diameter of end 18 in.
4. In a log 14 ft. long, having diameter of end 24 in.
5. In a log 15 ft. long, having diameter of end 20 in.
6. In a log 12 ft. long, having diameter of end 12 in.
7. In a log 13 ft. long, having diameter of end 15 in.
8. In a log 10 ft. long, having diameter of end 22 in.

Oak and Other Heavy Timber.

Large heavy timber of hard wood is generally sold by the ton, signifying 50 cu. ft., or 600 ft. board measure.

9. If oak timber is sold by the ton at a price equal to $100 a thousand feet board measure, what is the price per ton?

10. If oak timber is sold for $50 per ton, what will be the cost of enough to make a thousand feet board measure?

Capacity of Cisterns.

263. **1.** Find the number of gallons in a cubic foot.

A cubic foot contains 1728 cu. in.
A gallon contains 231 cu. in.

Therefore, the number of gallons is $\frac{1728}{231} = 7.48+$

2. Find the number of gallons a rectangular cistern $11' \times 5' \times 4'$ will hold.

Solution 1. $$\frac{\cancel{11} \times 5 \times 4 \times \overset{576}{\cancel{1728}}}{\underset{\underset{7}{\cancel{21}}}{\cancel{231}}} = \frac{11520}{7} = 1645\tfrac{5}{7} \text{ gal. } \textit{Ans.}$$

Solution 2.
$$11 \times 5 \times 4 \times 7\tfrac{1}{2} = 1650$$
$$\text{Subtract } \tfrac{1}{4} \text{ of } 0.01 \text{ of } 1650 = \underline{\quad 4.125}$$
$$1645.875 \textit{ Ans.}$$

In Solution 1 we divide the number of cubic inches in the cistern by 231, the number of cubic inches in a gallon.

In Solution 2 we allow $7\frac{1}{2}$ gallons to the cubic foot, and multiply this number by the number of cubic feet in the cistern. This gives an approximate value 1650, sufficiently near for most purposes. If a more nearly correct answer is required, *subtract $\frac{1}{4}$ of 0.01 of the number from the number of gallons.*

Exercise 135. — Written.

Reckoning $7\frac{1}{2}$ gallons to the cubic foot, find the capacity in gallons of:

1. A rectangular cistern $11' \times 7' \times 7'$.
2. A rectangular cistern $8' \times 8' \times 6'$.
3. A rectangular cistern $9' \times 7' \times 5'$.
4. A rectangular tank $2\frac{1}{2}' \times 2' \times 1'$.
5. A rectangular tank $6' \times 1\frac{1}{2}' \times 1'$.
6. A round cistern 7 ft. in diameter and 8 ft. deep.
7. A round cistern 8 ft. in diameter and 10 ft. deep.
8. A round cistern 20 ft. in diameter and 10 ft. deep.

Find the number of gallons of water in:

9. A well 3 ft. in diameter, depth of water 6 ft.
10. A well 4 ft. in diameter, depth of water 5 ft.
11. A well $3\frac{1}{2}$ ft. in diameter, depth of water 10 ft.
12. A well $2\frac{1}{2}$ ft. in diameter, depth of water 8 ft.

NOTE. If the number of *barrels* a cistern will hold is required, divide the number of gallons by 31.5; if the number of *hogsheads* is required, divide the number of gallons by 63.

Capacity of Bins.

264. Find the number of cubic feet in a bushel.

A bushel contains 2150.42 cu. in.
A cubic foot contains 1728 cu. in.

Therefore, a bushel is equal to $\frac{2150.42}{1728} = 1.24445$ cu. ft.

Add $\frac{1}{2}$ of 0.01 of 1.24445 to 1.24445 and we have 1.25. Hence,

265. To find the approximate number of bushels a bin will hold,

Take $\frac{4}{5}$ of the number of cubic feet in the bin, and add to the product $\frac{1}{2}$ of 0.01 of the product.

Find the number of bushels in a bin $10' \times 8' \times 5'$.

$$\begin{aligned} \tfrac{4}{5} \text{ of } 10 \times 8 \times 5 &= 320 \text{ bu.} \\ \tfrac{1}{2} \text{ of } 0.01 \text{ of } 320 \text{ bu.} &= \underline{1.6} \text{ bu.} \\ & 321.6 \text{ bu. } \textit{Ans.} \end{aligned}$$

266. To find the number of cubic feet required for a given number of bushels of grain,

Take $\frac{5}{4}$ of the number of bushels, and subtract $\frac{1}{2}$ of 0.01 of the product from the product.

Find the number of cubic feet required for 1000 bu.

$$\begin{aligned} \tfrac{5}{4} \text{ of } 1000 \text{ cu. ft.} &= 1250 \text{ cu. ft.} \\ \tfrac{1}{2} \text{ of } 0.01 \text{ of } 1250 \text{ cu. ft.} &= \underline{6.25} \text{ cu. ft.} \\ & 1243.75 \text{ cu. ft. } \textit{Ans.} \end{aligned}$$

Exercise 136. — Written.

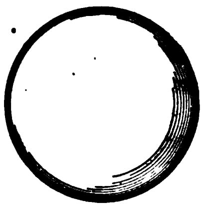
Find the number of bushels of grain required to fill:

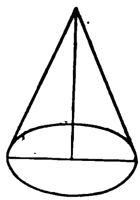
1. A bin $10' \times 6' \times 5'$.
2. A bin $12' \times 8' \times 6'$.
3. A bin $15' \times 10' \times 8'$.
4. A bin $20' \times 10' \times 8'$.
5. A bin $12\frac{1}{2}' \times 7\frac{1}{2}' \times 4'$.
6. A bin $8\frac{3}{4}' \times 8' \times 6\frac{1}{2}'$.

Find the number of cubic feet in a bin that will hold:

7. 1200 bu.
8. 2500 bu.
9. 6000 lb. of wheat.
10. 8600 lb. of barley.
11. 47,000 lb. of rye.
12. 51,000 lb. of corn.

Spheres and Cones.


Sphere.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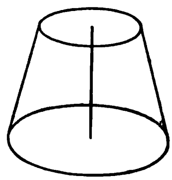

Frustum.

267. A **Sphere** is a solid bounded by a curved surface all points of which are equally distant from a point within called the centre.

268. A **Cone** is a solid bounded by a circle called the *base*, and a curved surface called the *lateral surface*, which ends in a point called the *vertex*.

269. A **Frustum of a cone** is the part of a cone left when the top is cut off by a plane parallel to the base.

270. The base of the cone and the section made by the cutting plane are called the **bases of the frustum.**

271. The **height** of a cone is the shortest distance from its vertex to its base, and the height of a frustum of a cone is the shortest distance between its bases.

272. To find the area of the surface of a sphere,

Multiply the square of the diameter by 3.1416.

273. To find the volume of a sphere,

Multiply the cube of the diameter by 0.5236 ($\frac{1}{6}$ of 3.1416).

274. To find the volume of a cone,

Find one third the product of the base and height.

1. Find the surface and volume of a sphere 4 in. in diameter.

$$\text{Surface} = 3.1416 \times 4^2 = 50.2656 \text{ sq. ft.}$$
$$\text{Volume} = 0.5236 \times 4^3 = 33.5104 \text{ cu. ft.}$$

2. Find the volume of a cone whose base contains 64 sq. in. and whose height is 21 inches.

$$\text{Volume} = \tfrac{1}{3}(64 \times 21) = 448 \text{ cu. in.}$$

275. To find the volume of the frustum of a cone,

To the square of half the sum of the diameters of its bases add one third the square of half their difference; multiply this sum by 0.7854 of the height of the frustum.

Find the volume of the frustum of a cone the diameters of whose upper and lower bases are 9 inches and 16 inches, respectively, and whose height is 10 inches.

Half the sum of diameters of bases is $12\frac{1}{2}$ inches.
Half the difference of the diameters is $3\frac{1}{2}$ inches.

$$(12\tfrac{1}{2})^2 + \tfrac{1}{3}(3\tfrac{1}{2})^2 = (156\tfrac{1}{4} + 4\tfrac{1}{12}) = 160\tfrac{1}{3} \text{ sq. in.}$$
$$0.7854 \text{ of } 10 \times 160\tfrac{1}{3} = 1259.26 \text{ cu. in. } \textit{Ans.}$$

Exercise 137. — Written.

Find the surface and volume of:

1. A sphere 6 inches in diameter.
2. A sphere 3 inches in diameter.
3. What part of the surface of a sphere 8 in. in diameter is the surface of a sphere 4 in. in diameter?

4. What part of the volume of a sphere 8 in. in diameter is the volume of a sphere 4 in. in diameter?

5. Find the cost of gilding a ball 60 inches in diameter at 6 cents a square inch.

6. Find the cost of copper-roofing a dome in the shape of a hemisphere (half a sphere) whose diameter is 60 feet at 30 cents per square foot.

7. Find the volume of a cone, if its height is 20 in. and the diameter of its base is 27 in.

8. Find the volume of the frustum of a cone the diameters of whose bases are 8 inches and 12 inches, respectively, and whose height is 12 inches.

9. How many cubic inches in a pail 12 in. deep, 16 in. wide at the top, and 12 in. wide at the bottom?

276. To find the number of quarts that round pans, tubs, and pails, with slanting sides will hold.

Find the capacity in cubic inches (by § 275), and divide this number by 57.75 for liquid quarts; by 67.2 for dry quarts.

Exercise 138. — Written.

1. Find the number of liquid quarts a round pan will hold if it is 10 in. across the top, 8 in. across the bottom, and 4 in. deep.

2. Find the number of liquid quarts a round tub will hold if it is 32 in. across the top, 29 in. across the bottom, and 10 in. deep.

3. Find the number of liquid quarts a round tub will hold if it is 22 in. across the top, 20 in. across the bottom, and 12 in. deep.

4. Find the number of dry quarts a round pail will hold if it is 12 in. across the top, 9 in. across the bottom, and 8 in. deep.

277. To find the capacity of any round vessel, like a cup, saucer, bowl, or tunnel.

If the vessel is spherical, it holds $\frac{2}{3}$ as much as a cylinder of the same diameter and depth; if conical, $\frac{1}{3}$ as much; if like an ordinary coffee-cup, $\frac{1}{2}$ as much.

EXERCISE 139. — WRITTEN.

1. Find the number of quarts a conical tunnel will hold if it is 9 in. across the top and 8 in. deep.
2. Find the number of pints a spherical bowl will hold if it is 5 in. across the top and $2\frac{1}{4}$ in. deep.
3. Find the number of pints a spherical bowl will hold if it is 4 in. across the top and $3\frac{1}{2}$ in. deep.
4. Find the capacity in pints of a coffee-cup 3 in. across the top and 3 in. deep.

278. To find the number of gallons a cask will hold,

Multiply by 0.65 the difference between the bung and head diameters expressed in inches, and add the product to the head diameter for the mean diameter.

Divide the product of the length of the cask expressed in inches and the square of the mean diameter by 294; the quotient is the number of gallons the cask will hold.

EXERCISE 140. — WRITTEN.

1. Find the number of gallons contained in a full cask whose bung diameter is 24 inches, head diameter 22 inches, and length 30 inches.
2. Find the number of gallons contained in a full cask whose bung diameter is 22 inches, head diameter 20 inches, and length 28 inches.
3. Find the number of gallons contained in a full cask whose bung diameter is 20 inches, head diameter 18 inches, and length 28 inches.

CHAPTER XII.

PERCENTAGE.

279. A **Percentage** of a number is the result obtained by taking a stated number of *hundredths* of it.

One *hundredth* of a number is called one *per cent* of it; two *hundredths,* two *per cent;* and so on.

280. A **Rate per cent** is a *fraction* whose denominator is 100, and whose numerator is the *given number of hundredths.* The methods of common fractions or of decimal fractions are used in the solution of all examples in Percentage. The *shortest* method is the *best* method.

281. This sign % stands for the words *per cent.*

Thus, 13% is $\frac{13}{100}$; $2\frac{1}{4}$% is $\frac{2\frac{1}{4}}{100}$; 867% is $\frac{867}{100}$.

To express a rate per cent as a common fraction.

282. Express $62\frac{1}{2}\%$ as a common fraction.

$$62\tfrac{1}{2}\% = \frac{62\frac{1}{2}}{100} = \frac{5}{8}.$$

283. Therefore, we have the following rule:

Write the rate for the numerator and 100 for the denominator, and reduce this fraction to its lowest terms.

EXERCISE 141. — WRITTEN.

Reduce to a common fraction:

1. 50%.	**6.** 30%.	**11.** $33\frac{1}{3}\%$.	**16.** $56\frac{1}{4}\%$.
2. 25%.	**7.** 60%.	**12.** $66\frac{2}{3}\%$.	**17.** $87\frac{1}{2}\%$.
3. 75%.	**8.** 40%.	**13.** $6\frac{1}{4}\%$.	**18.** $83\frac{1}{3}\%$.
4. 125%.	**9.** $8\frac{1}{3}\%$.	**14.** $12\frac{1}{2}\%$.	**19.** $162\frac{1}{2}\%$.
5. 20%.	**10.** $16\frac{2}{3}\%$.	**15.** $37\frac{1}{2}\%$.	**20.** $143\frac{3}{4}\%$.

To express a common fraction as a rate per cent.

284. Express $\frac{4}{5}$ as a rate per cent.

$$1 = \tfrac{100}{100} = 100\%.$$
$$\tfrac{4}{5} = \tfrac{4}{5} \text{ of } 100\% = 80\%.$$

285. Therefore, we have the following rule:

Divide 100 by the denominator of the fraction and multiply the quotient by the numerator.

EXERCISE 142. — WRITTEN.

Express as a rate per cent:

1. $\frac{1}{2}$.	**4.** $\frac{7}{25}$.	**7.** $\frac{1}{6}$.	**10.** $\frac{2}{3}$.	**13.** $\frac{13}{20}$.	**16.** $\frac{5}{12}$.	**19.** $2\frac{1}{4}$.
2. $\frac{1}{3}$.	**5.** $\frac{11}{50}$.	**8.** $\frac{3}{8}$.	**11.** $\frac{4}{9}$.	**14.** $\frac{4}{15}$.	**17.** $\frac{5}{16}$.	**20.** $3\frac{1}{2}$.
3. $\frac{1}{4}$.	**6.** $\frac{3}{10}$.	**9.** $\frac{3}{5}$.	**12.** $\frac{3}{7}$.	**15.** $\frac{9}{40}$.	**18.** $\frac{23}{16}$.	**21.** $7\frac{1}{5}$.

To express a decimal as a rate per cent.

286. 1. Express 0.3 as a rate per cent.

$$0.3 = 0.30 \text{ or } 30\%.$$

2. Express 0.5625 as a rate per cent.

$$0.5625 = 0.56\tfrac{25}{100} = 0.56\tfrac{1}{4} = 56\tfrac{1}{4}\%.$$

3. Express 0.00625 as a rate per cent.

$$0.00625 = 0.00\tfrac{625}{1000} = 0.00\tfrac{5}{8} = \tfrac{5}{8}\%.$$

287. Therefore, we have the following rule:

Write the decimal as hundredths, and the number expressing the hundredths is the rate per cent required.

NOTE. If the decimal has more than two places, the figures that follow the hundredths' place signify parts of 1%.

288. Problems in Percentage are conveniently arranged in **three classes.**

Class 1.

289. To find a percentage of a number.

1. Find $16\frac{1}{3}\%$ of 336.

$16\frac{1}{3}\% = 0.16\frac{1}{3}$.

$$\begin{array}{r} 336 \\ \underline{0.16\frac{1}{3}} \\ 112 \\ 2016 \\ \underline{336} \\ 54.88 \end{array}$$

54.88. *Ans.*

2. Find $16\frac{2}{3}\%$ of 336.

$16\frac{2}{3}\% = \frac{1}{6}$.

$\frac{1}{6}$ of $336 = 56$.

56. *Ans.*

290. Therefore, we have the following rule:

Multiply the number by the given rate per cent, expressed as a common fraction or as a decimal.

EXERCISE 143. — WRITTEN.

Find by using decimals:

1. 23% of 435.	**6.** 67% of 386.	**11.** 16% of 27.86.
2. 46% of 2783.	**7.** 8% of 0.37.	**12.** 9435% of 0.23.
3. 1% of 363.	**8.** 38% of 34.6.	**13.** 3% of 2.
4. 9% of 864.	**9.** 256% of 3.	**14.** 0.2% of 938.
5. 7% of 0.436.	**10.** $5\frac{2}{7}\%$ of 4.655.	**15.** 0.63% of 852.

Find by using common fractions:

16. $33\frac{1}{3}\%$ of 9.	**23.** $37\frac{1}{2}\%$ of 16.	**30.** 25% of 24.
17. 20% of 19.	**24.** $62\frac{1}{2}\%$ of 16.	**31.** $28\frac{4}{7}\%$ of 35.
18. 15% of 60.	**25.** 1% of 900.	**32.** 70% of 30.
19. 8% of 200.	**26.** $16\frac{2}{3}\%$ of 18.	**33.** $66\frac{2}{3}\%$ of 27.
20. $8\frac{1}{3}\%$ of 600.	**27.** $56\frac{1}{4}\%$ of 320.	**34.** $162\frac{1}{2}\%$ of 32.
21. $6\frac{1}{4}\%$ of 640.	**28.** $83\frac{1}{3}\%$ of 300.	**35.** $143\frac{3}{4}\%$ of 32.
22. $14\frac{2}{7}\%$ of 560.	**29.** $87\frac{1}{2}\%$ of 200.	**36.** $168\frac{3}{4}\%$ of 16.

Class 2.

To find a number when a percentage of the number and the rate per cent are given.

291. 1. If 14% of a number is 9646, what is the number?

SOLUTION. 14% of a number is 9646,
1% of the number is $\frac{1}{14}$ of 9646, or 689,
100% of the number is 100×689, or 68,900.

2. If $16\frac{2}{3}$% of a number is 432, what is the number?

SOLUTION. 432 is $16\frac{2}{3}$%, or $\frac{1}{6}$ of the required number.
Therefore the required number is 6×432, or 2592.

3. 1400 is $16\frac{2}{3}$% more than what number?

SOLUTION. 100 % of the number = the number.
$16\frac{2}{3}$% of the number = the increase.
$116\frac{2}{3}$%, or $\frac{7}{6}$ of the number = 1400.
$\frac{1}{6}$ of the number = $\frac{1}{7}$ of 1400, or 200.
Therefore, the number = 6×200, or 1200.

4. 1200 is 25% less than what number?

SOLUTION. 100% of the number = the number.
25% of the number = the decrease.
75%, or $\frac{3}{4}$ of the number = 1200.

If 1200 is $\frac{3}{4}$ of the number, $\frac{1}{4}$ of the number is $\frac{1}{3}$ of 1200, or 400.
Therefore the number is 4×400, or 1600.

5. An agent sold a hat that cost $2.40 so as to gain 25% after taking off from the marked price 20%. What was the marked price?

SOLUTION. Gain = 25%, or $\frac{1}{4}$ of $2.40 = $0.60.
Selling price = $2.40 + $0.60 = $3.
100% of marked price = marked price.
20% of marked price = per cent taken off.
80%, or $\frac{4}{5}$ of marked price = selling price, $3.

If $3 is $\frac{4}{5}$ of marked price, $\frac{1}{5}$ of marked price is $\frac{1}{4}$ of $3, or 75 cts., and marked price is 5×75 cts. = $3.75.

292. Therefore, we have the following rule:

Express the rate per cent as a fraction, divide the percentage by the numerator of this fraction and multiply the quotient by the denominator.

Exercise 144. — Oral.

1. 4 is $\frac{1}{3}$ of what number? 4 is $33\frac{1}{3}\%$ of what number?
2. 13 is $\frac{1}{2}$ of what number? 13 is 50% of what number?
3. 8 is $\frac{1}{4}$ of what number? 8 is 25% of what number?
4. 7 is $\frac{1}{6}$ of what number? 7 is $16\frac{2}{3}\%$ of what number?
5. 12 is $\frac{3}{4}$ of what number? 12 is 75% of what number?
6. 48 is $\frac{16}{100}$ of what number? 48 is 16% of what number?
7. 9 is $\frac{3}{8}$ of what number? 9 is $37\frac{1}{2}\%$ of what number?
8. 15 is $\frac{5}{8}$ of what number? 15 is $62\frac{1}{2}\%$ of what number?

Exercise 145. — Written.

1. \$87.89 is 17% of what sum of money?
2. 43% of a number is 374.1. What is the number?
3. Find the number of which 9.84 is $37\frac{1}{2}\%$.
4. What is the sum of which 28% is \$87.36?
5. 82 is $12\frac{1}{2}\%$ of what number?
6. 676 is 13% of what number?
7. 72 is 18% of what number?
8. 25% of 914.4 is 18% of what number?
9. A man sold a horse at a gain of \$48.62, which was 17% of the cost. Find the cost and the selling price.
10. A merchant dropped \$4.65 from his price on a suit of clothes by taking off 15% from his price. What was the price asked?

Class 3.

To find the rate per cent when a number and a percentage of the number are given.

293. **1.** What per cent of 5 is 4?

4 is $\frac{4}{5}$ of 5, and $\frac{4}{5} = \frac{80}{100}$, or 80%.

Therefore, 4 is 80% of 5.

2. What per cent of 874 is 227.24?

SOLUTION.

$$8.74 = 1\% \text{ of } 874.$$
$$227.24 = \frac{227.24}{8.74}\% \text{ of } 874.$$
$$= 26\% \text{ of } 874.$$

294. Therefore, we have the following rule:

Write the number as the denominator and the percentage as the numerator of a fraction, and reduce this fraction to hundredths. Or,

Divide the percentage of the number by one per cent of the number.

EXERCISE 146. — ORAL.

1. What part of 6 is 3? What per cent of 6 is 3?
2. What part of 8 is 2? What per cent of 8 is 2?
3. What part of 9 is 3? What per cent of 9 is 3?
4. What part of 24 is 4? What per cent of 24 is 4?
5. How many times 4 is 24? What per cent of 4 is 24?
6. What part of 35 is 7? What per cent of 35 is 7?
7. How many times 7 is 35? What per cent of 7 is 35?
8. What part of $\frac{1}{2}$ is $\frac{1}{8}$? What per cent of $\frac{1}{2}$ is $\frac{1}{8}$?
9. How many times $\frac{1}{8}$ is $\frac{1}{2}$? What per cent of $\frac{1}{8}$ is $\frac{1}{2}$?
10. What part of $\frac{3}{4}$ is $\frac{1}{2}$? What per cent of $\frac{3}{4}$ is $\frac{1}{2}$?

Exercise 147. — Written.

1. What per cent of 1472 is 460 ?
2. What per cent of 460 is 1472 ?
3. What per cent is 23.1 of 165 ?
4. What per cent of 927 is 64.89 ?
5. 26.79 is what per cent of 893 ?
6. Of 276 what per cent is 46 ?
7. A man bought a horse for $175 and sold him at a gain of $35. What was his gain per cent ?
8. 157.08 is what per cent of 374 ?
9. Find what per cent 42.38 is of 978.
10. A man sold a horse at a loss of 7% of the cost. If the actual loss was $43.40, find the cost.
11. A farmer had 375 bu. of potatoes. 37½ bu. decayed. What per cent remained ?
12. A merchant sold a piano for $350 which cost him $280. What was the gain and the gain per cent ?
13. An agent received $420 for selling 4800 tons of coal at $3.50 per ton. What was his charge per cent on the sum received for the coal ?
14. An agent charged $162.50 for selling a house. If the house was sold for $6500, what was his charge per cent ?

Find the gain per cent in population in each of the following cities from 1880 to 1890:

	Cities.	1880.	1890.
15.	New York,	1,206,594,	1,513,501.
16.	Chicago,	503,304,	1,099,850.
17.	Philadelphia,	846,981,	1,046,964.
18.	Brooklyn,	566,689,	806,343.
19.	Boston,	362,535,	448,477.

Commercial Discount.

295. Commercial Discount is a reduction from the list price of an article, from the amount of a bill of goods, or of a debt.

296. Discounts are reckoned at some common fraction of, or at some rate per cent of, *the amount from which the discount is made.*

If two or more discounts are quoted, the *first* denotes a discount off the *list price;* the *second* denotes a discount off *the remainder after the first discount is made;* the *third* denotes a discount off *the remainder after the second discount is made;* and so on.

Thus, discounts of 20 and $\frac{1}{3}$ off mean that 20% is to be deducted from the amount, and then from the remainder $\frac{1}{3}$ of it is to be taken.

NOTE. By varying the rate of discount the manufacturer can raise or lower the price of his goods without issuing a new catalogue.

1. Find the net amount of a bill of $160 after a discount of 20% is made.

Discount is 20%, or $\frac{1}{5}$ of $160 = $32.
Net amount $160 — $32, or $128.

2. Find the net amount of a bill of $462 with $\frac{1}{3}$ and 10 off.

$$\begin{array}{r|l} 3 & \$462 \\ & \underline{\ \ 154} \\ 10 & \$308 \\ & \underline{\ \ \ 30.80} \\ & \$277.20.\ \textit{Ans.} \end{array}$$

EXERCISE 148. — WRITTEN.

1. Find the discount at 14% on $8.50.
2. Find the discount at 8% on a coat marked $18.75.
3. A bicycle was marked $87.50; the dealer sold it at a discount of 12%. How much was the discount? How much did he receive for it?

4. A piano was listed at \$675. If sold at 35% discount, what was the selling price?

5. What is paid for a pane of glass listed at \$1.20 and sold at 25% and 10% off?

6. A gross of cartridges is listed at \$3.50 and sold at $\frac{1}{7}$ and 8% discount. Find the price paid.

7. An agent sold a \$3.50 book at 20% discount. What did he receive for it?

8. A man bought a bill of goods amounting to \$468; the merchant discounted the bill 4% for cash. How much was paid?

9. How much is paid for a dozen locks, if the list price is \$8.40 and if the discounts are $\frac{1}{4}$ and 10?

10. Find the net amount of a bill of \$289.20, subject to 25, 15, and $12\frac{1}{2}$ off.

11. The gross amount of a bill of goods was \$891, and the discounts were 25 and 15. Find the net amount.

12. Find the difference between a single discount of 50% and two successive discounts of 25% and 25% off a bill of \$1000.

13. An agent bought 25 sewing machines with 25, 10, and 10 off the list price of \$40 each, and sold them at 10% advance on the list price. Find his whole gain.

14. A merchant buys goods amounting by list price to \$722.40. He gets 25 and 10 off, and sells them at 5% advance on the list price. Find his whole gain.

Gain and Loss.

297. The **Gain** or **Loss** in business transactions is often computed as a per cent of the cost.

298. Per cent of commercial discount is always upon *price asked.* Per cent of gain or loss is always upon *price paid* or *cost.*

Exercise 149. — Written.

1. A merchant sold silk that cost $1.75 per yard at a loss of 4%. How much was the loss on each yard? What was the selling price?

2. An agent sold a sewing-machine at a gain of $33\frac{1}{3}$%; it cost him $27. Find the selling price.

3. A man bought a watch for $26.25, and sold it at a loss of 8%. What did he receive for it?

4. A man sold a cow at a gain of 18%. What was the amount received, the cost being $25?

5. A man bought a horse for $75 and a carriage and harness for $60. He sold them, gaining 8% on the horse and losing 12% on the carriage and harness. Find his net gain or loss.

6. A jeweller paid $4.50 each for clocks, and sold them at a gain of 16%. What was the selling price?

7. A wholesale merchant paid $2.50 a barrel for apples; he sold them to a retailer at a gain of 8%; the retailer sold them at a gain of $16\frac{2}{3}$%. What was the cost to the consumer?

8. A man bought a horse for $380, and sold it at a gain of 5%. How much did he receive for it?

9. An agent sold a piano that cost $320, gaining $27\frac{1}{2}$%. What was the selling price?

10. A boy sold a pair of skates at a gain of $22\frac{1}{2}$%. What did he receive for them if they cost him $1.20?

11. A grocer bought coffee at 32 cents. At what price must he sell it to gain $6\frac{1}{4}$%?

12. A clothier marked a suit of clothes that cost him $6.60 so as to gain 40%. He discounted his price $16\frac{2}{3}$%. How much did he receive for it?

13. A man built two houses at a cost of $3864 each. He sold one at a gain of 15% and the other at a loss of 3%. What was his net gain?

14. A wholesale hatter marked hats at $18 per dozen. He discounted this price 20% to a retailer who marked them to gain 25%. Find the retail price per hat.

15. A farmer received $16 per ton for his hay. It passed through the hands of a shipper, a commission merchant, and a retailer, who gained respectively $12\frac{1}{2}$%, $3\frac{1}{3}$%, and 20%. What was the retail price?

16. A carriage dealer had two carriages that cost him $178 each. He sold one at a gain of 15% and the other at a loss of 24%. Find his net loss.

17. A boy buys papers at 7 cents a dozen, and sells them at a gain of $71\frac{3}{7}$%. How many dozen will he have to sell to gain enough to buy a $5.50 suit of clothes?

Commission.

299. Commission is payment made by one person, called Principal or Factor, to another, the Agent, for the transaction of business.

300. Commission is usually a percentage of the money involved in the transaction. If goods are bought, it is a percentage of the cost; if sold, a percentage of the amount received; if a collection, a percentage of the amount collected.

Exercise 150. — Written.

1. A commission merchant sold 9000 bu. of wheat at 68 cents, charging 3% commission. How much was his commission? How much did he remit to his principal?

Note. In commercial transactions, final results are expressed in dollars and cents. When the number of mills is less than five, discard them; when five or more, count them as one cent.

2. An agent collected a debt of \$385.90, charging 4% commission. How much did he remit?

3. What is a real estate agent's commission at 5% for renting a store for one year at \$37 per month?

4. An agent receives 32% commission on books. He has three styles of binding, cloth at \$2.75, half morocco at \$3.50, and morocco at \$4.25. How much is his commission on each kind?

5. A real estate agent sells a farm of 360 acres at \$44.60 an acre, receiving a commission of 5% on the first thousand dollars and $2\frac{1}{2}$% commission on the remainder. Find his commission.

6. An agent buys an invoice of 89,500 lb. of coffee at 16 cents, 8% off for cash. What is his commission at 2%?

7. How many barrels of flour at \$3.50 can an agent buy with \$15,925 which sum includes his commission at 4%.

HINT. Find cost of one barrel including commission.

8. An agent bought 3000 bu. of corn at $37\frac{1}{2}$ cents, charging 2% commission and \$8.75 insurance. For how much a bushel must it be sold to gain 20% over the entire cost?

9. A collector discounts a bill 15% and receives 5% commission on the amount collected. How much does the principal receive from a bill of \$430.75?

10. An agent buys 12,000 bu. of potatoes, at 42 cents, commission $2\frac{1}{2}$%. He sells them at an advance of $28\frac{4}{7}$% on cost price, charging 3% commission. Find both commissions.

11. A dealer bought through his agent apples at \$2.40 per barrel, allowing $2\frac{1}{2}$% commission. How many barrels did he purchase if the cost and commission amounted to \$312.42? How much was the commission?

Insurance.

301. **Insurance** is promise of indemnity for loss by fire or other specified causes.

302. The **Policy** is the written agreement. The **Face** of the policy is the sum named therein which is to be paid to the holder in case of loss.

303. The **Premium** is the sum paid for the insurance. It is calculated as a percentage on the face of the policy.

Insurance agents are also called Underwriters.

Exercise 151. — Written.

1. A man insures his house for \$3250 at $1\frac{1}{5}\%$. What is the premium?

2. He insures his furniture at $\frac{3}{5}\%$ for \$1800. Find the premium.

3. He insures his barn for \$1275 at $1\frac{1}{2}\%$. Find the premium.

4. A merchant insures his store valued at \$4850 for $\frac{4}{5}$ of its value at $\frac{7}{8}\%$. What is the premium?

5. His goods are valued at \$8768. What is the cost of insuring his goods for $\frac{3}{4}$ of their value at $1\frac{3}{8}\%$?

6. A shipper insures a cargo of coal (3000 t.) valued at \$3.50 per ton for $\frac{3}{5}$ of its value at $\frac{5}{8}\%$. Find the premium.

7. Find the cost of insuring a stock of groceries valued at \$8964, risk being taken at $1\frac{1}{4}\%$ on $\frac{3}{5}$ of its value.

8. A man has a house worth \$5600. He insures it at $1\frac{1}{5}\%$ on $\frac{5}{7}$ of its value. Find the cost of insurance.

9. A man buys a lot for \$3000, and builds a house for \$6750. He insures the house for $\frac{5}{6}$ of its cost at $1\frac{3}{8}\%$. The house burns, and he sells the lot at 16% advance. Find his net loss.

Direct Taxes.

304. A **Tax** is money required from individuals for the support of the government or for other purposes.

305. **Direct Taxes** are laid on the person or on the value of the property he possesses.

306. A **Poll Tax** is a tax levied upon a person, and a **Property Tax** is a tax levied on property.

307. **Assessors** are officers whose duty it is to appraise the property belonging to each person to be taxed.

308. A **Tax Collector** is an officer who collects the taxes. He receives a salary or a per cent of the sum collected.

309. The **Treasurer** receives the money collected and is usually paid a salary.

310. The tax to be raised on the assessed valuation of property is the total amount of tax voted, diminished by the poll taxes and by such corporation taxes as are collected by the state and distributed to the several towns.

A tax of \$18,000 is levied upon a town which contains 800 polls, assessed at \$1.50 each, and which has taxable property assessed at \$1,200,000. The town receives from the state \$3600 as its share of the railroad tax, etc. Find the rate of taxation, and the tax paid by Brown, if his property is assessed at \$5960, and if he pays for 1 poll.

The amount of poll taxes = 800 × \$1.50 = \$1200
The amount from the state = \$3600
The sum from state and polls = \$4800

Sum levied on property = \$18,000 − \$4800 = \$13,200.
The rate = \$13,200 ÷ 1,200,000 = \$0.011.
That is, the tax is 11 mills on a dollar, or \$11 on \$1000.

To provide for contingencies, such as abatement of taxes, cost of collecting, etc., the assessors make the rate \$12 on \$1000.

Therefore, Brown's property tax is 0.012 of \$5960 = \$71.52.
His total tax = \$71.52 + \$1.50 = \$73.02.

311. To facilitate the making of taxes, the assessors usually prepare a table like the following, which is computed at 12 mills on a dollar:

TABLE.

PROP.	TAX.	PROP.	TAX.	PROP.	TAX.	PROP.	TAX.
$1	$0.012	$10	$0.12	$100	$1.20	$1000	$12.00
2	0.024	20	0.24	200	2.40	2000	24.00
3	0.036	30	0.36	300	3.60	3000	36.00
4	0.048	40	0.48	400	4.80	4000	48.00
5	0.060	50	0.60	500	6.00	5000	60.00
6	0.072	60	0.72	600	7.20	6000	72.00
7	0.084	70	0.84	700	8.40	7000	84.00
8	0.096	80	0.96	800	9.60	8000	96.00
9	0.108	90	1.08	900	10.80	9000	108.00

Find by the above table the tax of James Johnson, who pays for one poll and has property assessed at $9765.

Tax on $9000 = $108.00
Tax on 700 = 8.40
Tax on 60 = 0.72
Tax on 5 = 0.06
Tax on poll = 1.50
Total tax = $118.68

EXERCISE 152. — WRITTEN.

Make a table for tax rate of 17 mills, and:

1. Find the tax on $5000.
2. Find the tax on $800.
3. Find the tax on $380.
4. Find the tax on $469.
5. Find the tax on $97.
6. Find the tax on $3469.
7. Find the tax on $937.
8. Find the tax on $3248.
9. Find the tax on $18,345.
10. Find the tax on $39,846.

11. The tax rate is 14 mills. Mr. Brown's tax is $54.64. On how much property does he pay tax if his poll tax is $2?

12. How many dollars on $1000 must be levied on $486,500 to raise $5838 tax?

13. The assessed valuation of a school district was $246,750; the tax rate was 8 mills on a dollar. How much was left for school purposes, if the collector received 3% of the amount collected?

14. In a town $8500 tax was levied; 4% was uncollectable, and the collector's fee was 2½% of the amount collected. How much was left for the town?

15. How many mills on a dollar must be levied to raise a net tax of $16,005 on an assessed valuation of one and a half millions, allowing 3% for collecting?

Indirect Taxes.

312. **Indirect Taxes** are taxes upon articles of merchandise. They are **Customs** or **Duties,** taxes levied on certain imported goods; **Excises** or **Internal Revenue,** taxes levied on certain domestic goods.

313. These are called Indirect Taxes because while paid to the government by the person in whose possession they are first found, this tax forms a part of the price finally paid by the consumer, and is, therefore, a tax paid by him.

314. **Duties** are of two classes: *Specific* and *Ad Valorem.*

315. A **Specific Duty** is a definite sum levied on each unit by which the article is measured or weighed.

316. An **Ad Valorem Duty** is levied as a per cent of the cost of the article in the country where purchased. *Ad Valorem* means according to value.

317. On articles imported at a specific duty the following allowances are made: Tare is an allowance for the weight of the box or cask; breakage an allowance on glass bottles, etc.; leakage an allowance on liquids in casks. Duties are computed on the sum left *after making all deductions.*

Exercise 153. — Written.

(In these problems, the present tariff rates (1896) are quoted.)

1. A Boston merchant imported 400 bbl. of apples from New Brunswick. If they cost 75 cents a barrel in New Brunswick, what was the duty at 20%?

2. A merchant imported 1200 yd. of Brussels carpet, costing in England 3 shillings per yard. What was the duty at 40%? (A shilling is \$0.243$\frac{1}{3}$.)

3. Find the duty at 20% on 480 dozen bottles of olives, costing in Italy 4 liras per dozen. (A lira is \$0.193.)

4. Find the duty on 40 sets of china, costing in Dresden 50 marks per set, at 30%. (A mark is \$0.238.)

5. Find the duty on 120 boxes of castile soap (110 lb. each), costing 20 liras per cwt., at 20%, allowing 5% tare.

6. A dealer imported 24 bicycles from England. He paid £6 each, transportation 16*s.* each, and the duty 35%. Find the entire cost in U. S. money. (£1 = \$4.8665.)

7. A merchant imported 50 gross of penknives, costing 3*s.* per doz. in Sheffield. What was the duty at 15 cents per doz. specific and 25% ad valorem? (A gross = 12 dozen.)

8. A manufacturer produces oleomargarine at a cost of 3 cents per lb., pays the internal revenue tax of 15 cents per lb. What does he gain per cent by selling it at 20 cents per lb.?

9. A New York music dealer imported mandolins, costing in Germany 43 marks each. He paid a duty of 25%. At what price each must he sell them to gain 14$\frac{2}{7}$% on the cost?

10. Find the duty on 500 boxes of cigars, gross weight 482 lb., costing 75 cents per box in Havana. Rate of duty \$4 per lb., and 25% of the cost in Havana. Tare 5%.

Interest.

318. **Interest** is money paid for the use of money.

319. The **Principal** is the sum loaned.

320. The **Rate of Interest** is some per cent, called **rate per cent,** of the principal for **one year.**

321. The **Amount** is the sum total of principal and interest.

NOTE. In the applications of percentage thus far considered, no reference has been made to time. In all applications of percentage that relate to *the use of money* the elements of *time* and *rate* are considered.

Find the interest on \$840 at 5% for 3 yr. 6 mo.

3 yr. 6 mo. = $3\frac{1}{2}$ yr.
Interest for 1 yr. = 5% of \$840 = \$42.
Interest for $3\frac{1}{2}$ yr. = $3\frac{1}{2} \times$ \$42 = \$147.

EXERCISE 154. — WRITTEN.

Find the interest on:

1. \$730 for 2 yr. at 4%.
2. \$962 for 5 yr. at 6%.
3. \$625 for $2\frac{3}{4}$ yr. at 8%.
4. \$350 for $3\frac{1}{2}$ yr. at 7%.
5. \$240 for 4 yr. 3 mo. at $4\frac{1}{2}$%.
6. \$125 for 2 yr. 8 mo. at 6%.
7. \$333 for 4 yr. 10 mo. at 5%.
8. \$560 for 3 yr. 4 mo. at 5%.
9. \$426 for 1 yr. 7 mo. at 6%.
10. \$26.40 for 9 mo. at 7%.

Find the amount of:

11. \$402 for 3 yr. 4 mo. at 6%.
12. \$521 for 2 yr. 4 mo. at $4\frac{1}{2}$%.
13. \$325 for 2 yr. 5 mo. at 5%.
14. \$650 for 1 yr. 9 mo. at $3\frac{1}{2}$%.
15. \$462 for 2 yr. 7 mo. at 6%.
16. \$605 for 3 yr. 1 mo. at 7%.

Six Per Cent Method.

322. At 6% what is the interest on one dollar for one year?

How many months will it take $1 to yield 1 cent interest? 2 cents interest?

How many cents interest will $1 yield in 6 mo.? 8 mo.? 10 mo.? 16 mo.? 7 mo.? 11 mo.? 3 mo.? 14 mo.? 4 mo.? 9 mo.? 5 mo.?

What is the simplest way to find the interest on one dollar at six per cent for any number of months?

Again, how many months will it take $1 to yield 1 cent interest?

How many days to yield 1 cent interest?

How many days to yield 1 mill? 2 mills? How many mills will $1 yield in 18 days? 30 days? 48 days? 108 days? 27 days? 34 days? 25 days? 9 days?

What relation has the number of mills interest to the number of days time?

What relation has the number of cents interest to the number of months time?

What is the simplest way to find the interest on one dollar at six per cent for any number of months and days? For years, months, and days?

Find the interest on $320 for 3 yr. 9 mo. 20 dy. at 6%.

Int. on $1.		Int. on $320.	Condensed Work.
For 3 yr. = 3 × 6c.	= $0.18	$0.228⅓	3 yr. 9 mo. 20 dy.
For 9 mo. = 9 × ½c.	= 0.045	320	$0.18 0.045 0.003⅓
For 20 dy. = 20 × ⅙ m.	= 0.003⅓	106⅔	0.045
For time required	= $0.228⅓	456	0.003⅓
		684	$0.228⅓
		$73.07 *Ans.*	320

323. **Therefore, to find interest at 6% for years, months, and days, we have the following rule:**

Multiply 6 cents by the number of years, ½ a cent by the number of months, ⅙ of a mill by the number of days. Find the sum, and multiply this sum by the number of dollars in the principal.

Exercise 155. — Written.

Find the interest at 6% on:

1. $850 for 2 yr. 8 mo.
2. $27 for 3 yr. 9 mo.
3. $390 from Jan. 6, 1892 to July 12, 1896.
4. $450 from Aug. 4, 1895 to Feb. 28, 1899.
5. $675.20 from Apr. 9, 1894 to Sept. 24, 1900.
6. $36.50 from July 7, 1896 to Oct. 18, 1900.

324. Short time notes generally run for 1, 2, 3, or 4 months; or for 30, 60, or 90 days.

325. To find interest at 6% for time expressed in months,

Put the decimal point two places to the left in the principal, and multiply by half the number of months.

326. To find interest at 6% for time expressed in days,

Put the decimal point three places to the left in the principal, and multiply by one sixth of the number of days.

Thus, the interest at 6% on $600 for 4 mo. is 2 × $6.00 = $12; and for 30 dy. is 5 × $0.600 = $3.00.

Exercise 156. — Written.

Find the interest at 6% on:

1. $2650 for 1 mo.; 2 mo.; 3 mo.; 4 mo.
2. $1768 for 1 mo.; 2 mo.; 3 mo.; 4 mo.
3. $2172 for 30 dy.; 60 dy.; 90 dy.
4. $2548 for 30 dy.; 60 dy.; 90 dy.

Six Per Cent Method for Other Rates.

327. Find the interest on $73.42 for 5 yr. 8 mo. 16 dy. at $7\frac{1}{2}\%$.

5 yr.	8 mo.	16 dy.		
$0.30	0.04	$0.002\frac{2}{3}$		$\$0.342\frac{2}{3}$
0.04				73.42
$0.002\frac{2}{3}$			6)	$25.158
$\$0.342\frac{2}{3}$				4.193
			Multiply by	$7\frac{1}{2}$
				$31.45

328. Therefore, we have the following rule:

Divide the interest at six per cent by six and multiply the quotient by the given rate. Or,

Take such a part of the interest at six per cent as the given rate is of six per cent.

The interest at 8% is $\frac{8}{6}$ or $1\frac{1}{3}$ of the interest at 6%; we add to the interest at 6% $\frac{1}{3}$ of itself. The interest at $4\frac{1}{2}\%$ is $\frac{3}{4}$ of the interest at 6%. Since $\frac{3}{4} = 1 - \frac{1}{4}$, we subtract from the interest at 6% $\frac{1}{4}$ of itself.

For 7%, we add to the interest at 6% $\frac{1}{6}$ of itself; for $7\frac{1}{2}\%$, we add $\frac{1}{4}$; for 9%, we add $\frac{1}{2}$; for 4%, we subtract $\frac{1}{3}$; for 5%, we subtract $\frac{1}{6}$.

Exercise 157. — Written.

Find the interest on:

1. $340 for 3 yr. 6 mo. at 5%.
2. $7250 for 1 yr. 7 mo. at $4\frac{1}{2}\%$.
3. $980 for 2 yr. 10 mo. 15 dy. at 7%.
4. $275.65 for 2 yr. 9 mo. 12 dy. at 4%.
5. $73.42 for 5 yr. 8 mo. 17 dy. at $4\frac{1}{2}\%$.
6. $97.35 for 3 yr. 7 mo. 20 dy. at 7%.
7. $46.25 for 3 yr. 7 mo. 4 dy. at 8%.

Find the amount of:

8. $1250 for 1 yr. 1 mo. at 5%.
9. $132 for 2 yr. 11 mo. 15 dy. at 6%.
10. $370 for 2 yr. 3 mo. 18 dy. at $4\frac{1}{2}\%$.
11. $595.63 for 8 mo. at $3\frac{1}{2}\%$.

Promissory Notes.

329. A **Promissory Note** (called simply a *Note*) is a promise to pay a specified sum of money on demand or at a specified time.

A note may be made payable *to bearer*, to a *person named in the note*, or to the person named or *his order*.

The **Maker** of a note is the person who signs it.

The **Payee** is the person to whom the note is payable.

The **Holder** of a note is the person who has lawful possession of it.

The **Face** of a note is the sum of money named in it.

A **Negotiable Note** is a note that can be sold and transferred to any one else by the holder of the note. A note is negotiable unless made payable to the *payee only*.

An **Endorser** of a note is a person who writes his name on the back of the note.

Without Recourse. An endorser becomes responsible for the payment of a note unless he writes above his signature the words *without recourse*.

(Forms of Notes.)

1. *\$375.25.* *Boston, Mass., June 1, 1896.*

Six months after date, I promise to pay Benjamin Parker Three Hundred Seventy-five and $\frac{25}{100}$ Dollars, with interest at 5%, for value received. *James Overton.*

Who is the *maker* of this note? Who is the *payee?* Is the note *negotiable?*

2. *\$375.25.* *Boston, Mass., June 1, 1896.*

Six months after date, I promise to pay Benjamin Parker, or bearer, Three Hundred Seventy-five and $\frac{25}{100}$ Dollars, with interest at 5%, for value received. *James Overton.*

3. *\$375.25.* *Boston, Mass., June 1, 1896.*

Six months after date, I promise to pay Benjamin Parker, or order, Three Hundred Seventy-five and $\frac{25}{100}$ Dollars, with interest at 5%, for value received. *James Overton.*

A note that contains the words "or bearer" can be collected by any one who has lawful possession of it.

A note that contains the words "or order" becomes payable to bearer when the payee merely writes his name across the back. This is called an *Endorsement in blank.*

In form 3, if Benjamin Parker sells the note to James Whitney he writes on the back "Pay to the order of James Whitney" and signs his name, or else he simply endorses in blank.

Demand Notes are written "On demand I promise to pay," etc., and are due whenever payment is demanded. Other notes are called *Time Notes.* The day on which a time note is legally due is called the *day of its maturity.*

Notes containing the words *with interest* bear interest from date until paid. Notes not containing these words begin to bear interest at the legal rate, if not paid when due.

Legal rate of interest is the rate established by law in the state where the note is made.

Notes should contain the words "value received," otherwise it may be necessary to prove that the note was given for a valuable consideration; and the *face* of a note, except the cents, should be expressed in words.

Exercise 158. — Written.

Find the day of maturity and amount due, having given:

	Face of Note.	Date of Note.	Time.	Rate of Interest.
1.	$350.	June 3, 1896.	60 dy.	6%.
2.	$570.	Apr. 25, 1896.	6 mo.	5%.
3.	$460.	Apr. 4, 1896.	1 yr.	4½%.
4.	$328.45.	July 2, 1896.	3 mo.	6%.
5.	$536.25.	July 18, 1896.	90 dy.	7%.
6.	$840.54.	May 27, 1896.	1 yr.	5%.
7.	$900.75.	May 28, 1896.	4 mo.	7½%.
8.	$324.70.	Aug. 1, 1896.	7 mo.	6½%.
9.	$367.40.	Sept. 21, 1896.	5 mo.	5½%.

Find the amount due Sept. 22, 1896, on the following demand notes:

10. $408.45. New York, N. Y., May 15, 1896.

On demand, I promise to pay George A. Wood, or order, Four Hundred Eight and $\frac{45}{100}$ Dollars, with interest at $4\frac{1}{2}\%$. Value received. James Sinclair.

11. $1070.60. Boston, Mass., Apr. 29, 1896.

On demand, I promise to pay James Goodwin, or order, Ten Hundred Seventy and $\frac{60}{100}$ Dollars, with interest at 5%. Value received. John Hilton.

12. $2053.37. Concord, N. H., Apr. 15, 1896.

On demand, I promise to pay Francis Warren, or order, Two Thousand Fifty-three and $\frac{37}{100}$ Dollars, with interest at 6%. Value received. Alfred Gibson.

Partial Payments.

330. Partial Payments, as the term implies, are payments of a part of a note.

There are several methods of computing interest upon such notes, all recognizing the principle that interest should stop upon the amount paid.

331. Merchants' Rule (used when settlement is made within a year):

1. *Find the amount of the note at date of settlement without regarding payments.*

2. *Find the amount of each payment with interest from date of payment to date of settlement.*

3. *Subtract the sum of the payment amounts from the total amount.*

A man holds a note of $380, dated Aug. 16, 1895, on which the following payments are endorsed: $120, Nov. 4, 1895; $70, Mar. 18, 1896; $145, May 1, 1896. Settlement was made June 25, 1896. Find balance due, interest at 6%.

Time from Aug. 16 to June 25 is 10 mo. 9 dy. Int. on $1 = $0.0515.
Time from Nov. 4 to June 25 is 7 mo. 21 dy. Int. on 1 = 0.0385.
Time from Mar. 18 to June 25 is 3 mo. 7 dy. Int. on 1 = $0.016\frac{1}{6}$.
Time from May 1 to June 25 is 1 mo. 24 dy. Int. on 1 = 0.009.

Int. on $380 = 380 × $0.0515 = $19.57.	Amount is		$399.57
Int. on 120 = 120 × 0.0385 = 4.62.	Amount is	$124.62	
Int. on 70 = 70 × $0.016\frac{1}{6}$ = 1.13.	Amount is	71.13	
Int. on 145 = 145 × 0.009 = 1.31.	Amount is	146.31	
	Total payment amounts		$342.06
	Balance due June 25, 1896		$ 57.51

Exercise 159. — Written.

Find balance due:

1. Note of $230. Date, May 9, 1895. Settlement Feb. 24, 1896. Payments: $40, July 1, 1895; $75, Sept. 16, 1895; $50, Jan. 2, 1896. Rate of interest 6%.

2. Note of $375 given Mar. 3, 1894. Payments: $30, May 8, 1894; $65, Aug. 16, 1894; $110, Nov. 20, 1894. Rate of interest 5%. Settlement Jan. 1, 1895.

3. Note of $80 given Nov. 1, 1894, bearing interest at 6%. Payments: $10 on the first day of each month following. How much interest must be paid with the last payment?

4. Note of $600 given May 16, 1896, bearing interest at 5%. Payments: $50, June 1; $25, July 3; $50, July 10. Settlement Nov. 12, 1896. Find balance due.

332. United States Rule (established by act of Congress):

1. *Find the amount of the principal to the time when the payment, or sum of the payments, is equal to or greater than the interest.*

2. *From this amount deduct the payment, or sum of the payments.*

3. *Consider the remainder as a new principal, and proceed as before.*

$760. Boston, Mass., July 16, 1888.

Three months after date, I promise to pay Henry W. Street, or order, Seven Hundred Sixty Dollars. Value received. James Johnson.

Payments endorsed on the back: Jan. 10, 1889, $200; Aug. 4, 1891, $75; May 1, 1892, $568. What is due Oct. 7, 1893?

yr.	mo.	dy.			
1889	1	10		$760	Principal.
1888	10	16		0.014	
	2	24	0.014.	3040	
				760	
				$10.640	Int.
				760	
				$770.64	Amount.
				200	Payment.
1891	8	4		$570.64	New principal.
1889	1	10		0.154	
2	6	24	0.154.	228256	
				285320	
				57064	
				$87.87856	Interest.

Interest exceeds payment.

Since the interest is greater than the payment, we find the interest on the same principal from Jan. 10, 1889, to May 1, 1892.

yr.	mo.	dy.			
1892	5	1		$570.64	Same principal.
1889	1	10		0.1985	
3	3	21	0.1985.	285320	
				456512	
				513576	
				57064	
				$113.272040	Interest.
				570.64	
				$683.91	Amount.
			$75 + $568 =	$643.	Payments.
				$40.91	New principal.
1893	10	7		0.086	
1892	5	1		24546	
1	5	6	0.086.	32728	
				$3.51826	Interest.
				40.91	
				$44.43	Due Oct. 7, 1893.

EXERCISE 160. — WRITTEN.

1. Note of $425. Date, Oct. 10, 1892. Interest from date at 6%. Payments: $100, March 14, 1893; $85, July 8, 1893; $125, Feb. 24, 1894. What was due Oct. 10, 1894?

2. Note of $360. Date, Jan. 10, 1893. Interest, 8%. Payments: $75, Sept. 16, 1893; $20, Oct. 8, 1894; $160, Dec. 1, 1894. What was due Jan. 10, 1895?

3. $375. Boston, May 20, 1894.

Six months after date, I promise to pay Oliver Higgins Three Hundred Seventy-five Dollars, with interest at 5%. Value received. William Julian.

Endorsements: Nov. 20, 1894, $25; Apr. 6, 1895, $75; Aug. 20, 1895, $140. What is due Mar. 12, 1896?

4. $250. New York, Aug. 24, 1892.

One year after date, I promise to pay, for value received, Ira Hoyt, or order, Two Hundred Fifty Dollars.

James Patterson.

Payments: Aug. 24, 1893, $100; Nov. 12, 1893, $40; Feb. 3, 1894, $80; May 10, 1894, $25. Paid in full, Apr. 16, 1895. What was the last payment?

Bank Discount.

333. A **Draft** or **Bill of Exchange** is a written order directing one person to pay a specified sum of money to another.

334. A **Commercial Draft** is a draft payable at a specified time after sight (or date).

When the person on whom a commercial draft is drawn accepts a draft, he writes the word "Accepted," with the date, across the *face*, and signs his name. The draft is then called an *acceptance*, and the *acceptor* is responsible for its payment.

An acceptance is of the nature of a promissory note, the acceptor and maker having respectively the same responsibility for payment as the maker and endorser of a promissory note.

335. A **Check** is a draft upon a bank where the maker (or drawer) has money deposited.

336. A **Certified Check** is a check upon which the cashier of a bank has endorsed in red ink the word "Certified," with date, name of bank, and his signature as cashier. The bank is then responsible for its payment.

337. **Bank Discount** is a deduction from the amount of a note or draft for cashing it before it is due.

338. The **Term of Discount** is the time from the day of discounting the note to and including the day that it is legally due, called the **day of maturity.**

NOTE. In some states, banks include in the term of discount the day of discounting the note as well as the day of maturity.

339. Days of Grace are **three days** allowed in addition to the time stated in the note before the note is legally due, unless the note contains the words "without grace," or is made in states which have abolished days of grace by statute.

340. A **Protest** is a notice in writing by a notary public to the endorsers that a note has not been paid. If a note is not protested at the end of the day of its maturity the endorsers are released from their obligation.

341. Bank discount is found by *computing the interest* for the term of discount at a specified rate on the amount of the note.

342. If a note draws interest, the amount of the note is the *face and interest.*

343. Exchange is a charge sometimes made for collecting, in case the place of payment of the note or draft is not the place of discount. The rate of exchange is generally from $\frac{1}{8}$ to $\frac{1}{2}$ of 1% of *the face value* (a fraction of \$100 being reckoned as \$100); or a number of cents on \$1000.

344. The **Proceeds** of a note is the amount of the note less the discount and exchange.

Find the day of maturity, the time to run, the discount, and the proceeds of the following notes:

1. \$540. Concord, N. H., Jan. 12, 1896.

Sixty days after date, without grace, I promise to pay S. L. Gage, or order, Five Hundred Forty Dollars, for value received. John Munroe.

Discounted at 6% Feb. 1.

Counting 60 days from Jan. 12, we have 19 in Jan., 29 in Feb., and 12 in Mar. Therefore, the note is due Mar. 12.

The time to run (from Feb. 1, day of discount) is 28 days in Feb. and 12 in Mar. Therefore, the time to run is 40 days.

The discount is the interest on \$540 for 40 days; and this is $6\frac{2}{3} \times \$0.540 = \$3.60$. (§ 326).

The proceeds is $\$540 - \$3.60 = \$536.40$.

2. $600. Concord, N. H., May 18, 1896.

Four months from date, we jointly and severally promise to pay Charles Farnsworth, or order, Six Hundred Dollars, value received, with interest at six per cent.

James Parkhurst,
William P. Hill.

Discounted at 6%, June 3, 1896.

$$\begin{array}{r} \$0.600 \\ \underline{20\frac{1}{2}} \\ 300 \\ \underline{1200} \\ \end{array}$$

Interest on note 4 mo. 3 dy., or 123 dy. = $12.30

Amount of note = $612.30.

Day of maturity, Sept. 21.

Time to run, June, 27 days; July, 31 days; Aug., 31 days; Sept. 21 days = 110 days.

Discount on $612.30 at 6% for 110 days is $18\frac{1}{3} \times \$0.6123$, or $11.23.

Proceeds is $612.30 − $11.23 = $601.07.

NOTE 1. In Boston, and in many other places, when the time a note has to run is expressed in *months*, the term of discount is computed for this number of months (plus 3 days of grace in states where grace is allowed), and not for the exact number of days contained in the months.

NOTE 2. When a note falls due on Sunday or a legal holiday it becomes due on the business day before, but in Massachusetts and many other states it becomes due by special statute on the day following.

EXERCISE 161. — WRITTEN.

Find the day of maturity and the proceeds of the following notes, without grace:

1. Note of $750, given Apr. 16, 1896 for 1 yr., with interest at 5%. Discounted Jan. 1, 1897, at 6%.

2. $450, Dec. 6, 1895, for four months. Discounted at 5% same day.

3. Same note, with interest at 6%, discounted Feb. 1, 1896, at 5%.

4. \$365, Aug. 17, 1896, for 90 days. Discounted at date at 7%.

5. \$2000, July 3, 1896, for 60 days, with interest at 6%. Discounted at date, at 5%.

6. \$175.20, Feb. 13, 1896, for 8 mo. with interest at 5%. Discounted Aug. 25, at 6%.

7. \$500, Sept. 4, 1896, for 90 days. Discounted on its date, at 5%.

8. Find the day of maturity and the proceeds of the notes in examples 6 and 7 with grace.

9. Find the proceeds of the notes in examples 6 and 7, without grace, if discounted in Delaware, Maryland, Missouri, Pennsylvania, or the District of Columbia, where banks include in the term of discount *the day of discount* in addition to the number of days the note has to run.

Find the proceeds of the following drafts with grace:

10. Draft for \$740 at 90 days; rate of discount 6%; exchange $\frac{1}{8}$%.

The discount for 93 days is \$11.47; and the exchange is $\frac{1}{8}$% of \$800, or \$1 (every fraction of \$100 being reckoned as \$100). The total discount, therefore, is \$12.47.

11. Draft for \$900 at 60 days; rate of discount 6%; exchange $\frac{1}{8}$%.

12. Draft for \$1000 at 30 days; rate of discount 5%; exchange $\frac{1}{4}$%.

13. Draft for \$1200 at 90 days; rate of discount 5%; exchange $\frac{1}{8}$%.

14. Draft for \$360 at 90 days; rate of discount 6%; exchange $\frac{1}{4}$%.

15. Draft for \$2450 at 4 months; rate of discount $4\frac{1}{2}$%; exchange $\frac{1}{4}$%.

16. Draft for \$3150 at 3 months; rate of discount $5\frac{1}{2}$%; exchange $\frac{1}{8}$%.

Exact Interest.

345. By the ordinary method interest is computed on a basis of 30 days for a month, or 360 days for a year.

A year has 365 days. The interest for a year counted by days would thus be $\frac{365}{360}$, or $\frac{73}{72}$ of the true interest.

The exact interest for any number of days is, therefore, found by diminishing interest found by the ordinary method by $\frac{1}{73}$ of itself.

346. To find $\frac{1}{73}$ of a number,

Move the decimal point two places to the left, add a third three times, each time one place further to the right. Carry only to three decimal places.

Find $\frac{1}{73}$ of $2987.63.

$2987.63	
$29.876	
9.958	
995	
99	
$40.928	$40.93. *Ans.*

Find the exact interest on $320 from Apr. 4 to Sept. 19, at 6%.

26	$0.320
31	28
30	$8.96
31	$0.089
31	29
19	2
6)168	$0.12
28	$8.84 *Ans.*

Exact interest is used by trust companies and in all calculations of national and state governments.

NOTE. The pupil should remember that the rule for exact interest applies only to periods less than one year. For periods containing years, months, and days find the interest for the years, then for the months and days by the above rule, and add the results.

Exercise 162. — Written.

Find the exact interest at 6%:

1. On $2475 for 126 days.
2. On $4560 for 90 days.
3. On $837.24 from Feb. 21, 1895, to July 4, 1895.
4. On $726.35 from May 2, 1895, to Nov. 20, 1895.
5. On $350 from May 22, 1895, for 4 months.
6. On $983.83 from Aug. 1, 1896, for 1 year 7 months.

Annual Interest.

347. Annual Interest is simple interest on the principal and on each year's interest from the time each interest is due until settlement.

Find the interest upon $400 for 4 yr. 7 mo. 20 dy. at 5%, payable annually.

Simple interest on $400 for the whole time = $92.78.

Interest due the 1st year, $20, draws interest	3 yr.	7 mo.	20 dy.
" " " 2d " $20, " "	2 yr.	7 mo.	20 dy.
" " " 3d " $20, " "	1 yr.	7 mo.	20 dy.
" " " 4th " $20, " "		7 mo.	20 dy.
Interest upon the interest = Interest on $20 for	8 yr.	6 mo.	20 dy.

Interest on $20 for 8 yr. 6 mo. 20 dy. = $8.56.

The annual interest = $92.78 + $8.56 = $101.34.

Exercise 163. — Written.

1. Find the amount at annual interest of $250 for 3 yr. 4 mo. 15 dy. at 6%.
2. Find the interest due on $140 in 3 yr. 8 mo. 10 dy., interest at 8%, payable annually.
3. Find the amount of $700 at 6% annual interest for 4 yr. 6 mo. 12 dy.
4. Find the interest due on $341 for 3 yr. 10 mo. 18 dy. at 5%, payable annually.

5. Find the interest due on \$650.50 for 2 yr. 6 mo. 20 dy. at $4\frac{1}{2}\%$, payable annually.

6. Find the amount due May 21, 1898, on a note dated April 3, 1895, for \$1246.25 at 5% annual interest.

Compound Interest.

348. Interest is compounded when it is added to and becomes a part of the principal at specified intervals. Interest is compounded annually, semi-annually, or quarterly, according to agreement. Interest is understood to be compounded annually unless otherwise stated.

Find the amount of \$60 at compound interest for three years at 5%. Find also the compound interest.

Interest for 1st year = 5% of \$60 = \$3. Amount = \$63.
Interest for 2d year = 5% of \$63 = \$3.15. Amount = \$66.15.
Interest for 3d year = 5% of \$66.15 = \$3.31. Amount = \$69.46.
\$69.46 (compound amt.) − \$60 (principal) = \$9.46 (compound int.).

Exercise 164. — Written.

1. What is the amount of \$340 at 6% compound interest for 3 yr.?

2. Find the compound interest on \$100 at 4% for 4 yr.

3. Find the compound interest on \$560 for 2 yr. at 6%, interest compounded semi-annually.

Note. The interest is 3% every six months.

4. Find the amount of \$347.40 for 3 yr. 4 mo. at 4%, compound interest.

Note. Find the amount for 3 years at compound interest, and then find the interest on this amount for 4 mo. and add it to the amount for 3 years.

5. Find the compound interest at 8% on \$500 for 1 yr. 2 mo., interest payable quarterly.

6. Find the compound interest at 6% on \$1000 for 1 yr. 8 mo., interest payable semi-annually.

Problems in Interest.

349. In interest, four factors are considered: Principal, Rate Per Cent, Time, and Interest or Amount.

In the computation of interest in business, the first three factors are given to find the fourth.

But any one of these may be found if the other three are given, as will be seen by the following examples:

1. What principal will in 2 yr. 10 mo. 24 dy. give \$222.72 interest at 8%?

SOLUTION. Interest at 8% on \$1 for 2 yr. 10 mo. 24 dy. is \$0.232.

Since \$0.232 is the interest on \$1,

\$222.72 is the interest on $\$\frac{222.720}{0.232}$, or \$960. *Ans.*

2. What principal will amount to \$570 in 2 yr. 4 mo. at 6%?

SOLUTION. The amount of \$1 for 2 yr. 4 mo. at 6% is \$1.14.

Since \$1.14 is the amount of \$1,

\$570 is the amount of $\$\frac{570}{1.14}$, or \$500. *Ans.*

3. In what time will \$540 at 6% yield \$109.80 interest?

SOLUTION. Interest on \$540 at 6% for 1 yr. is \$32.40.

Since \$32.40 is the interest for 1 yr.,

\$109.80 is the interest for $\frac{109.80}{32.40}$ yr., or $3\frac{7}{18}$ yr.

3 yr. 4 mo. 20 dy. *Ans.*

4. At what rate will \$600 yield \$81 interest in 3 years?

SOLUTION. Interest on \$600 at 1% for 3 yr. is \$18.

Since \$18 is the interest at 1%,

\$81 is the interest at $\frac{81}{18}$%, or $4\frac{1}{2}$%. *Ans.*

350. To find the required factor in interest problems, whether principal, time, or rate,

Divide the given interest by the interest obtained when the required factor is represented by a unit.

The *unit* of the above rule is \$1 in finding the principal; 1 yr. in finding the time; and 1% in finding the rate.

EXERCISE 165. — WRITTEN.

1. At what rate will \$320 in 1 yr. 2 mo. 12 dy. yield \$19.20 interest?

2. What principal will in 3 yr. 6 mo. at 4% yield \$9.59 interest?

3. The interest on \$49.60 for a certain period at 5% is \$4.64. Find time.

4. The interest on \$87.50 for 2 yr. 8 mo. 24 dy. is \$14.35. Find rate.

5. \$120 put at interest for 3 yr. 5 mo. 16 dy. amounts to \$144.92. Find rate.

6. How long must \$84.80 be put on interest at 5½% to amount to \$102.29?

7. What principal in 5 mo. 18 dy. at 6% will yield \$3.64 interest?

8. How long will it take \$1 or any other sum to double itself at 4%? at 5%? at 6%?

9. For what sum did I give my note at a bank so that when discounted for 90 dy. without grace at 6% I received \$3152?

10. Find the time in which the interest on \$225 will be \$24 at 4%.

11. Find the principal that will amount to \$2800 in 3 yr. at 4%.

12. Find the rate when the interest on \$326 for 5 yr. is \$73.35.

Present Worth and True Discount.

351. The **Present Worth** of a debt is a sum, which, if put at interest, will amount to the debt when due.

352. **True Discount** is the difference between a sum of money due at some future time and the present worth of that sum.

Find the present worth of \$824 due in 7 mo. 6 dy., money being worth 5%.

SOLUTION. Amount of \$1 at 5% for 7 mo. 6 dy. is \$1.03.
Since \$1.03 is the amount of \$1,

\$824 is the amount of $\$\frac{824}{1.03}$, or \$800. *Ans.*

The true discount is \$824 − \$800 = \$24.

353. To find the present worth of a given sum of money due at a stated future time,

Divide the given sum by the number representing the amount of \$1 for the given time and rate.

EXERCISE 166. — WRITTEN.

1. Find the present worth and true discount of \$600 due in 5 mo., money being worth 6%.
2. Find the bank discount at 5% on \$600 due in 5 mo., without grace.
3. Find the present worth and the true discount of \$100 due in 2 yr. 7 mo. 15 dy., money being worth $4\frac{1}{2}$%.
4. Find the true discount April 20, 1896, on \$840 due in 4 mo., money being worth 5%.
5. Find the difference between the present worth and the bank proceeds of \$540 Aug. 3, 1896, with 3 mo. to run, money being worth 4%.
6. If money is worth 5%, find the difference between the present worth and the bank proceeds of a note for \$250 due in one year without interest.

Stocks and Bonds.

354. A **Stock Company** is an association of persons under the laws of the state in which the company is organized for the purpose of doing a specified business.

355. The **Stock** represents the capital invested in the business, and is issued in the form of certificates, each certifying that the person named in the certificate owns the stated number of shares of stock.

356. Bonds are written obligations under seal given by a company; or by a municipal or state government; or by the national government; in which an agreement is made to pay a specified amount on or before a specified date with interest payable annually, semi-annually, or quarterly.

357. Stocks and bonds are at **par** when they sell for their **face value**; at a **premium** when they sell for **more than their face value**; and at a **discount** when they sell for **less than their face value.**

358. Mortgage bonds are bonds secured by mortgage, but **Debenture bonds** are simply notes under seal.

359. Coupons are certificates of interest attached to bonds. These coupons are cut off as they become due and given up on receipt of the interest represented by them.

360. Bonds are **named** by giving the name of the Corporation issuing them and the rate of interest they bear. The date on which they become due is also given when necessary.

Thus, U. S. 4's, 1907 means United States Bonds bearing 4 per cent interest and due in 1907. U. S. 4's, 1925 means United States 4% Bonds due in 1925.

361. Persons who buy and sell stocks and bonds are called **Stock brokers,** and their commission is called **brokerage.** Brokerage is $\frac{1}{4}$% or $\frac{1}{8}$% on a share, reckoned at $100, and the same per cent on every $100 of the face value of every bond bought.

1. What will be the annual income from 420 shares of gas stock paying an annual dividend of $7\frac{1}{2}\%$?

SOLUTION. 1 share at $7\frac{1}{2}\%$ yields $\$7\frac{1}{2}$.
420 shares at $7\frac{1}{2}\%$ yield $420 \times \$7\frac{1}{2} = \$3150$.

2. How much must be invested in 6% stock at 107, to yield an annual income of $240, brokerage $\frac{1}{4}$?

SOLUTION. $6 is the dividend from 1 share.

$\$240$ dividend requires $\frac{240}{6}$, or 40 shares.

$$\begin{aligned} \text{Price of 40 shares is } 40 \times \$107 &= \$4280 \\ \text{Brokerage on 40 shares is } 40 \times \$0.25 &= \underline{\quad 10} \\ \text{Total cost} &= \$4290 \end{aligned}$$

3. What per cent does my investment yield if I buy Dominion Coal 6's at 92?

SOLUTION. Every $92 paid for the bonds yields $6 interest. The question becomes, what per cent of $92 is $6? (§ 294).

The following are stock quotations from a New York paper at date of this writing:

			Assumed Dividend.
1.	Adams Express Co.,	$150\frac{1}{2}$	8%
2.	Chicago & Northwestern Ry.,	$105\frac{1}{2}$	6%
3.	Illinois Central R.R.,	97	5%
4.	Lake Shore & Michigan Southern Ry.,	150	6%
5.	Michigan Central R.R.,	$97\frac{1}{2}$	4%
6.	New Jersey Central R.R.,	107	6%
7.	New York Central R.R.,	98	4%
8.	Pullman Palace Car Co.,	$163\frac{1}{2}$	8%
9.	Western Union Telegraph Co.,	86	5%
10.	New York, New Haven & Hartford R.R.,	170	8%

EXERCISE 167. — WRITTEN.

1. How much will 360 shares of No. 6 cost? How much will they yield per annum? What per cent do they yield on the cost?

2. How much must be invested in Illinois Central to yield an annual income of $220?

3. A man owns 90 shares of No. 2. What annual income does he receive?

4. What per cent on the investment does No. 4 pay? No. 10? No. 2? No. 6?

5. A man has $50,699.25 to invest. Which stock will yield the larger annual income, No. 3 or No. 5, allowing $\frac{1}{8}$ brokerage in each case? How much larger?

6. A man invested equal amounts in No. 1 and No. 10, paying $\frac{1}{8}$ brokerage on each share. Which yielded the larger income?

7. Which will pay the better income on the amount invested, No. 6 or No. 9? What per cent does each pay on the investment, reckoning brokerage $\frac{1}{8}$?

8. Which will pay the better income on the investment, No. 2 or No. 8? What per cent does each pay on the investment, reckoning brokerage $\frac{1}{8}$?

9. What annual rate of dividend would be just 6% on the cost of No. 4, not reckoning brokerage?

10. How much should one pay for stock paying 6% so that the income would be just 5% of the amount paid?

11. How much should one pay for $7\frac{1}{2}$% stock so that the income would be just 6% on the investment?

Exercise 168. — Written.

Review Problems.

1. What is 7% of $486?

2. $35.05 is 4% of what sum?

3. What per cent of $415 is $37.35?

4. A merchant sold cloth that cost him $1.50 per yard, gaining $16\frac{2}{3}$%. For what price did he sell it?

5. A salesman discounted a hat 22 cents, which was $5\frac{1}{2}$% on the price. What was the price?

6. The premium for insuring a house for $2300 was $11.50. What was the rate?

7. An agent sold a quantity of flour at 2% commission, receiving $73.86 for his commission. How much did he get for the flour?

8. Find the interest on $475 from Aug. 14, 1894, to Feb. 2, 1896, at 4%.

9. A man bought 4 Union Pacific Ry. bonds of $1000 each at 107, paying $\frac{1}{4}$ brokerage. What did they cost him?

10. How long must $360 be on interest at 5% to amount to $443.88?

11. The property of a town is assessed at $1,600,000. The tax levy is $7 on $1000. If the collector receives 2% and the treasurer $1\frac{1}{2}$% of the sum collected, how much will be left for the town?

12. Write a note for $275 at 90 days with interest at 6%. Date Aug. 9, 1892. Endorsed June 3, 1893, a payment of $10; Sept. 21, 1893, $120. Find what is due Feb. 1, 1894.

13. An agent received $974.61, which sum included money to invest in wheat at 50 cents per bu., and his commission at 2%. He sold the wheat at $67\frac{1}{2}$ cents per bu., charging 3% commission. Find his entire commission.

14. A note for $450 without interest, dated March 5, 1896, to run one year without grace, is discounted at a bank June 29 at 5%. Find the proceeds.

15. Find the present worth of a note of $500 due in one year, money being worth 5%.

16. A man bought land at $37.50 an acre and sold it at $44.25. What per cent did he gain?

17. A Chicago firm draws on a firm in Trenton at 60 days for $978. The draft is accepted. How much does the Chicago firm receive from its bank, exchange being $\frac{1}{4}$%, and rate of interest 6%?

18. If $120 yields a certain amount of interest in 3 yr. 9 mo. at 6%, what principal will yield the same interest in 2 yr. 3 mo.?

19. $98.70 is what per cent of $7.05?

20. Find the exact interest on $563 from Aug. 14, 1896, to May 10, 1897, at 5%.

21. A note for $250 without interest is dated July 6, 1896, payable in 1 yr. without grace. What can you get at a bank for it Jan. 13, 1897, if the rate of discount is 5%?

22. Suppose the above note bears interest at 6%. What will you receive for it?

23. Find the bank discount on a note of $2000 due in 90 days without grace, discounted at 5%.

24. $2400. Hartford, Conn., Feb. 1, 1892.

Three months after date, I promise to pay to the order of Samuel Hill, Twenty-four hundred dollars at the First National Bank of Hartford. Value received.

Benjamin Simpson.

Endorsed, July 16, 1892, $500; Dec. 4, 1893, $75.

What is due April 7, 1894, the rate of interest being 6%?

25. A man sold a piano for $243, gaining 8%. What did it cost?

26. An agent received $435.69 from his principal with which to pay for flour and his commission of 3% for buying. How much was the commission?

27. Find the simple interest at 5% on a note of $450 from June 10, 1896, to May 1, 1900.

28. Find the compound interest on the note in example 27.

29. Find the annual interest on the note in example 27.

EXERCISE 169. — WRITTEN.

Review Problems.

1. An agent's commission at $2\frac{1}{2}\%$ for selling 600 bu. of wheat was \$7.50. What was the price per bushel?

2. The premium for insuring a house at $1\frac{3}{4}\%$ was \$45.50. For how much was the house insured?

3. What was the amount of a bill upon which a collector received \$14.76 for collecting at $4\frac{1}{2}\%$?

4. A man paid \$94.64 for insuring his house at $\frac{1}{8}\%$ on $\frac{2}{3}$ of its value. Find the value of the house.

5. A farmer sold his farm for \$1296 less than it cost him, which was a loss of 18%. Find the cost.

6. $37\frac{1}{2}\%$ discount from the list price of a sewing-machine was \$14.25. What was the list price?

7. The interest at 6% for one year on a certain sum was \$50.40. What was the sum at interest?

8. A man paid \$38.40 for insuring his house at $1\frac{1}{5}\%$. For how much was the policy written?

9. A man sold a horse at a gain of \$37, and this was 16% of the cost. Find the cost.

10. A merchant added 20% to the cost price of an article in making his selling price, and this was an advance of \$3 on the cost. He dropped 25% from his price in selling. How much did he gain or lose?

11. A merchant's assets amount to \$6082.20. This will pay 62% of his debts. How much does he owe?

12. A man buys a bankrupt stock and pays \$1950 for it. The amount paid was 65% of the original cost. Find the original cost.

13. A lady bought a book for 75 cents less than the price, which was a discount of $16\frac{2}{3}\%$. What would it have cost her at 20% and 10% off?

14. A merchant bought hats at $1.50 each, and sold them at $1.75. Find his gain per cent.

15. A one-year note of $1000, without interest, dated March 2, 1896, is discounted at a bank Sept. 29, at 5%. Find the proceeds.

16. A ninety-day note for $275, dated Aug. 9, 1894, bearing interest at 6% from date, has payments of $10 June 3, 1895, and $120 Sept. 21, 1895. What was due Jan. 1, 1896?

17. A merchant marked an article so as to gain 30%. He dropped $4.50 from his price, and made a profit of only 18%. What did the article cost him?

18. A merchant sold a hat for $2.99, gaining 15%. Find the cost.

19. An agent received $8935.20 to buy cotton and pay his commission at 2%. Find his commission.

20. A merchant sells cloth at $7.41 per yard, gaining 14%. What per cent would he gain by selling at $7.67?

21. An agent sold corn, and remitted $7992.80 after deducting his commission of 3%. How much did he get for the corn?

22. A farmer sold his cow for $18.90, which was at a loss of 16%. What did she cost him?

23. For how much must a cargo worth $12,115.50 be insured so that the indemnity in case of loss shall be the value of the cargo and the amount paid as premium, the rate being $1\frac{1}{2}\%$?

HINT. 100% of policy = policy (property and premium).
$1\frac{1}{2}\%$ of policy = premium.
Therefore, $98\frac{1}{2}\%$ of policy = property.

24. At what price must you mark a hat costing $1.50 so you can discount the price 20% and still make 12%?

EXERCISE 170. — WRITTEN.

Review Problems.

1. The premium for insuring a house for $\frac{5}{6}$ of its value at $\frac{3}{4}\%$ was \$39 per year. Find the value of the house.

2. A note of \$210 was given June 21, 1895, bearing interest at 6%, due in one year without grace. Find the bank proceeds April 14, 1896, if the rate of discount is 5%.

3. Find the exact interest at 6% on \$325 from the date of Sept. 21, 1895 to Jan. 1, 1896.

4. A man bought a lot for \$1200, and built a house for \$1920. He insured the house for $\frac{2}{3}$ of its value at $\frac{3}{4}\%$. The house burned, and the lot was sold for \$1328. How much was his loss or gain?

5. A school had 42 pupils. During a month of 20 school days the average daily attendance was 39. What was the per cent of attendance?

6. What price must you mark a hat that cost \$1.25 so as to make 12% after discounting your price 30%?

7. The net tax used in a city was \$50,592; the assessed valuation was \$3,400,000; 7% was lost from the amount levied in delinquencies and cost of collection. How many mills on the dollar were levied?

8. A man buys 8% stock at a premium of 20%. What per cent does he receive on his investment?

9. Find the true discount Aug. 6, 1896, on a note of \$150 due Aug. 6, 1901, if money is worth 6%.

10. A merchant buys woolen goods in Europe at \$9 a piece, pays a duty of $16\frac{2}{3}\%$, and sells them at \$12.60 a piece. What per cent on the entire cost does he gain?

11. How much will 26 shares of stock cost at 7% discount, brokerage $\frac{1}{4}$?

12. A note for 90 days, without grace, at 5%, was discounted at a bank. The amount received from the bank was $256.10. What was the face of the note?

13. An agent sold a sewing machine for 20% less than his catalogue price, and yet gained 20% on its cost. If the cost of the sewing machine was $25, what was the catalogue price?

14. What amount must be invested in stocks at 85, yielding a semi-annual dividend of $2\frac{1}{4}$%, to give an annual income of $2475?

15. At 7% for a certain time $240 amounts to $281.44. Find the time.

16. I take a note for $475 without interest to a bank to-day and receive $468.90 for it, paying 7% discount. How long has it to run?

17. If New York paper is at $\frac{1}{2}$% premium in Chicago, what will be paid there for a ninety-day draft on New York for $540, money being worth 7%?

18. Find the amount of $803 at 8% exact interest from July 7 to Nov. 28.

19. A note for $360 was given Aug. 12, 1890, due 6 mo. after date. Payments were made as follows: Feb. 12, 1891, $110; June 24, 1892, $50; Sept. 6, 1895, $40. Find amount to settle May 15, 1896, interest being 6%.

20. A merchant dropped 8% on his price and yet made 15% on an article that cost him $10.40. What was his asking price?

21. A man has $5081.67 at interest. What is his daily income from it at 6%?

CHAPTER XIII.

PROPORTION.

362. Ratio. The *ratio* of two numbers is their *relative magnitude,* expressed by the fraction which has the first number for numerator and the second number for denominator.

Thus, the ratio of 2 to 3, commonly written 2 : 3, is expressed by the fraction $\frac{2}{3}$.

363. Antecedent and Consequent. The first term of a ratio is the *antecedent,* and the second term the *consequent.*

364. If both terms of a ratio are multiplied or divided by the same number, the value of the ratio is not altered.

Thus, if both terms of the ratio $2\frac{1}{2} : 3\frac{1}{3}$ are multiplied by 6, the resulting ratio is $15 : 20$, and the two ratios are equal, for $\frac{2\frac{1}{2}}{3\frac{1}{3}} = \frac{15}{20}$.

Since $\frac{15}{20} = \frac{3}{4}$, the simplest expression for $2\frac{1}{2} : 3\frac{1}{3}$ is $3 : 4$.

365. If the numerator and denominator of a fraction are interchanged, the fraction is *inverted;* likewise, if the antecedent and consequent of a ratio are interchanged, the resulting ratio is the *inverse* of the given ratio.

Thus, if the fraction $\frac{4}{5}$ is inverted, the resulting fraction is $\frac{5}{4}$, and the inverse of the ratio $4 : 5$ is $5 : 4$.

366. If two *quantities* are expressed in the *same unit,* their ratio is the same as the ratio of the two *numbers* by which they are expressed.

Thus, the quantity \$7 is the same fraction of \$9 as 7 is of 9.

367. Since ratio is simply *relative magnitude,* two quantities *different in kind* cannot form the terms of a ratio;

and two quantities of the same kind must be expressed in a *common unit* before they can form the terms of a ratio.

Thus, no ratio exists between $5 and 20 dy.; and the ratio of 3 t. to 5000 lb. can be expressed only when *both* quantities are written as tons or as pounds.

368. When two ratios are equal, the four terms form a *proportion*, and are called *proportionals*.

Thus, 6, 3, 18, 9 are in proportion; for $\frac{6}{3} = \frac{18}{9}$.

369. A **Proportion,** therefore, is an expression of equality between two ratios, and is written by putting the sign = or a double colon between the ratios.

Thus, $6 : 3 = 18 : 9$, or $6 : 3 :: 18 : 9$, means, and is read, the ratio of 6 to 3 is equal to the ratio of 18 to 9.

370. The *first* and *last* terms of a proportion are the **extremes,** and the two *middle* terms are the **means.**

371. Test of a Proportion. When four numbers are in proportion, the product of the extremes is equal to the product of the means.

Thus, in the proportion $5 : 3 :: 15 : 9$, $5 \times 9 = 3 \times 15$.

372. Either extreme, therefore, is equal to the product of the means divided by the other extreme; and either mean is equal to the product of the extremes divided by the other mean.

Hence, if three terms of a proportion are given, the fourth may be found.

1. Find the missing term of the proportion $6 : 9 :: 18 : ?$.

The product of the means divided by the given extreme is

$$\frac{9 \times 18}{6} = 27.$$

2. Find the missing term of the proportion $20 : 24 :: ? : 30$.

The product of the extremes divided by the given mean is

$$\frac{20 \times 30}{24} = 25,$$

Exercise 171. — Written.

Find the missing term:

1. $24:16::?:12$.
2. $15:?::35:21$.
3. $6:32::15:?$.
4. $?:8::15:20$.
5. $15:?::10:8$.
6. $22:10::55:?$.
7. $6:4::9:?$.
8. $5:9::?:12$.
9. $?:18::45:32$.
10. $6:9::12:?$.

373. Rule of Three. When three terms of a problem in proportion are given, the method of finding the fourth term is called the *Rule of Three.*

It is usual to arrange the quantities (that is, to *state* the question) so that the quantity required for the answer may be the *fourth* term. Hence the quantity which *corresponds* to that of the required answer must be the *third* term.

1. If 5 t. of hay cost $87.50, what will 21 t. cost?

Since the *cost* of 21 t. is required, $87.50 is the third term.

Since 21 t. will cost *more* than 5 t., 21 t. is the second term and 5 t. the first term.

That is, 5 t. : 21 t. : : $87.50 : What quantity?

A difficulty presents itself here, inasmuch as no meaning can be given to the product of the means ($87.50 multiplied by 21 t.). Since, however, the ratio of 5 t. : 21 t. = the ratio of 5 : 21, the ratio 5 : 21 may be put in place of 5 t. : 21 t.

Then 5 : 21 : : $87.50 : What quantity?

That is, What quantity $= \dfrac{21 \times \$87.50}{5}$?

$367.50. *Ans.*

2. When a post 11.5 ft. high casts a shadow on level ground 17.4 ft. long, a neighboring steeple casts a shadow 63.7 yd. long. How high is the steeple?

Height is required; 11.5 ft. is therefore the third term.

Since the *shadow* of the steeple is the *longer*, the *height* of the steeple must be the *greater;* therefore the second term must be the

greater of the two remaining quantities expressed in the same *unit.* 63.7 yd. = 191.1 ft. Therefore,

	Shadow.	Shadow.	Height.	Height.
	17.4 ft. :	191.1 ft. : :	11.5 ft. :	What?
or,	17.4 :	191.1 : :	11.5 ft. :	What?

That is, height of steeple $= \frac{191.1 \times 11.5 \text{ ft.}}{17.4} = 126.3$ ft. *Ans.*

374. To solve problems by the Rule of Three,

Make that quantity which is of the same kind as the required answer the third term.

If, from the nature of the question, the answer will be greater than the third term, make the greater of the two remaining quantities the second term; if the answer will be smaller than the third term, make the smaller of these quantities the second term, and the other the first term.

Divide the product of the second and third terms by the first term, and the quotient will be the answer required.

EXERCISE 172. — WRITTEN.

1. If 17 yards of silk cost $34, what will be the cost of 20 yards at the same rate?
2. How long will 12 men take to do a piece of work that 8 men can do in 9 days?
3. If 12 bushels of wheat can be bought for $8, how many bushels can be bought for $100?
4. If the railway fare for a journey of 50 miles is $1.25, what will be the fare for a journey of 130 miles?
5. If 8 men require 75 days to finish a piece of work, how many men could do it in 40 days?
6. How many men in 19 days would do a piece of work which 209 men can do in 10 days?
7. How far can 36 tons be carried for the money paid for carrying 54 tons 144 miles?

8. A quantity of provisions will last 189 men 4 months, how long will the same provisions last 252 men?

9. If 102 acres of land produce 1326 bushels of wheat, how much did 71 acres of it produce?

10. If 9.45 acres of pasture are sufficient for 25 sheep, how many acres will be sufficient for 360 sheep?

11. If a train makes 177 miles 120 rods in 4 hours, what is the average rate per hour?

12. If a bankrupt's debts are $5344, and he is able to pay 75 cents on a dollar, what are his assets worth?

13. When a shadow 8 ft. 6 in. long is cast by a post 5 ft. 6 in. high, how high is a steeple that casts a shadow 221 ft. long?

14. When a tree 38 ft. high casts a shadow 40 ft. long, how long is the shadow of a tree 57 feet high?

15. If $18\frac{7}{8}$ acres are sold for $900, what will 160 acres cost at the same rate?

16. If $1\frac{1}{2}$ bushels of wheat are sold for 93 cents, how many bushels can be bought for $6.51?

17. If a train travels 319 miles in 9 hr. 40 min., how long will it be in traveling 231 miles?

18. How many yards of cloth $\frac{5}{8}$ yd. wide will be required to line 35 yards $1\frac{3}{4}$ yd. wide?

19. A bankrupt owes $4000 and has only $840. How much can he pay on a dollar?

20. If the railway fare between two places 56 miles apart is $1.40, what must be paid for a journey of 320 miles at the same rate?

21. Gun-metal is composed of 1 part of tin to $5\frac{1}{2}$ parts of copper by weight. How many pounds of tin must be added to $210\frac{3}{8}$ pounds of copper to make gun-metal?

22. How many pounds of tin will there be in $232\frac{3}{8}$ pounds of gun-metal composed of 1 part of tin to $5\frac{1}{2}$ parts of copper?

Compound Proportion.

375. Compound Ratio. A ratio is said to be *compounded* of two or more given ratios, when it is expressed by a fraction which is the product of the fractions representing the given ratios.

Thus the ratios 2 : 3 and 7 : 11 are represented by the fractions $\frac{2}{3}$ and $\frac{7}{11}$; and the ratio 14 : 33, which is represented by $\frac{14}{33}$ (the product of $\frac{2}{3}$ and $\frac{7}{11}$), is said to be compounded of the ratios 2 : 3 and 7 : 11.

376. Compound Proportion. A proportion which has one of its ratios a compound ratio is called a *compound proportion.*

In stating problems in compound proportion the quantity which corresponds to the answer required is made the third term. Each *pair* of the remaining quantities is then considered *separately* with reference to the answer required. The process will be understood by the following example:

If four men mow 15 A. in 5 dy. of 14 hr., in how many days of 13 hr. can 7 men mow $19\frac{1}{2}$ A.?

As the answer is to be in days, make 5 dy. the third term.

I. *It will require less* days for 7 men to mow 15 A. than for 4 men. Therefore make 7 the first term and 4 the second.

II. *It will require more* days for the same number of men to mow $19\frac{1}{2}$ A. than 15 A.

Therefore make 15 the first term and $19\frac{1}{2}$ the second.

III. *It will require more* days of 13 hr. than of 14 hr.

Therefore make 13 the first term and 14 the second.

Hence the statement is

$$\left.\begin{array}{l} 7:4 \\ 15:19.5 \\ 13:14 \end{array}\right. :: 5 \text{ days} : \text{What?}$$

or

$$\frac{4 \times 19.5 \times 14 \times 5 \text{ days}}{7 \times 15 \times 13}.$$

This, simplified by cancellation, gives 4 days.

EXERCISE 173. — WRITTEN.

1. If 4 men earn $144 in 12 days, how much will 6 men earn in 10 days at the same rate?

2. If 15 men earn $750 in 25 days, in how many days will 20 men earn $600?

3. If a man walks 120 miles in 5 days, walking 8 hours a day, in how many days will he walk 300 miles walking 10 hours a day?

4. If 12 bushels of oats are sufficient for 3 horses 10 days, how many bushels will be sufficient for 15 horses 5 days?

5. If 3 workmen can board 4 weeks for $54, how many can board 13 weeks for $585?

6. If 10 men can cut 120 cords of wood in 8 days, how many cords can 20 men cut in 6 days?

7. If 12 men earn $324 in 7 days, how much will 10 men earn in 78 days?

8. If 12 men can dig a ditch 96 rods long in 8 days, in how many days will 6 men dig a ditch 66 rods long of the same width and depth?

9. If 4 men build 66 rods of wall in 11 days, in how many days will 6 men build 72 rods?

10. How many days will it take 15 men to cut 405 cords of wood, if 13 men can cut 780 cords in 40 days?

11. If the foreman of a shop earns $30 in 6 days working 10 hours a day, how many hours a day must he work to earn $46 in 8 days at the same rate per hour?

12. If 36 men earn $1296 in 18 days, how much will 42 men earn in 87 days?

13. If it costs $2.40 to carry 20 cwt. 50 miles, what will it cost to carry 40 cwt. 40 miles at the same rate?

Proportional Parts.

377. If it is required to divide a quantity into parts proportional to 3, 4, 5, the *numbers* 3, 4, 5 may be taken to represent the *parts,* and then the *whole* will be represented by $3 + 4 + 5$; that is, by 12.

1. Divide \$391 into parts proportional to 5, 7, and 11.

The whole quantity will be represented by $5 + 7 + 11 = 23$.

Therefore the respective parts will be $\frac{5}{23}$, $\frac{7}{23}$, $\frac{11}{23}$ of \$391.

\$85, \$119, \$187. ***Ans.***

2. Divide \$248 into parts proportional to $\frac{1}{10}$, $\frac{1}{15}$, $\frac{1}{25}$.

Multiply the fractions by 150, L.C.M. of their denominators. The results are 15, 10, 6. Hence the parts will be represented by the numbers 15, 10, 6, and the whole by 31.

Therefore the respective parts will be $\frac{15}{31}$, $\frac{10}{31}$, $\frac{6}{31}$ of \$248.

\$120, \$80, \$48. ***Ans.***

Exercise 174. — Written.

1. Divide 1000 into parts proportional to 10, 12, 13, 15.
2. Divide 570 into parts proportional to $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$.
3. Divide a profit of \$6817 among three partners, if the first owns $\frac{4}{17}$, the second $\frac{6}{17}$, and the third $\frac{7}{17}$ of the joint stock.
4. Four men invest \$4000, \$2300, \$1900, \$1600 respectively in a joint business. Find their respective liabilities in a loss of \$627.20.
5. Three partners claim respectively $\frac{2}{9}$, $\frac{11}{18}$, and $\frac{7}{18}$ of \$1320. Give to each his proportional share.
6. James has \$4.50 in 5-cent, 10-cent, 25-cent, and 50-cent pieces, the same number of each. How many pieces of each kind has he?
7. Divide 64 cents between George and Henry so that their shares shall be in proportion to $\frac{1}{7}$, $\frac{1}{9}$.

Partnership.

378. Partnership is separated into *simple* and *compound.*

In simple partnership the capital of each partner is invested for *the same time.*

In compound partnership *the time* for which the capital of each partner is invested is taken into account, as well as *the amount of the capital;* and the division of profits and losses is made proportionally to the amount of the capital and the time it is invested.

A and B enter into partnership. A puts in $2000 for 2 yr., and B puts in $3000 for 1 yr. Their profits are $1400. What is the share of each?

The use of $2000 for 2 yr. is equal to the use of 2 × $2000 for 1 yr. Hence their profits must be divided in the ratio $4000 to $3000; that is, 4 : 3. A, $800; B, $600. *Ans.*

EXERCISE 175. — WRITTEN.

1. A and B engage in trade for one year. A puts in $6000 and B $4000. They gain $1000. What is each one's share of the gain?

2. A, B, and C rented a pasture for $441. A put in 18 cows for 5 months; B, 24 cows for 4 months; and C, 36 cows for 3 months. How much of the rent ought each to pay?

3. A and B enter into partnership. A puts in $1800 and B $1500. At the end of 4 months B puts in $500 more. Their profit at the end of the year is $872. What is each one's share of the profit?

4. A and B are in partnership for two years. A at first put in $2500 and B $3000. At the end of 9 months A took out $500 and B put in $500. They gained in the two years $2640. What is each one's share of the gain?

Averages.

379. The **Average** of several numbers is a number which can be put in place of each of them without altering their sum.

Find the average weight of four turkeys, weighing respectively 10 lb., 11 lb., 12 lb., and 13 lb.

The turkeys together weigh (10 + 11 + 12 + 13) lb. = 46 lb. Hence, the average weight of the four turkeys is $\frac{1}{4}$ of 46 lb., or $11\frac{1}{2}$ lb.

380. To find the average of a number of quantities of the same kind,

Divide the sum of the quantities by the number of them.

Exercise 176. — Written.

1. There were 125 pupils at school on Monday, 130 on Tuesday, 128 on Wednesday, 132 on Thursday, and 125 on Friday. What was the average daily attendance?

2. A grocer mixed 20 lb. of coffee costing 28 cents a pound and 25 lb. costing 25 cents a pound, with 12 lb. of chicory costing 8 cents a pound. What is the cost per pound of the mixture?

3. A merchant's receipts were: Monday, \$329.63; Tuesday, \$279.87; Wednesday, \$337.50; Thursday, \$319.68; Friday, \$256.33; Saturday, \$497.62. What were his average daily receipts?

4. A grocer mixed 50 lb. of tea at 35 cents a pound, 70 lb. at 40 cents a pound, and 80 lb. at 42 cents a pound. He sold the mixture at 58 cents a pound. How much did he gain on the whole?

5. A goldsmith combined 2 oz. of gold 16 carats fine, 2 oz. 18 carats fine, and 6 oz. 22 carats fine. What is the fineness of the composition?

Note. Pure gold is 24 carats fine. Gold 22 carats fine has 22 parts gold and 2 parts alloy (silver or copper).

Average of Payments.

381. The **Term of Credit** is the time allowed for the payment of a debt.

A owes B $250 due in 3 mo., and $350 due in 5 mo. What is the average term of credit?

The use of $250 for 3 mo. equals the use for 1 mo. of $750.
The use of $350 for 5 mo. equals the use for 1 mo. of $1750.
The use of $600 for $4\frac{1}{6}$ mo. equals the use for 1 mo. of $2500.

The *number* of months required, $4\frac{1}{6}$, is found by dividing the $2500 by $600. Hence,

382. To find the average term of credit for payments due at different times,

Multiply each payment by its term of credit, and divide the sum of the products by the sum of the payments.

EXERCISE 177. — WRITTEN.

1. Find the average time for the payment of $600 due in 3 mo., $1000 due in 6 mo., $400 due in 9 mo.

2. Find the average time for the payment of $2000 due in 3 mo., $1500 due in 4 mo., $2500 due in 8 mo.

3. Find the average time for the payment of $3000 due in 4 mo., $1500 due in 6 mo., $1000 due in 12 mo., $500 due in 8 mo.

4. Find the average time for the payment of $4000 due Jan. 1; $3000 due May 1; $2000 due July 1; and $3000 due Sept. 1; reckoning the time from Jan. 1.

5. A owes B $2000, payable at the end of 8 mo. He pays him $400 at the end of 3 mo. and $600 at the end of 6 mo. At what time after the 8 mo. is the balance due?

6. On the first day of July, 1896, A purchases of B $300 worth of goods on 3 mo. and $500 on 4 mo. He gives one note in payment. At what date should the note be made payable?

CHAPTER XIV.

Powers and Roots.

383. The **Square** of a number is the product of *two* factors, each equal to this number.

Thus, the squares of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100.

384. The **Square Root** of a number is one of the *two equal factors* of the number.

Thus, the square roots of 1, 4, 9, 16, 25, 36, 49, 64, 81, 100,
are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

385. The square root of a number is indicated by the *radical sign* $\sqrt{\ }$, or by the fraction $\frac{1}{2}$ written above and to the right of the number.

Thus, $\sqrt{27}$, or $27^{\frac{1}{2}}$, means the square root of 27.

386. Since $35 = 30 + 5$, the square of 35 may be obtained as follows:

$$\begin{array}{l} 30 + 5 \\ \underline{30 + 5} \\ 30^2 + (30 \times 5) \\ \underline{\quad + (30 \times 5) + 5^2} \\ 30^2 + 2(30 \times 5) + 5^2 \end{array} \qquad \begin{array}{r} 30^2 = 900 \\ 2(30 \times 5) = 300 \\ 5^2 = \underline{\ \ 25} \\ 35^2 = 1225 \end{array}$$

387. Hence, since every number consisting of two or more figures may be regarded as composed of tens and units,

The square of a number will contain the square of the tens + twice the tens × the units + the square of the units.

Square Root.

388. The first step in extracting the square root of a number is to mark off the figures of the number into groups.

Since $1 = 1^2$, $100 = 10^2$, $10{,}000 = 100^2$, and so on, it is evident that the square root of any number between 1 and 100 lies between 1 and 10; of any number between 100 and 10,000 lies between 10 and 100. In other words, the square root of any integral number expressed by *one* or *two* figures is a number of *one* figure; expressed by *three* or *four* figures is a number of *two* figures, and so on.

If, therefore, an integral number be divided into groups of two figures each, from the right to the left, the number of figures in the root will be equal to the number of groups of figures. The last division to the left may have one or two figures.

Find the square root of 1225.

SOLUTION. The first group 12, contains the square of the tens' number of the root.

The greatest square in 12 is 9, and the square root of 9 is 3. Hence. 3 is the tens' figure of the root.

12 25(35
9
65)3 25
3 25

The square of the tens is subtracted, and the remainder contains twice the tens × the units + the square of the units. Twice the 3 tens is 6 tens, and 6 tens is contained in the 32 tens of the remainder 5 times. Hence, 5 is the units' figure of the root. Since twice the tens × the units + the square of the units is equal to (twice the tens + the units) × the units, the 5 units are annexed to the 6 tens, and the result, 65, is multiplied by 5.

389. The same method will apply to numbers of more than two groups of figures, by considering *the part of the root already found as so many tens* with respect to the next figure.

Extract the square root of 7,890,481.

7 89 04 81(2809
4
48)3 89
3 84
5609)5 04 81
5 04 81

SOLUTION. When the third group, 04, is brought down, and the divisor, 56, formed, the next figure of the root is 0, because 56 is not contained in 50. Therefore, 0 is placed both in the root and the divisor, and the next two figures, 81, are brought down.

390. If the square root of a number have decimal places, the number itself will have *twice* as many.

Thus, if 0.11 be the square root of some number, the number will be $(0.11)^2 = 0.11 \times 0.11 = 0.0121$. Hence, if a given number contains a decimal, we divide it into groups of two figures each, beginning at the decimal point and marking toward the left for the integral number, and toward the right for the decimal. The last group of the decimal must have *two* figures, a cipher being annexed if necessary.

Extract the square root of 52.2729.

52.27 29(7.23
49
142)3 27
2 84
1443) 43 29
43 29

SOLUTION. It will be seen from the groups of figures that the root will have one integral place and two decimal places.

391. If a number is not a perfect square, ciphers may be annexed, and an *approximate* value of the root found.

Extract to six places of decimals the square root of 19.

19.00 00 00(4.358899
16
83)3 00
2 49
865) 51 00
43 25
8708)7 75 00
6 96 64
8716) 78 360
69 728
8 6320
7 8444
78760

SOLUTION. In this example, after finding four figures of the root, the other three are found by common division. The rule in such cases is, that one less than the number of figures already obtained may be found without error by division, the divisor to be employed being twice the part of the root already found.

392. The square root of a common fraction is found by extracting the square root of the numerator and denominator. But, when the denominator is not a perfect square, we must multiply both terms of the fraction by such a number as will make the denominator a perfect square, or reduce the fraction to a decimal and then extract the root.

RULE FOR SQUARE ROOT. *Separate the number into groups of two figures each, beginning at the units.*

Find the greatest square in the left-hand group and write its root for the first figure of the required root.

Square this root, subtract the result from the left-hand group, and to the remainder annex the next group for a dividend.

For a partial divisor, double the root already found, considered as tens, and divide the dividend by it. The quotient (or the quotient diminished) will be the next figure of the root.

To this partial divisor add the last figure of the root for a complete divisor. Multiply this complete divisor by the last figure of the root, subtract the product from the dividend, and to the remainder annex the next group for a new dividend.

Proceed in this manner until all the groups have been thus annexed. The result will be the square root required.

NOTE 1. When the number is not a perfect square annex groups of zeros and continue the process.

NOTE 2. If a given number contains a decimal, divide it into groups of two figures each, beginning at the decimal point and marking toward the left for the integral number and toward the right for the decimal number.

Be careful to have the last group on the right of the decimal point contain two figures, annexing a zero when necessary.

EXERCISE 178. — WRITTEN.

Find the square root of:

1. 190,969.	**8.** 43.267.	**15.** $\frac{729}{961}$.
2. 743,044.	**9.** 872.6783.	**16.** $\frac{225}{4489}$.
3. 401,956.	**10.** 0.00755.	**17.** $\frac{1849}{3249}$.
4. 1075.84.	**11.** 1272.9.	**18.** $\frac{1}{5}$.
5. 60.3729.	**12.** 3263.927.	**19.** $\frac{3}{14}$.
6. 4.347225.	**13.** 0.12773.	**20.** $\frac{7}{8}$.
7. 0.76790169.	**14.** 86.9432.	**21.** $\frac{19}{23}$.

Cube Root.

393. The **Cube** of a number is the product of *three* factors, each equal to the number.

Thus, the cubes of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
are 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000.

394. The **Cube Root** of a number is one of the *three equal factors* of the number.

Thus, the cube roots of 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000,
are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

395. The cube root of a number is indicated by $\sqrt[3]{\ }$, or by the fraction $\frac{1}{3}$ written above and to the right of the number.

Thus, $\sqrt[3]{343}$, or $343^{\frac{1}{3}}$, means the cube root of 343.

396. Since $35 = 30 + 5$, the cube of 35 may be obtained thus:

$$\begin{array}{l}
30 + 5 \\
\underline{30 + 5} \\
30^2 + (30 \times 5) \\
\underline{\quad + (30 \times 5) + 5^2} \\
30^2 + 2(30 \times 5) + 5^2 \\
\underline{30 + 5} \\
30^3 + 2(30^2 \times 5) + (30 \times 5^2) \\
\underline{\quad + (30^2 \times 5) + 2(30 \times 5^2) + 5^3} \\
30^3 + 3(30^2 \times 5) + 3(30 \times 5^2) + 5^3
\end{array}$$

$$\begin{array}{r}
30^3 = 27{,}000 \\
3(30^2 \times 5) = 13{,}500 \\
3(30 \times 5^2) = 2{,}250 \\
5^3 = \underline{\quad 125} \\
35^3 = 42{,}875
\end{array}$$

Hence, the cube of any number composed of tens and units contains four parts:

I. *The cube of the tens.*

II. *Three times the product of the square of the tens by the units.*

III. *Three times the product of the tens by the square of the units.*

IV. *The cube of the units.*

397. In extracting the cube root of a number, the first step is to mark off the figures of the number into groups.

Since $1 = 1^3$, $1000 = 10^3$, $1,000,000 = 100^3$, and so on, it follows that the cube root of any integral number between 1 and 1000, that is, of any integral number that has *one*, *two*, or *three* figures, is a number of *one* figure; and that the cube root of any integral number between 1000 and 1,000,000, that is, of any integral number that has *four*, *five*, or *six* figures, is a number of *two* figures, and so on.

If, therefore, an integral number be divided into groups of three figures each, from right to left, the number of figures in the root will be equal to the number of groups. The last division to the left may consist of one, two, or three figures.

Extract the cube root of 42,875.

SOLUTION. Since 42,875 consists of two groups, the cube root will consist of two figures.

The first group, 42, contains the cube of the tens' number of the root.

The greatest cube in 42 is 27, and the cube root of 27 is 3. Hence, 3 is the tens' figure of the root.

$$
\begin{array}{r|l}
 & 42\ 875(35 \\
 & \underline{27} \\
3 \times 30^2 = 2700 & 15\ 875 \\
3 \times (30 \times 5) = 450 & \\
5^2 = \underline{25} & \\
3175 & \underline{15\ 875}
\end{array}
$$

The remainder, 15,875, resulting from subtracting the cube of the tens, will contain three times the product of the square of the tens by the units + three times the product of the tens by the square of the units + the cube of the units.

Each of these three parts contains the units' number as a factor.

Hence the 15,875 consists of two factors, one of which is the units' number of the root; and the other factor is three times the square of the tens + three times the product of the tens by the units + the square of the units. The largest part of this second factor is three times the square of the tens.

And, if the 158 hundreds of the remainder be divided by the $3 \times 30^2 = 27$ hundreds, the quotient will be the units' number of the root.

The second factor can now be completed by adding to the 2700 $3 \times (30 \times 5) = 450$ and $5^2 = 25$.

398. The same method will apply to numbers of more than two groups of figures, by considering the part of the root already found as *so many tens with respect to the next figure of the root.*

Extract the cube root of 57,512,456.

		57 512 456 (386
		27
$3 \times 30^2 =$	2700	30 512
$3 \times (30 \times 8) =$	720	
$8^2 =$	64	
	3484	27 872
		2 640 456
$3 \times 380^2 =$	433200	
$3 \times (380 \times 6) =$	6840	
$6^2 =$	36	
	440076	2 640 456

399. If the cube root of a number have decimal places, the number itself will have *three times* as many.

Thus, if 0.11 is the cube root of a number, the number is $0.11 \times 0.11 \times 0.11 = 0.001331$. Hence, if a given number contains a decimal, we divide the figures of the number into groups of three figures each, by beginning at the decimal point and marking toward the left for the integral number, and toward the right for the decimal. We must be careful to have the last group on the right of the decimal point contain *three* figures, annexing ciphers when necessary.

Extract the cube root of 187.149248.

		187.149 248 (5.72
		125
$3 \times 50^2 =$	7500	62 149
$3 \times (50 \times 7) =$	1050	
$7^2 =$	49	
	8599	60 193
		1 956 248
$3 \times 570^2 =$	974700	
$3 \times (570 \times 2) =$	3420	
$2^2 =$	4	
	978124	1 956 248

It will be seen from the groups of figures that the root will have one integral and two decimal places, and therefore the decimal point must be placed in the root as soon as one figure of the root is obtained.

400. If the given number is not a perfect cube, ciphers may be annexed, and a value of the root may be found as near to the *true* value as we please.

Extract the cube root of 1250.6894.

		1 250.689 400 (10.77
		1
$3 \times 10^2 =$	300	250

Since 300 is not contained in 250, the next figure of the root is 0.

$3 \times 100^2 =$	30000	250 689
$3 \times (100 \times 7) =$	2100	
$7^2 =$	49	
	32149	225 043
		25 646 400
$3 \times 1070^2 =$	3434700	
$3 \times (1070 \times 7) =$	22470	
$7^2 =$	49	
	3457219	24 200 533
		1 445 867

401. The following is a short method.

Extract the cube root of 5 to five places of decimals.

		5.000 (1.70997
		1
$3 \times 10^2 =$	300	4 000
$3\,(10 \times 7) =$	210	
$7^2 =$	49	
	559	3 913
	259	87 000 000
$3 \times 1700^2 =$	8670000	
$3\,(1700 \times 9) =$	45900	
$9^2 =$	81	
	8715981	78 443 829
	45981	8 556 1710
$3 \times 1709^2 =$	8762043	7 885 8387
		670 33230
		613 34301

After the first two figures of the root are found, the next trial divisor is obtained by bringing down the sum of the 210 and 49 obtained in completing the preceding divisor, then adding the three lines connected by the brace, and annexing two ciphers to the result.

It is seen at a glance that, when the trial divisor is increased by 3 times the 17 tens of the root, it will be greater than 87000; so that 0 is placed in the root, and 3×1700^2 is obtained by annexing two ciphers to the 86,700. Again: the trial divisor is obtained by bringing down the sum of the 45,900 and 81, which was obtained in completing the preceding divisor, then adding the three lines connected by the brace, and annexing two ciphers to the result.

The last two figures of the root are found by division. The rule in such cases is, that two less than the number of figures already obtained may be found without error by division, the divisor to be employed being three times the square of the part of the root already found.

402. The cube root of a common fraction is found by taking the cube root of the numerator and denominator; but, if the denominator is not a perfect cube, it is generally best to reduce the fraction to a decimal, and then extract the root.

RULE FOR CUBE ROOT. *Separate the number into groups of three figures each, beginning at the units.*

Find the greatest cube in the left-hand group and write its root for the first figure of the required root.

Cube this root, subtract the result from the left-hand group, and to the remainder annex the next group for a dividend.

For a partial divisor, take three times the square of the root already found, considered as tens, and divide the dividend by it. The quotient (or the quotient diminished) will be the second figure of the root.

To this partial divisor add three times the product of the first figure of the root considered as tens by the second figure, and also the square of the second figure. This sum will be the complete divisor.

Multiply the complete divisor by the second figure of the root, subtract the product from the dividend, and to the remainder annex the next group for a new dividend.

Proceed in this manner until all the groups have been annexed. The result will be the cube root required.

NOTE. If a given number contains a decimal, the last group on the right of the decimal point must be made to contain three figures, by annexing one or two zeros if necessary.

EXERCISE 179. — WRITTEN.

Find the cube root of :

1. 636,056.	**7.** 71.296.	**13.** $\frac{2197}{4913}$.
2. 2,048,383.	**8.** 643.25.	**14.** $\frac{6859}{12167}$.
3. 47.832147.	**9.** 7.1296.	**15.** $\frac{1}{2}$.
4. 11.390625.	**10.** 0.75475.	**16.** $\frac{5}{8}$.
5. 87,528.384.	**11.** 1,127,632.	**17.** 2.
6. 0.000912673.	**12.** 21.782.	**18.** 3.

Geometrical Representation of Square and Cube Roots.

The square of $(30 + 5) = 30^2 + 2(30 \times 5) + 5^2$. § 386.

The 30^2 may be represented by a square (Fig. 1) 30 in. on a side.

The $2(30 \times 5)$ may be represented by two strips 30 in. long and 5 in. wide, of Fig. 2, which are added to two adjacent sides of Fig. 1.

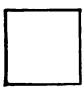


FIG. 1.

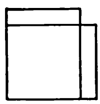


FIG. 2.

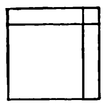


FIG. 3.

The 5^2 may be represented by the small square of Fig. 3 required to make Fig. 2 a complete square.

In extracting the square root of 1225, the large square, which is 30 in. on a side, is first removed, and a surface of 325 sq. in. remains.

This surface consists of two equal rectangles, each 30 in. long, and a small square whose side is equal to the width of the rectangles.

The width of the rectangles is found by dividing the 325 sq. in. by the sum of their lengths, that is, by 60, which gives 5 in.

Hence, the entire length of the surfaces added is 30 in. + 30 in. + 5 in. = 65 in., and the width is 5 in.

Therefore the total area is (65 × 5) sq. in. = 325 sq. in.

The cube of $(30 + 5) = 30^3 + 3(30^2 \times 5) + 3(30 \times 5^2) + 5^3$. § 396.

The 30^3 may be represented by a cube whose edge is 30 in. (Fig. 1.)

The $3(30^2 \times 5)$ may be represented by three equal rectangular solids, each 30 in. long, 30 in. wide, and 5 in. thick, to be added to three adjacent faces of Fig. 1.

The $3(30 \times 5^2)$ may be represented by three equal rectangular solids, 30 in. long, 5 in. wide, and 5 in. thick, to be added to Fig. 2.

The 5^3 may be represented by the small cube required to complete the cube of Fig. 3.

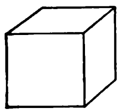


FIG. 1.

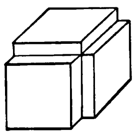


FIG. 2.

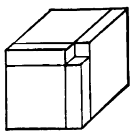


FIG. 3.

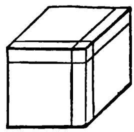


FIG. 4.

In extracting the cube root of 42,875, the large cube (Fig. 1), whose edge is 30 in., is first removed.

There remain (42,875 − 27,000) cu. in. = 15,875 cu. in.

The greatest part of this is contained in the three rectangular solids which are added to Fig. 1, and which are each 30 in. long and 30 in. wide.

The thickness of these solids is found by dividing the 15,875 cu. in. by the sum of the three faces, each of which is 30 in. square; that is, by 2700 sq. in. The result is 5 in.

There are also the three rectangular solids which are added to Fig. 2, and which are 30 in. long and 5 in. wide; and a cube which is added to Fig. 3, and which is 5 in. long and 5 in. wide.

Hence, the sum of the products of two dimensions of all these solids is

For the larger rectangular solids,	$3(30 \times 30)$ sq. in. =	2700 sq. in.
For the smaller rectangular solids,	$3(30 \times 5)$ sq. in. =	450 sq. in.
For the small cube,	(5×5) sq. in. =	25 sq. in.
		3175 sq. in.

This number multiplied by the third dimension gives (5 × 3175) cu. in. = 15,875 cu. in.

Exercise 180. — Written.

Applications of Square and Cube Roots.

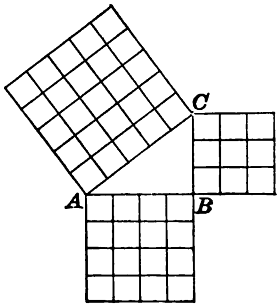
The length of a side of a square is found by extracting the square root of its area.

1. Find in yards the length of the side of a square field that contains 4 A. 36 sq. rd.

2. Find in rods the length of the side of a square piece of wood land that contains 4698 A. 9 sq. rd.

3. A rectangular field is 945 yd. long and 420 yd. wide. Find the side of a square field that has the same area.

4. A town lot in the shape of a square contains 13,225 sq. ft. Find its side.



A triangle that has a right angle is called a right triangle; the side opposite the right angle is called the hypotenuse; and the other two sides are called the base and perpendicular.

In a right triangle the square of the hypotenuse (AC) is equal to the sum of the squares of the other two sides. Therefore the hypotenuse is equal to the square root of the sum of the squares of the other two sides; and the base or the perpendicular is equal to the square root of the difference of the squares of the hypotenuse and the other side.

5. The base of a right triangle is 82 ft., and the perpendicular 35 ft. Find the hypotenuse.

6. The base of a right triangle is 52 ft., and the hypotenuse 65 ft. Find the perpendicular.

7. The perpendicular of a right triangle is 72 ft., and the hypotenuse 75 ft. Find the base.

8. Find the length of the longest straight line that can be drawn on the floor of a room 15 ft. square.

The area of any triangle is equal to the square root of half the sum of the sides multiplied in succession by the three remainders obtained by subtracting each side separately from the half sum of the sides.

9. Find the area of a triangle whose sides are respectively 5 in., 6 in., and 7 in.

The half sum of the sides is $\frac{5+6+7}{2}$, or 9.

$$9-5=4, \qquad 9-6=3, \qquad 9-7=2.$$

$$\sqrt{9 \times 4 \times 3 \times 2} = \sqrt{216} = 14.696+$$

14.696 sq. in. *Ans.*

10. Find the area in square rods of a triangular field whose sides are respectively 22 rd., 28 rd., and 34 rd.

11. Find the area of a triangle whose sides are respectively 3 in., 4 in., and 6 in.

12. What is the area of a triangular field whose sides are respectively 360 yd., 385 yd., and 315 yd.?

13. Find the area of a triangular field whose sides are respectively 450 ft., 500 ft., and 550 ft.

The edge of a cube is found by extracting the cube root of its volume.

14. Find the edge of a cube that contains 1728 cu. in.

15. Find the edge of a cube whose volume is equal to the volume of a rectangular solid $16' \times 2' \times 2'$.

16. Find the edge of a cubical cistern that will hold as many gallons as a rectangular cistern $12' \times 8' \times 5\frac{1}{3}'$.

17. Find the edge of a cubical vessel that will hold 1 ton of water.

403. Similar Figures are figures that have the same shape.

The areas of similar figures are to each other as the squares of their corresponding dimensions, and their volumes are to each other as the cubes of their corresponding dimensions.

The corresponding dimensions of similar figures are to each other as the square roots of their areas, or as the cube roots of their volumes.

Exercise 181. — Written.

1. The edge of a cube is 2 in. and the edge of another is 4 in. How many times the total surface of the smaller is the total surface of the larger? How many times the volume of the smaller is the volume of the larger?

2. The volume of a rectangular solid is 1728 cu. in., and the volume of a similar solid is 13,824 cu. in. How many times a dimension of the smaller solid is the corresponding dimension of the larger?

3. How many apples 1 inch in diameter are equivalent to an apple 2 inches in diameter, if the apples are of the same shape and kind?

4. The surfaces of two mounds of exactly the same shape are as 36 : 9. Find the ratio of their heights.

5. A rectangular lot of land has a front of 160 feet and is worth $1000. Find the value of a similar lot which has twice the front and twice the depth.

6. Find the diameter of an iron ball that will weigh 27 times as much as an iron ball 2 inches in diameter.

7. The weights of iron cylinders of the same shape are as 8 to 27. Find the ratio of their heights.

CHAPTER XV.

Metric Measures.

404. The **Metric System** is a system of weights and measures expressed in the *decimal scale.*

405. The **Standard Meter,** as defined by law, is the length of a bar of very hard metal carefully preserved at Paris, accurate copies of which are furnished to the governments of all civilized nations.

406. The principal units of the metric system are:

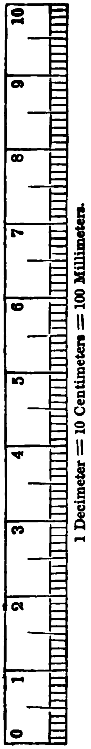
The **meter** (^{m}) for lengths;
The **square meter** (^{qm}) for surfaces;
The **cubic meter** (^{cbm}) for large volumes;
The **liter** (^{l}) (*lee'-ter*) for smaller volumes;
The **gram** (^{g}) for weights.

407. All these units are divided and multiplied decimally, and the size of the measures thus produced is shown by one of six prefixes; namely, *deka,* meaning 10; *hekto,* meaning 100; *kilo,* meaning 1000; and *deci,* meaning 0.1; *centi,* meaning 0.01; *milli,* meaning 0.001.

408. But, as in United States money, we seldom speak of anything else than dollars and cents, so in other measures it is only those printed in **black letter** in this chapter that are in common use.

Note. A meter is a trifle more than 39.37 inches, and all the units of the system are derived from the meter. All the compound names are accented on the first syllable; thus. *mil'limeter.* The teacher should be supplied with a meter stick, a liter, and a cubic centimeter.

Units of Length.



409. The principal unit of length is the **meter.**

TABLE.

10 **millimeters** (^{mm})	= 1 **centimeter** (^{cm}).
10 **centimeters**	= 1 decimeter (^{dm}).
10 decimeters	= 1 **meter** (^{m}).
10 **meters**	= 1 dekameter (^{dkm}).
10 dekameters	= 1 hektometer (^{hm}).
10 hektometers	= 1 **kilometer** (^{km}).

410. Any one of these measures may be expressed in terms of another measure by *simply moving the decimal point to the right or left.*

Thus, 17,856,342^{mm} may be written as **kilo**-meters by observing that **milli**-meters are changed to meters by moving the point **three** places to the left, and meters to **kilo**-meters by carrying it **three** places further, making, in all, **six** places.

Therefore, 17,856,342^{mm} = 17.856342^{km}.

Again, 4.876326^{km} may be written as **centi**-meters, by observing that **kilo**-meters are changed to meters by moving the point **three** places to the right, and meters to **centi**-meters by moving it **two** places further, making, in all, **five** places.

Therefore, 4.876326^{km} = 487,632.6^{cm}.

411. The rule, therefore, for this conversion, is:

First count the number of places needed to convert the given measure into terms of the principal unit; then the number needed to convert the principal into the required unit.

412. Before adding or subtracting, the quantities must be written in the same unit of measure.

Exercise 182. — Written.

1. Change 7245^{m} to kilometers; to centimeters.
2. Change $35,865^{mm}$ to meters; to decimeters.
3. Write 7.94^{m} as centimeters; as millimeters.
4. Change 12.56^{km} to meters; to dekameters.

Find in meters:

5. $0.354^{m} + 376^{cm} + 5375^{mm} + 0.123^{km}$.
6. $0.729^{km} - 3269^{dm}$.
7. $5.327^{cm} - 24.87^{mm}$.
8. 12×0.055^{km}.
9. 40.3×93.62^{cm}.
10. 5.4×670^{mm}.
11. $760^{mm} \div 0.02$.
12. $0.45^{km} \div 25.625$.
13. $388,000^{mm} \div 0.097$.

14. At $1.75 a meter, find the cost of 6.4^{m} of cloth.

15. At $5.20 a meter, find the cost of 80^{cm} of cloth.

16. The lengths of two roads are 244.08^{m} and 2.45^{km}. How many meters is one road longer than the other?

17. If sound goes 340.5^{m} a second, how far is a cannon from a man who hears the report 9 seconds after seeing the flash?

18. If 15^{m} of velvet cost $150, find the cost of 80^{cm}.

19. If a man walks 6^{km} an hour, how many minutes will it take him to walk 500 meters?

20. A meter is 39.37 inches. If the mercury in a French barometer stands 760^{mm} high, how many inches high will it be in an English barometer?

21. A railway train leaves Paris at 8.40 in the evening and arrives at Geneva, a distance of 626 kilometers, at 10.40 in the morning. What is its average rate per hour in kilometers?

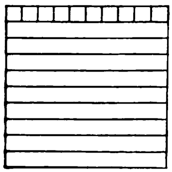
22. One kilometer is 0.6214 of a mile. Reduce 472 miles to kilometers.

23. Thirty-two meters are equal to 35 yards. Find the number of meters in a mile.

Units of Surface.

413. The principal unit of surface is a **square meter** ($^{\text{qm}}$).

414. In square measure the multiplication and division of units is by hundreds and hundredths, instead of by tens and tenths. Suppose the square in the margin to represent a **square meter.** It is divided into ten equal horizontal bands, and each band is one tenth of the square meter. Each band can be divided, as the upper one is, into ten little squares measuring one tenth of a meter on a side. Each of these squares will be 0.1 of the band, or 0.01 of the whole square. The **square meter,** therefore, contains 10×10 or 100 **square decimeters.**



If the square meter were divided into 100 equal horizontal bands, each band would be 0.01 of the square; and if each of the 100 bands were divided into 100 squares, that is, into 100 square centimeters, the whole square would contain 100×100 or 10,000 square centimeters. **A square meter,** therefore, contains 10,000 **square centimeters.**

In like manner, a **square meter** contains 1,000,000 **square millimeters.**

TABLE.

100 square millimeters ($^{\text{qmm}}$)	= 1 **square centimeter** ($^{\text{qcm}}$).
100 **square centimeters**	= 1 square decimeter ($^{\text{qdm}}$).
100 square decimeters	= 1 **square meter** ($^{\text{qm}}$).
100 **square meters**	= 1 square dekameter ($^{\text{qdkm}}$).
100 square dekameters	= 1 **square hektometer** ($^{\text{qhm}}$).
100 **square hektometers**	= 1 **square kilometer** ($^{\text{qkm}}$).

415. It will be observed that in **linear measure** each unit is 10 times as large as the next smaller unit, but in **surface measure** each unit is 100 times as large as the next smaller unit.

416. In the measurement of land, the **square dekameter** is called an **ar** (^{a}), the **square hektometer** is called a **hektar** (^{ha}), the **square meter** is called a **centar** (^{ca}).

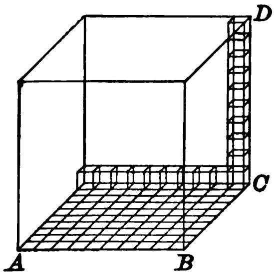
EXERCISE 183. — WRITTEN.

1. Change 2.6^{qkm} to hektars; to ars; to centars.
2. Change 3^{qhm} 2^{qdkm} 7^{qm} to square meters.
3. Change $250{,}000^{qm}$ to hektars.
4. Change $240{,}000^{ca}$ to hektars.
5. If a field contains 5000^{ca}, how many ars does it contain? What part of a hektar?
6. Find the cost of 1.9^{ha} of land at $1 an ar.
7. How many ars in 147^{a} and 3561^{ca}?
8. How many square meters must be added to $13{,}453^{qm}$ to obtain 2^{ha} 19^{a}?
9. How many hektars are there in a rectangular field 200 meters long and 180 meters wide?
10. At 20 cents a square meter, what will be the cost of painting a rectangular surface 24.6^{m} long and 10.5^{m} wide?
11. How many bricks 20^{cm} long and 10^{cm} wide will it take to pave a sidewalk 3^{m} wide and 2^{km} long?
12. How many meters of carpet 75^{cm} wide are required to make 54^{qm}?
13. How many meters of carpet 70^{cm} wide will be required for a room 7^{m} long and 6^{m} wide, if the strips run across the room and 2^{m} are allowed for waste in matching the pattern?
14. Find the area of a triangular field whose base is 950 meters and altitude 360 meters.
15. A lot of land containing 62.5 ars is sold for 25 cents a square meter. What does the whole lot bring?
16. If 34 ars of land are sold for $1020, what is the price per square meter?

Units of Volume.

417. The principal unit of volume is a **cubic meter** (^{cbm}).

418. The cubic meter can be divided into 10 layers, each a meter square and a decimeter thick. Each layer will, therefore, be 0.1 of a cubic meter.



Again, each layer can be divided into 10 equal parts. Each part will, therefore, be 0.1 of the layer, or 0.01 of the meter, and will be a decimeter square and a meter long.

Also, each one of these parts can be divided into 10 equal parts, each of which will be a cubic decimeter, and will be 0.1 of 0.01, that is, 0.001 of the cubic meter.

The **cubic meter,** therefore, contains 1000 **cubic decimeters.**

The **cubic decimeter** contains 1000 **cubic centimeters.**

The **cubic centimeter** contains 1000 **cubic millimeters.**

Each unit in **cubic measure,** therefore, is **1000** times as large as the next smaller unit.

TABLE.

1000 cubic millimeters (^{cmm})	= 1 cubic centimeter (^{ccm}).
1000 cubic centimeters	= 1 cubic decimeter (^{cdm}).
1000 cubic decimeters	= 1 cubic meter (^{cbm}).

419. In measuring wood, the cubic meter is called a **ster** (st).

TABLE.

10 decisters (^{dst})	= 1 ster (st).
10 sters	= 1 dekaster (^{dkst}).

EXERCISE 184. — WRITTEN.

1. Change 2^{cbm} to cubic decimeters.

2. Change $64{,}000^{cdm}$ to cubic meters.

3. Change $46{,}000^{ccm}$ to cubic decimeters.

4. How many cubic centimeters in a block $10^{cm} \times 9^{cm} \times 8^{cm}$?

5. How many cubic decimeters will a bin hold that is 11^{m} long, 5.37^{m} wide, and 3.02^{m} deep?

6. A bin 10^{m} long, 5^{m} wide, and 2.7^{m} deep, is full of wheat worth $0.02 a cubic decimeter. What is the whole worth?

7. How many cubic decimeters of air in a room 7.6^{m} long, 6.25^{m} wide, and 3.2^{m} high?

8. How many sters of wood in a pile 8^{m} long, 1.5^{m} high, and 1^{m} wide?

9. How many cubic meters in a wall 30^{m} long, 6.8^{m} high, and 65^{cm} thick?

10. How many cubic meters of earth must be removed to dig a ditch 70^{m} long, 85^{cm} wide, and 855^{mm} deep?

11. A pile of wood 5.5^{m} long, 3^{m} wide, and 2^{m} high, supplies 3 fires. How long will it last if each fire consumes 0.25 ster a day?

12. If each gas burner consumes 120 cubic decimeters of gas an hour, find the cost of lighting a room with 4 of these burners for 20 evenings, 4 hours an evening, at $0.045 per cubic meter of gas.

13. A pile of wood 4^{m} long and 2^{m} wide contains 14 sters. How high is the pile?

14. Find the cost of an oak beam 4 meters long and 0.5 of a meter square at the end, at $25 a cubic meter.

15. A cistern is 4.4^{m} long and 2.3^{m} wide. After a rain it was found that the water in the cistern had risen 52 centimeters. How many cubic meters of water had run into the cistern?

Units of Capacity.

420. In measuring liquids, grain, etc., the principal unit is the **liter.**

421. The **Liter** is a **cubic decimeter;** that is, a cube whose edge is a decimeter long.

422. In **Measures of Capacity,** each unit is **10** times as large as the next smaller unit.

TABLE.

10 milliliters (^{ml})	=	1 centiliter (^{cl}).
10 centiliters	=	1 deciliter (^{dl}).
10 deciliters	=	1 liter (^{l}).
10 liters	=	1 dekaliter (^{dkl}).
10 dekaliters	=	1 hektoliter (^{hl}).
10 hektoliters	=	1 kiloliter (^{kl}).

EXERCISE 185. — WRITTEN.

1. How many hektoliters of air in a room 6^m long, 5^m wide, and 3^m high?

2. At 5 cents a liter, what will a hektoliter of vinegar cost?

3. If a bin is 10^m long, 4^m wide, and $1\frac{1}{2}^m$ high, how many hektoliters of corn will it hold? How many liters?

4. What must be the depth of a bin 10^m long and 6^m wide to hold 1800^{hl} of grain?

5. If the value of a hektoliter of wheat is $1.70, what is the value of a dekaliter?

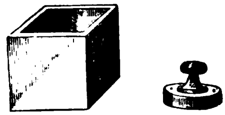
6. How many hektoliters of wheat are contained in 79 sacks, if each sack contains 1 hektoliter and 1 dekaliter?

7. The mean depth of a pond is 1.5^m, and its area is 3 hektars. How many kiloliters of water does it contain?

8. The height of a cylindrical vessel is 53 centimeters, and the area of its base is 155 square centimeters. How many liters will it hold?

Units of Weight.

423. The units of weight are the weights of units of pure water taken at its greatest density, that is, a little before freezing.



Cubic Centimeter. Gram Weight.

The principal unit is the **gram,** which is the weight of a **cubic centimeter** of water.

In **weight,** each unit is 10 times as large as the next smaller unit.

TABLE.

10 milligrams (^{mg})	= 1 centigram (^{cg}).
10 centigrams	= 1 decigram (^{dg}).
10 decigrams	= 1 gram (^{g}).
10 grams	= 1 dekagram (^{dkg}).
10 dekagrams	= 1 hektogram (^{hg}).
10 hektograms	= 1 kilogram (^{kg}).
1000 kilograms	= 1 metric ton (^{t}).

424. A **cubic centimeter** of water weighs a **gram.**
A **liter** of water weighs a **kilogram.**
A **cubic meter** of water weighs a **ton** (1000^{kg}).

EXERCISE 186. — WRITTEN.

1. A kilogram of sea water contains 50^{g} of salt. How much salt in 5600 grams of sea water?

2. If a kilogram of butter is worth 55 cents, find the cost of $10^{kg}\ 5^{hg}\ 3^{dkg}$.

3. Find the cost of 3.75^{kg} of oil at 12 cents a hektogram.

4. From a barrel of sugar containing 100^{kg} of sugar, there were sold 3 parcels of 3.75^{kg} each, and 5 parcels of 4.25^{kg} each. How much was left in the barrel?

5. At \$18 a metric ton, what will 300^{kg} of hay cost?

Specific Gravity.

425. The **specific gravity** of a substance is the *number* found by dividing the weight of the substance by the weight of an equal bulk of water.

426. The specific gravity of a substance therefore is the *number* that expresses the weight of a **cubic centimeter** of it in **grams,** of a **liter** in **kilograms,** of a **cubic meter** in **tons.**

427. The **volume** of a body is found by dividing *its weight by its specific gravity.*

Exercise 187. — Written.

1. A bar of iron 75^{cm} long, 4^{cm} wide, 1^{cm} thick has a specific gravity of 7.8. Find its weight in kilograms.

2. A piece of iron weighing 120^{kg} is made into a bar 6^{cm} wide and 2^{cm} thick. Find its length, if the specific gravity of the iron is 7.8.

3. If a body weighs 5.75^{kg} in air and 4.25^{kg} in water, what is its specific gravity ?

Note. A body immersed in a liquid weighs less than in air by the weight of the liquid that it displaces.

4. What is the weight of a load of 200 paving stones, 12^{cm} long, 8^{cm} wide, and 15^{cm} deep, if the specific gravity of the stone is 2.72 ?

5. A vessel full of oil, specific gravity 0.905, weighs 6.5^{kg}. The vessel when empty weighs 450^{g}. How many liters will the vessel hold ?

6. A piece of zinc, specific gravity 7.19, weighs 350^{g}. Find its volume in cubic centimeters.

Exercise 188. — Written.

Review Problems.

1. How many kilograms of water will a cistern hold that is 180^{cm} long, 120^{cm} wide, and 75^{cm} deep?

2. If a metric ton of raw sugar costs $72, what is the price of a kilogram?

3. If a barrel full of water weighs 123 kilograms, and empty weighs 10 kilograms, how many liters will it hold?

4. How many sacks each holding 1.2^{hl} can be filled from 2880^{hl} of wheat?

5. How many hektars are there in a piece of land that is 150^{m} square?

6. At $5 a metric ton for coal, what will it cost to keep a fire through January if the average consumption is 40 kilograms a day?

7. If 4957 cakes of maple sugar weigh 7782 kilograms 49 dekagrams, find the average weight of a cake.

8. If 15 kilograms of copper, worth 25 cents a kilogram, are melted with 3 kilograms of tin, worth 40 cents a kilogram, what is the value of 1 kilogram of the composition?

9. The space for children in a schoolroom is 9^{m} long and 7^{m} wide. How many children can occupy it, if 150^{qdm} are allowed each child?

10. A keg of olive oil weighs 40.5^{kg}. The keg empty weighs 3.9^{kg}, and holds just 40 liters. What is the weight of a liter of the oil?

11. A liter of air weighs 1.292^{g}. Find the weight of air in a room 6.8^{m} long, 5.2^{m} wide, and 3^{m} high.

12. Find the weight of water required to fill a tank 2.3^{m} long, 98^{cm} wide, and 56^{cm} deep.

13. If 1^{ccm} of oil weighs 925^{mg}, what is the value of 50 liters of this oil at $1 a kilogram?

428. Table of Equivalents.

Length.

Meter	$= \begin{cases} 39.37043 \text{ in.} \\ 1.09362 \text{ yd.} \end{cases}$	Inch	$= 2.53998^{\text{cm}}$.
		Yard	$= 0.91439^{\text{m}}$.
Kilometer	= 0.62138 mi.	Mile	$= 1.60933^{\text{km}}$.

Surface.

Sq. meter	$= \begin{cases} 1550.031 \text{ sq. in.} \\ 1.19601 \text{ sq. yd.} \end{cases}$	Sq. inch	$= 6.45148^{\text{qcm}}$.
		Sq. yard	$= 0.83611^{\text{qm}}$.
Hektar	= 2.47110 A.	Acre	$= 0.40468^{\text{ha}}$.

Volume.

Cu. centimeter	= 0.06103 cu. in.	Cu. inch	$= 16.38662^{\text{ccm}}$,
Cu. meter	= 1.30799 cu. yd.	Cu. yard	$= 0.76453^{\text{cbm}}$.
Ster	= 0.27590 cord.	Cord	$= 3.62446^{\text{st}}$.

Capacity.

Liter	$= \begin{cases} 1.05671 \text{ liquid qt.} \\ 0.90810 \text{ dry qt.} \end{cases}$	Liquid quart	$= 0.94633^{\text{l}}$.
		Dry quart	$= 1.10119^{\text{l}}$.

Weight.

Milligram	= 0.015432 gr.	Grain	$= 0.06480^{\text{g}}$.
Gram	= 15.43235 gr.	Ounce av.	$= 28.34954^{\text{g}}$.
Kilogram	= 2.20462 lb. av.	Ounce troy	$= 31.10350^{\text{g}}$.
Metric ton	= 2204.62 lb. av.	Pound av.	$= 0.45359^{\text{kg}}$.

Approximate Equivalents.

Meter	= 1.1 yd.	Yard	$= 0.9^{\text{m}}$.
Kilometer	$= \frac{5}{8}$ mi.	Mile	$= 1.6^{\text{km}}$.
Sq. meter	$= 1\frac{1}{5}$ sq. yd.	Sq. yard	$= \frac{5}{6}^{\text{qm}}$.
Hektar	$= 2\frac{1}{2}$ A.	Acre	$= \frac{2}{5}^{\text{ha}}$.
Cu. centimeter	$= \frac{1}{16}$ cu. in.	Cu. inch	$= 16^{\text{ccm}}$.
Cu. meter	= 1.3 cu. yd.	Cu. yard	$= \frac{10}{13}^{\text{cbm}}$.
Ster	$= \frac{3}{11}$ cord.	Cord	$= 3\frac{2}{3}$ sters.
Liter	$= \begin{cases} 1\frac{1}{16} \text{ liq. qt.} \\ \frac{9}{10} \text{ dry qt.} \end{cases}$	Liquid quart	$= \frac{16}{17}$ liter.
		Dry quart	$= 1\frac{1}{9}$ liters.
Hektoliter	$= 2\frac{5}{6}$ bu.	Bushel	$= \frac{6}{17}^{\text{hl}}$.
Gram	$= 15\frac{1}{2}$ gr.	Pound av.	$= \frac{5}{11}^{\text{kg}}$.
Kilogram	$= 2\frac{1}{5}$ lb. av.	Pound troy	$= \frac{5}{13}^{\text{kg}}$.

EXERCISE 189. — WRITTEN.

(In the following problems use the Approximate Equivalents.)

1. Find in acres the area of a rectangular field whose length is 100^{m} and breadth 50^{m}.

2. Find the number of cubic meters in a box 5 ft. long, 4 ft. wide, 3 ft. deep.

3. Find the number of cubic yards in a box 3^{m} long, 1.25^{m} wide, and 50^{cm} deep.

4. If a man walks 100^{m} a minute, what is his rate in miles per hour?

5. How many steps 2 ft. 8 in. long will a man take in walking a kilometer?

6. A pile of wood is 1.2^{m} wide, 9^{m} long, and 2^{m} high. How much is it worth at $4.50 a cord?

7. If a train goes at the rate of 50^{km} an hour, how many miles will it go in 1 hr. 30 min.?

8. At 18 cents a liter, what will be the price of 100 qt. of olive oil?

9. If the specific gravity of sea water is 1.026, and that of the best alcohol is 0.792, find the weight of a hektoliter of each in pounds and in kilograms.

10. A man bought 300 bu. of wheat at 60 cents a bushel and sold it at $2.10 a hektoliter. How much did he gain?

11. If the diameter of a ball is 52^{cm}, find the surface and volume of the ball in inches.

12. A dry quart contains 67.2 cu. in. A liter contains 61.026 cu. in. If a vessel holds 100 dry quarts, how many liters does it hold?

CHAPTER XVI.

Miscellaneous Examples. (By Topics.)

Integral Numbers.

Exercise 190. — Oral.

1. Add 4, 2, 7, 3, 8, 2, 1, 7, 5, 1, 6, 6.
2. Add 5, 3, 1, 7, 3, 6, 3, 7, 9, 3, 7, 7.
3. Add 4, 7, 2, 5, 5, 2, 9, 6, 2, 8, 7, 5.
4. Add 2, 8, 8, 4, 2, 9, 3, 8, 6, 9, 6, 4.
5. Add 2, 6, 8, 7, 1, 2, 8, 5, 3, 2, 8, 9.
6. Subtract 16 from 66; 33 from 42.
7. Subtract 54 from 82; 58 from 100.
8. Subtract 28 from 54; 87 from 124.
9. Multiply 124 by 25; 16 by 125.
10. Multiply 62 by 50; 84 by 250.
11. Multiply 412 by 8; 64 by 75.
12. Multiply 109 by 9; 77 by 99.
13. Multiply 116 by 25; 88 by 375.
14. Multiply 56 by 625; 48 by 375.
15. Multiply 128 by 12; 324 by 11.
16. Multiply 316 by 40; 428 by 60.
17. Find the quotient of $19 \div 4$; of $40 \div 19$.
18. Find the quotient of $375 \div 75$; of $375 \div 125$.
19. Find the quotient of $36 \div 13$; of $115 \div 12$.
20. Find the quotient of $362 \div 25$; of $40 \div 11$.
21. Find the quotient of $71 \div 15$; of $132 \div 12$.

22. A checker board has 8 rows of squares and 8 squares in a row. How many squares has it in all?

23. What will 11 qt. of berries cost at 12 cents a quart?

24. If 6 men can do a piece of work in 9 days, in how many days can one man do it?

25. If 7 men can do a piece of work in 8 days, in how many days can one man do it?

26. If 6 men earn $72 in 6 days, how much does each man earn a day?

27. If 3 oranges are worth 7 apples, how many apples are 12 oranges worth?

28. If 6 oranges are sold for 18 cents, and 6 cents are gained, what is the cost of each orange?

29. A man earns $8 while a boy earns $5. How many dollars have both together earned when the man has earned $32?

30. A man earns $9 while a boy earns $5. How many dollars has the boy earned when the man has earned $36?

31. How many men in 4 days can do a piece of work that requires 3 men 12 days?

32. How many men in 3 days can do a piece of work that requires 9 men 7 days?

33. The art of printing was discovered about the year 1450. How many years from that date to the year 1900?

34. A swallow destroys 500 insects daily. How many insects does a swallow destroy in a week?

35. A dozen dozen is called a gross. How many steel pens are there in a gross?

36. A salt dealer puts up 1500 lb. of salt in boxes of 20 lb. each. How many boxes has he?

37. How many 6-lb. packages of buckwheat flour can be made from 1200 lb. of flour?

38. A fruit dealer sold 240 oranges at a profit of 5 cents on each dozen. What was his entire profit?

EXERCISE 191. — WRITTEN.

1. Add 5 thousand 3 hundred 79, 3 thousand 4, 7 thousand 7, 7 hundred 70, 54 hundred 11.

2. Add six billion sixty thousand six, seven million nine hundred ninety-nine thousand nine, forty-two million eighty-nine thousand eight hundred eight.

3. The distance from Boston to Springfield is 98 miles; from Springfield to New Haven, 62 miles; from New Haven to New York, 76 miles. How many miles is it from Boston to New York by this route?

4. New Hampshire furnished 12,497 soldiers for the Revolution; Massachusetts, 67,907; Rhode Island, 5908; Connecticut, 31,939. How many soldiers did these four states furnish?

5. What number must be added to 4872 to make 8021?

6. What number must be taken from 5301 to leave 4255?

7. What number increased by 63,915 makes a million?

8. If 169 tons of steel rails are required for a mile of railroad, how many tons are required for 449 miles?

9. If sound travels 1120 ft. a second, how far distant is a cloud when the thunderclap follows a flash of lightning in 11 seconds?

10. If 12 men can do a piece of work in 10 hours, how many hours will it take 8 men to do the same work?

11. If 9 tons of hay cost $135, what is the cost of 87 tons?

12. In how many hours will a cistern holding 1480 gallons be filled by a pipe that discharges into it 185 gallons an hour?

13. What is the nearest number to 7083 that will contain 372 without a remainder?

14. What number subtracted 88 times from 8739 will leave 27 for a remainder?

15. How many times can 29 be subtracted from 493?

16. A certain number exceeds 12 dozen and 9 by 105; another number falls short of 198 by 9 dozen and 4. Find the product of these two numbers.

17. If 17 oxen cost $1190, and an ox is worth 14 sheep, what is the value of a sheep?

18. Shem was 98 years old at the time of the flood, 2348 B.C., and 600 years old when he died. Abraham was born in 1996 B.C. How old was Abraham when Shem died?

19. Five firkins of butter contain respectively 42 pounds, 46 pounds, 50 pounds, 43 pounds, and 49 pounds. What is the average number of pounds to a firkin?

20. A publisher sold 2000 copies of a book for $1500, and thereby made a profit of $300. Find the cost and the selling price of each copy, and the profit on a hundred copies.

21. The dividend is 3860; the quotient, 142; and the remainder, 26. Find the divisor.

22. Multiply three million three by one hundred thousand one.

23. A man had 75 yd. of linen, worth 44 cents a yard, made into shirts. It takes 3 yd. for a shirt, and the cost of making is 50 cents a shirt. Find the entire cost.

24. A man paid $29 for a suit of clothes, $17 for an overcoat, $6 for a hat, $7 for a pair of boots, $12 for underclothes, and $19 for other articles. How much did he pay for the whole?

25. What sum taken 19 times will amount to $11,419?

26. If $38,057 is divided into 19 equal parts, what is the sum in each part?

27. If $130,634 is divided into 98 equal parts, what is the value of 79 of them?

28. The length of the Rhone is about 800,000 meters. How many hours does it take the water to flow from the source to the mouth at the rate of 3600 meters per hour?

Decimal Fractions.

Exercise 192. — Oral.

1. A man pays $1.38 for 3 gallons of molasses. What is the price per gallon?

2. A man buys 100 lb. of granulated sugar for $5.25, and pays for it in maple sugar at 10 cents a pound. How many pounds of maple sugar does it take?

3. A man buys a cask of vinegar for $8.40, paying 20 cents a gallon. How many gallons does the cask hold?

4. A man has $37 and spends $5.32. How much money has he left?

5. To enclose a certain lot, 225 yards of fence are needed. What will be the cost of fencing at $0.50 a yard?

6. If 0.7 of a ton of coal is worth $6.30, what will be the cost of 10.5 tons?

7. If 12 penknives cost $9, and are sold at $0.90 each, what is the whole gain?

8. If 12 yd. of velvet cost $150, what will 5 yd. cost at the same rate?

9. What will be the cost of 3.5 cd. of white oak wood at the rate of $7 per cord?

10. A man bought goods to the amount of $6.56. He gave the salesman a ten-dollar bill. How much money should he receive?

11. If 0.25 of a farm costs $500, what will 0.7 of it cost?

12. For the roof of a building 7000 tiles are used. What is the cost at $9 per M?

13. A contractor estimates that 1500 thousand bricks will be required for a certain building. What is the cost at $7.50 per M?

14. What must be paid for 4000 paving-stones at $9.50 a hundred?

15. A lumber dealer paid $4.50 a thousand for cedar shingles, and sold them for $5.25. What did he gain on 40 thousand?

16. An errand boy receives $2.75 a week. In how many weeks will he earn $11?

17. How many cords of pine wood at $3.50 a cord must be given for 14 yards of broadcloth at $2.25 a yard?

18. Find the cost of 20,000 laths at $0.35 a hundred.

19. A man earns $12 a week, and spends on the average $9.50. In how many weeks will he save $100?

20. If there are 11 rainy days in a month, and the amount of rain is 4.51 inches, what is the daily average of rainfall for the rainy days?

21. On a certain day the thermometer was at the highest 51.4°, and at the lowest 29.8°. What was the range of temperature?

22. Find the weight in tons of organic matter in 70 tons of superphosphate, if 0.16 of its weight is organic matter.

23. When potatoes are worth $0.40 a bushel, and corn $0.60 a bushel, how many bushels of potatoes must be given in exchange for 50 bushels of corn?

24. If the salt water which is obtained from the bottom of a salt mine contains 0.08 of its weight of pure rock salt, what weight of salt water is it necessary to evaporate to obtain 100 lb. of rock salt?

25. How many quarts of berries at $0.12 a quart will it take to pay for 4 yards of cloth at $0.165 a yard?

26. A fruit dealer bought 200 apples at the rate of four for a cent, and 200 at five for a cent. He sold them all at the rate of five for three cents. What did he gain?

27. If the dividend is $4.50 a share, how many shares of stock must a man own to receive a dividend of $900?

28. If a bankrupt owes $3000 and has assets of $750, how much on a dollar will his creditors receive?

Exercise 193. — Written.

Find the value of:

1. $5.412 + 2.7 + 8.4936 + 0.8247$.
2. $1.01 + 20.2 + 15 + 1.548 + 3.0004$.
3. $52.7 + 240 + 34.003 + 12.1557 + 3.0087$.
4. $8.052 - 3.687$.
5. $5.77 - 3.583$.
6. $6.92 - 3.537$.
7. $3.9009 - 0.099$.
8. $291.3 - 41.71294$.
9. $0.517 - 0.4157$.
10. 4.41×6.9.
11. 2.307×3.02.
12. 70.56×0.341.
13. 6.402×0.208.
14. 423.667×8.704.
15. 6.21×7.04.
16. $140.616 \div 3.906$.
17. $0.00768 \div 4.8$.
18. $11.65606 \div 0.127$.
19. $21.04 \div 4000$.
20. $78.125 \div 500$.
21. $361.8 \div 720$.

22. A gallon contains 231 cubic inches. How many gallons are there in one cubic foot?

23. How many cubic feet are there in a bushel, 2150.42 cu. in.?

24. If 881.68 bushels of corn are put into bags holding 2.14 bushels each, how many bags are there?

25. At $18.75 a ton, how many tons of hay can be bought for $356.25?

26. Of 100 lb. of beans, sugar and gum form 61.1 lb., other vegetable matter 31.55 lb., and moisture 5 lb. The remainder is mineral matter. How many pounds of mineral matter are there in 100 lb. of beans?

27. In a certain month, the maximum height of the barometer was 30.353 in., and the minimum height was 29.378 in. Find the difference.

28. A cubic inch of pure water weighs 252.458 grains. Find the weight in grains of a cylindrical inch which is 0.7854 of a cubic inch.

29. A stick 5.6 ft. long standing upright casts a shadow 10 ft. long. What is the height of a column which at the same time casts a shadow 65 ft. long?

30. If 3.5 bushels of wheat furnish 140 lb. of flour, and 140 lb. of flour make 182.2 lb. of bread, how much flour and how much bread will 100 lb. of wheat make?

31. 11.75 tons of coal cost $82.25. What is the cost of 21.4 tons?

32. A grain merchant spent $271.35 for equal quantities of rye at $0.60 and wheat at $0.75 a bushel. How many bushels of each did he buy?

33. John James had $1000 deposited in a bank. He drew checks for the following amounts: $150, $130.42, $30.37, $37.75, $60.43. What sum remained to his credit?

34. If 720 hoops can be made from a cord of hoop poles, what is the value of a cord, reckoning the hoops at $12 a thousand and the cost of manufacture at $4.25?

35. An agent charged $5.85 for collecting a bill of $260. What was his charge per dollar?

36. Bronze consists of 1 part of tin to 4.25 parts of copper. What weight of copper must be added to 330.7 lb. of tin to make bronze?

37. Gun metal is composed of 1 part of tin to 5.5 parts of copper. What weight of tin must be added to 841.5 lb. of copper to make gun metal?

38. How many pounds of copper will there be in 232.375 lb. of gun metal, composed of 1 lb. tin to 5.5 lb. copper?

39. How many bottles each holding 0.8 of a quart will 8.5 barrels of vinegar fill, reckoning 31.5 gallons to the barrel, and what decimal of a bottle will there be over?

40. If 0.35 of a share in a mining company is worth $31.15, what is the value of 15 shares?

41. A man sold a horse for $94.50, which was 0.05 more than he paid for it. What did he pay for it?

Common Fractions.

EXERCISE 194. — ORAL.

1. Four times $4\frac{3}{4}$ is how many times five?

2. Seven times $6\frac{2}{7}$ is how many times four?

3. Five times $5\frac{2}{5}$ is how many times nine?

4. Seven times $8\frac{1}{7}$ is how many times nineteen?

5. Eight times $8\frac{7}{8}$ is how many times seven?

6. Nine times $6\frac{2}{3}$ is how many times ten?

7. Seven times $7\frac{5}{7}$ is how many times nine?

8. Bought $4\frac{4}{5}$ lb. of sugar at 5 cents a pound, and paid for it with eggs at 18 cents a dozen. How many dozen did it take?

9. Bought $7\frac{7}{8}$ lb. of rice at 8 cents a pound, and paid for it with berries at 9 cents a quart. How many quarts of berries did it take?

10. How many barrels of flour at $5.50 a barrel must be given for $4\frac{5}{7}$ long tons of coal at $7 a ton?

11. A farmer planted $2\frac{1}{2}$ A. of ground to potatoes, and $8\frac{5}{8}$ A. to corn. How many acres did he plant?

12. One half of 12 is three fourths of what number?

13. One fifth of 20 is four fifths of what number?

14. One half of 144 is eight ninths of what number?

15. One third of 168 is seven eighths of what number?

16. $\frac{5}{9}$ of 72 is four elevenths of what number?

17. $\frac{3}{8}$ of 16 is what part of $\frac{5}{9}$ of 45?

18. $\frac{2}{3}$ of 18 is what part of $\frac{3}{11}$ of 77?

19. $\frac{6}{7}$ of 28 is what part of $\frac{2}{3}$ of 105?

20. $\frac{3}{5}$ of 10 is what part of $\frac{4}{7}$ of 21?

21. How many ninths in $6\frac{2}{9}$? $8\frac{7}{9}$? $5\frac{5}{9}$?

22. How many sevenths in $3\frac{3}{7}$? $5\frac{4}{7}$? $8\frac{5}{7}$?

23. How many eighths in $3\frac{7}{8}$? $4\frac{5}{8}$? $6\frac{3}{8}$?

How many whole units and what fraction of a unit in:

24. $\frac{19}{4}$? $\frac{27}{4}$? $\frac{31}{4}$? $\frac{13}{4}$? $\frac{15}{4}$? $\frac{22}{4}$? $\frac{44}{4}$? $\frac{49}{4}$?

25. $\frac{7}{5}$? $\frac{17}{5}$? $\frac{29}{5}$? $\frac{38}{5}$? $\frac{35}{5}$? $\frac{44}{5}$? $\frac{19}{5}$? $\frac{16}{5}$?

26. $\frac{12}{6}$? $\frac{15}{6}$? $\frac{29}{6}$? $\frac{54}{6}$? $\frac{56}{6}$? $\frac{63}{6}$? $\frac{37}{6}$? $\frac{42}{6}$?

27. $\frac{35}{7}$? $\frac{45}{7}$? $\frac{55}{7}$? $\frac{65}{7}$? $\frac{56}{7}$? $\frac{72}{7}$? $\frac{49}{7}$? $\frac{29}{7}$?

28. $\frac{22}{8}$? $\frac{39}{8}$? $\frac{43}{8}$? $\frac{54}{8}$? $\frac{57}{8}$? $\frac{69}{8}$? $\frac{76}{8}$? $\frac{35}{8}$?

29. $\frac{54}{9}$? $\frac{62}{9}$? $\frac{35}{9}$? $\frac{65}{9}$? $\frac{74}{9}$? $\frac{86}{9}$? $\frac{91}{9}$? $\frac{103}{9}$?

Find the value of:

30. $\frac{2}{3}$ of $\frac{3}{5}$. $\frac{3}{4}$ of $\frac{8}{9}$. $\frac{2}{3}$ of $\frac{7}{8}$. $\frac{3}{4}$ of $\frac{3}{7}$.

31. $\frac{3}{5}$ of $\frac{5}{8}$. $\frac{2}{5}$ of $\frac{5}{7}$. $\frac{4}{7}$ of $\frac{6}{8}$. $\frac{5}{9}$ of $\frac{7}{10}$.

32. $\frac{5}{7}$ of $\frac{8}{9}$. $\frac{3}{5}$ of $\frac{5}{11}$. $\frac{3}{8}$ of $\frac{2}{3}$. $\frac{5}{7}$ of 16.

33. $16 \times \frac{5}{6}$. $14 \times \frac{2}{7}$. $15 \times \frac{7}{8}$. $12 \times \frac{4}{3}$.

34. $\frac{3}{8}$ of 16. $14 \times \frac{11}{7}$. $24 \times \frac{5}{6}$. $\frac{6}{5}$ of 45.

35. $\frac{1}{6} \div \frac{1}{5}$. $\frac{1}{5} \div \frac{1}{4}$. $\frac{1}{4} \div \frac{1}{3}$. $\frac{1}{3} \div \frac{1}{2}$.

36. $\frac{4}{5} \div \frac{3}{4}$. $\frac{4}{5} \div \frac{4}{3}$. $\frac{4}{5} \div \frac{6}{5}$. $\frac{3}{4} \div \frac{2}{3}$.

37. $\frac{3}{4} \div \frac{3}{8}$. $\frac{3}{2} \div \frac{2}{3}$. $\frac{11}{3} \div \frac{5}{6}$. $2\frac{4}{5} \div 7$.

38. $8\frac{3}{4} \div 6$. $6\frac{6}{7} \div 12$. $8\frac{4}{7} \div 20$. $9 \div 1\frac{2}{7}$.

39. $1\frac{2}{7} \div 9$. $12 \div 3\frac{3}{4}$. $3\frac{3}{4} \div 12$. $22 \div 4\frac{8}{9}$.

40. $\frac{1}{2} + \frac{3}{4}$. $\frac{3}{4} + \frac{3}{5}$. $\frac{2}{3} + \frac{4}{5}$. $\frac{5}{6} + \frac{3}{5}$.

41. $\frac{7}{9} + \frac{7}{8}$. $\frac{6}{7} + \frac{7}{9}$. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{6}$. $1\frac{1}{3} + 2\frac{3}{4}$.

42. $3\frac{5}{6} + 4\frac{2}{3}$. $4\frac{1}{7} + 3\frac{1}{3}$. $5\frac{1}{8} + 4\frac{1}{9}$. $2\frac{1}{3} + 4\frac{5}{8}$.

43. $\frac{4}{5} - \frac{1}{3}$. $\frac{3}{7} - \frac{1}{4}$. $\frac{7}{9} - \frac{1}{2}$. $\frac{1}{2} - \frac{2}{9}$.

44. $\frac{5}{7} - \frac{3}{8}$. $\frac{6}{7} - \frac{3}{4}$. $\frac{5}{6} - \frac{4}{5}$. $1 - \frac{5}{6}$.

45. $7\frac{3}{5} - 2\frac{1}{3}$. $7\frac{2}{3} - 5\frac{1}{4}$. $8\frac{1}{4} - 4\frac{5}{6}$. $8\frac{2}{3} - 4\frac{4}{5}$.

46. A man sold a wagon for $45, which was $\frac{5}{8}$ of its cost. What was its cost?

47. A man bought a wagon. He paid down $20, and gave his note for the balance, which was $\frac{4}{5}$ of the price. What was the price of the wagon?

48. A boy sold a knife for 25 cents, and lost $\frac{2}{7}$ of the cost. What was the cost?

49. If $\frac{4}{5}$ of a barrel of sugar costs \$12, how many barrels of flour at \$5 a barrel will pay for the barrel of sugar?

50. Sold a horse for \$100, which was $\frac{4}{5}$ of his cost. If I paid for him with cloth at \$4 a yard, how many yards did I give?

51. If a contractor pays a mason \$10 for $2\frac{2}{3}$ days' work, how much does the mason receive per day?

52. What will 18 lb. of coffee cost at $33\frac{1}{3}$ cents a pound?

53. Paid \$21 for coffee at $37\frac{1}{2}$ cents a pound. How many pounds did I buy?

54. A man had $9\frac{3}{8}$ yd. of cloth, and sold $5\frac{2}{3}$ yd. How many yards did he have left?

55. Bought 50 yd. of muslin at $12\frac{1}{2}$ cents a yard. How many dollars did the muslin cost?

56. $\frac{2}{3}$ of 24 is 4 more than $\frac{4}{5}$ of what number?

57. $\frac{4}{5}$ of 10 is 6 less than how many thirds of 21?

58. $\frac{2}{3}$ of 36 increased by $\frac{4}{5}$ of 20 are how many?

59. $\frac{5}{9}$ of 45 diminished by $\frac{2}{3}$ of 21 are how many?

60. From a jar of butter containing $17\frac{1}{2}$ lb. of butter, $8\frac{3}{4}$ lb. have been sold. How many pounds remain?

61. Express in their lowest terms: $\frac{12}{18}$, $\frac{9}{12}$, $\frac{18}{20}$, $\frac{20}{36}$, $\frac{40}{56}$, $\frac{42}{63}$, $\frac{84}{96}$.

62. Change to improper fractions: $8\frac{4}{9}$, $6\frac{4}{5}$, $5\frac{5}{8}$, $9\frac{3}{7}$, $11\frac{2}{3}$.

63. Reduce $\frac{5}{6}$ to 12ths; to 18ths; to 24ths; to 36ths.

64. If $1\frac{1}{4}$ pk. are sold from a bushel of cranberries, what part of a bushel remains?

65. What part of $\frac{3}{4}$ is $\frac{2}{3}$? of $\frac{7}{8}$ is $\frac{5}{6}$? of $\frac{5}{6}$ is $\frac{3}{4}$?

66. If $\frac{7}{8}$ of a yard of cloth costs 42 cents, find the cost of $4\frac{1}{6}$ yards.

67. A farmer divided equally among his three sons $\frac{1}{2}$ of his farm. What part of the farm did each son receive?

68. How many bushels of carrots at $\$\frac{3}{8}$ a bushel can be bought for $\$5\frac{1}{4}$?

69. Three fourths of a cord of wood at \$6 a cord will pay for what part of a ton of coal at \$9 a ton?

70. When $2\frac{1}{2}$ qt. have been taken from a gallon of milk, what part of a gallon has been taken?

71. If two thirds of a yard of silk can be bought for $\$\frac{3}{4}$, how many yards can be bought for $\$6\frac{3}{4}$?

72. A carpet dealer sold $\frac{3}{4}$ of $\frac{2}{3}$ of a roll of carpet. What part of the roll was left?

73. How many pigs at $\$2\frac{1}{2}$ each can be bought for \$35?

74. How many barrels of apples will 44 bu. make, if $2\frac{3}{4}$ bu. make a barrel?

75. From a bin of potatoes containing 50 bu., $5\frac{1}{2}$, $2\frac{1}{2}$, and $4\frac{1}{2}$ bu. were sold. What part of the potatoes was left in the bin?

76. A man can do a certain piece of work in 5 hours, and a boy can do the same work in 8 hours. What part of the work can each do in an hour? What part can both together do? How many hours will it take both together to do the work?

77. After a fruit dealer has sold $\frac{2}{3}$ of $\frac{3}{4}$ of a bushel of berries, he sells the remainder at 12 cents a quart. What does he receive for the remainder?

78. A baker paid \$30 for $\frac{6}{7}$ of a hogshead of molasses. What is the value of $\frac{1}{4}$ of the remainder?

79. After a drover has sold $\frac{2}{3}$ of his sheep and $\frac{1}{3}$ of the remainder, he had 40 left. How many sheep had he at first?

80. A man invests $\frac{1}{2}$ of his money in land, $\frac{1}{5}$ in bank stock, and has \$6000 left. How much money had he?

81. Reduce $\frac{4}{5}$, $\frac{7}{8}$, $\frac{9}{16}$ to decimal fractions.

82. If $\frac{5}{9}$ of a ton of rye straw costs \$10, how many tons can be bought for \$126?

EXERCISE 195. — WRITTEN.

1. By how much is $\frac{19}{24}$ greater than $\frac{7}{9}$?

2. A cistern can be filled by a pipe in 30 minutes, and can be emptied by another pipe in 20 minutes. If the cistern is full and both pipes are open, in how long will the cistern be emptied?

3. Reduce to simplest form $\frac{2}{3}$ of $\frac{7}{8}$ of $9\frac{1}{3}$.

4. A man paid \$120 for keeping 2 horses 12 weeks. What would it cost at the same rate to keep 1 horse 5 days?

5. Reduce $\frac{4}{5}$, $\frac{7}{16}$, $\frac{3}{8}$, and $\frac{3}{4}$ to decimal fractions, and find their sum.

6. If 70 lb. of tea are put up in 1 lb., $\frac{1}{2}$ lb., $\frac{1}{4}$ lb. packages, an equal number of each, how many packages of each weight will there be?

7. If a man's weekly income is \$$16\frac{1}{2}$, his weekly expenses \$$21\frac{2}{3}$, and his money on hand is \$$147\frac{1}{4}$, how many weeks can he live without falling into debt?

8. A can mow a field in 8 days, B in 10 days, and B's son in 15 days. How many days will it take them together to mow the field?

9. A flour dealer bought 120 barrels of flour at \$$4\frac{4}{5}$ a barrel. He sold 94 barrels at \$$5\frac{1}{4}$, and the remainder at \$$4\frac{1}{2}$. What did he gain?

10. From 160 A. of land, $42\frac{1}{4}$ were sold to one man and $\frac{1}{3}$ of the remainder to another. How many acres remained unsold?

11. If $\frac{5}{9}$ of a ton of coal costs \$3, how many tons can be bought for \$$141\frac{3}{5}$?

12. A bankrupt's available property can be sold for \$16,760, and this will pay $62\frac{1}{2}$ cents on every dollar he owes. How much does he owe?

13. A loaf of bread weighing 2 lb. is sold for 5 cents

when flour is worth $4.90. For how much should it sell when flour is worth $3.92 a barrel?

14. If $437.80 is paid for 5500 ft. of rosewood, what is the price per thousand?

15. A farmer gives to his son $\frac{12}{25}$ of his farm, and the remainder to his daughter. The difference between their shares is $3\frac{3}{5}$ acres. How many acres does the daughter receive?

16. If $4\frac{4}{7}$ long tons of cannel coal cost $64, required the cost of $11\frac{5}{7}$ tons.

17. Of a certain farm $\frac{1}{5}$ is pasture, $\frac{3}{8}$ land suitable for cultivation, and the remainder wood land. What part of the farm is wood land?

18. If a man breathes 17 times a minute, and takes in at each breath $\frac{5}{7}$ of a quart of air, how many quarts of air does he require in 2 hours?

19. $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{3}{7}$ of a number added together make 120. What is the number?

20. A proper fraction has 51 for its numerator; and when reduced to its lowest terms it is $\frac{3}{8}$. What is its denominator?

21. If $\frac{3}{5}$ of a ship is worth $27,000, what is $\frac{5}{8}$ of the ship worth?

22. If $2\frac{3}{8}$ yd. of cloth are required to make a pair of trousers, how many pairs can be made from $56\frac{1}{4}$ yd., and how much cloth would be left?

23. What is the value of seven pieces of cloth, each containing $35\frac{3}{4}$ yd., at $2.91 per yard?

24. A dry-goods dealer bought 260 yd. of cloth at $\$1\frac{3}{4}$ per yard, and sold $\frac{3}{4}$ of it at $2 per yard. For how much per yard must he sell the remainder to make a clear profit of $130 on the whole transaction?

25. A meter is equal to $3\frac{28}{100}$ ft., nearly. **Express 1 ft.** as the fraction of a meter.

26. A workman spends $\frac{1}{3}$ of his income for food, $\frac{1}{8}$ for clothing and lodging, and $\frac{1}{10}$ for other purposes. He saves $371 per year. What is his income?

27. A man can plant a field in 10 days. With the assistance of his son he can plant it in 6 days. How long will it take his son alone to plant it?

28. The rates of the express and mail trains are 40 and 28 miles per hour, respectively. What time is saved by taking the express train for a journey of 150 miles?

29. From a tank full of water, $\frac{2}{3}$ of the water was drawn when the tank lacked 35 gal. of being half full. What is the capacity of the tank?

30. A trader bought wheat at 75 cts. a bushel, and sold it at 69 cts. a bushel. How many cents did he lose on every dollar he paid?

31. How many minutes will it take a pipe supplying water at the rate of $2\frac{5}{6}$ gal. per minute to fill a cistern holding $114\frac{3}{4}$ gal.?

32. How much must be paid for $6\frac{3}{4}$ lb. of sugar at the rate of 18 lb. for a dollar?

33. How many bushels of wheat can be bought for $\$53\frac{1}{4}$, at the rate of $29\frac{2}{3}$ bu. for $\$22\frac{1}{4}$?

34. A man sold $\frac{1}{3}$ of his gas stock to A, $\frac{1}{6}$ of it to B, and had remaining 189 shares. How many shares had he at first, and how many shares did he sell?

35. A boy lost $\frac{2}{3}$ of his kite string, and then added 40 ft. He found then that his string was just three fourths its original length. What was its original length?

36. A man gave $12\frac{3}{4}$ bu. of potatoes at 40 cts. a bushel for butter worth 20 cts. a pound. How much butter did he get?

37. A man worked $10\frac{3}{10}$ days. He paid for his board $\frac{2}{5}$ of his earnings, and had left $12.36. What were his daily wages?

38. If 6 pigs cost $\$8\frac{1}{4}$, how many pigs can be bought for $\$57\frac{3}{4}$?

39. A man invested $\frac{3}{4}$ of his money in railroad stock, and had $2015 left. How much money had he?

40. What must I pay for 19 arithmetics at the rate of $4.55 for 7?

41. If $27\frac{1}{2}$ tons of anthracite coal cost $\$178\frac{3}{4}$, what is the price per ton?

42. If a man's wages are $\$3\frac{1}{2}$ per day, and his daily expenses are $\$1\frac{3}{4}$, how many days must he work to pay for a suit of clothes worth $56?

43. If $\frac{7}{8}$ of a ton of hay cost $15, what will be the cost of $21\frac{3}{4}$ tons?

44. A paid $\frac{3}{4}$ of his money for a farm, $\frac{4}{5}$ of the remainder for a house, and had $2000 left. How much money had he at first?

45. A grocer invests $\frac{1}{3}$ of his money in flour, $\frac{1}{6}$ in sugar, and $\frac{1}{12}$ in tea and coffee. What part of his money had he remaining?

46. Two workmen earning the same money spend, respectively, each week $\frac{5}{7}$ and $\frac{11}{16}$ of their wages. Which will have more money at the end of the year?

47. A carpenter cuts off $\frac{3}{5}$ of a plank, and then $\frac{9}{14}$ of the remainder. What fraction of the whole will be left?

48. Multiply $3\frac{1}{2}$ of $2\frac{1}{2}$ by $3\frac{1}{3} + 2\frac{1}{2}$, and this product by $3\frac{1}{3} - 2\frac{1}{2}$.

49. Two men can reap a field separately in 6 days and 4 days. In what time can they reap it together?

50. If the wheel of a railway car is $\frac{22}{7}$ of $2\frac{3}{4}$ feet in circumference, how many times in a minute does it turn when the train is going 30 miles per hour?

51. Of the prismatic spectrum, red occupies $\frac{1}{8}$, orange $\frac{3}{40}$, and yellow $\frac{2}{15}$. What part of the whole spectrum is occupied by the other colors?

Compound Quantities.

EXERCISE 196. — ORAL.

1. How many rods in 66 ft.? How many yards?

2. What part of a yard is 1 ft. 6 in.? 2 ft. 6 in.?

3. What part of a rod is $8\frac{1}{4}$ ft.? $4\frac{1}{8}$ ft.?

4. How many rods in $\frac{3}{8}$ mi.? in $\frac{7}{10}$ mi.?

5. What part of a square yard is $2\frac{1}{4}$ sq. ft.?

6. What part of an acre is 60 sq. rd.?

7. How many yards in one surveyors' chain?

8. How many surveyors' chains make a mile?

9. What part of a mile is 30 chains?

10. What part of a chain is 50 lk.? 75 lk.?

11. How many inches are there in 1 lk.?

12. How many cubic yards in 81 cu. ft.?

13. What part of a cubic yard is 9 cu. ft.?

14. How many baskets holding $2\frac{1}{2}$ pk. each will 5 bu. of apples fill?

15. 5 gallons of water will fill how many pint bottles?

16. A 2-gallon can lacks 3 qt. of being full. What part of the can is empty?

17. How many 4-oz. packages can be put up from $4\frac{1}{2}$ lb. of nutmegs?

18. What part of a pound is $\frac{5}{8}$ oz.?

19. How many pounds of butter at $0.25 will pay for a ton of hay at $16?

20. If one ounce of gold is worth $20.67, what is the value of a pennyweight to the nearest cent?

21. At the rate of 3 mi. per hour, how far will a man walk in 50 min.?

22. If a man walks $\frac{1}{4}$ mi. in 5 min., how many hours will it take him to walk 6 mi.?

EXERCISE 197. — WRITTEN.

1. Reduce to feet 5 mi. 5 yd. 2 ft.

2. Reduce to higher denominations 43,735 ft.

3. Reduce 67,314 sq. yd. to higher denominations.

4. How many pint packages can a seedsman make from 2 bu. 2 pk. 6 qt. of peas?

5. Find the difference in pounds between 25 long tons of coal and 25 short tons.

6. What part of a day is 12 hr. 30 min. 30 sec.?

7. The length of a lunar month is 29 dy. 12 hr. 44 min. 3 sec., and the length of a solar year is 365 dy. 5 hr. 48 min. 48 sec., nearly. How many lunar months are there in 19 solar years?

8. Find $\frac{2}{3}$ of 5 mi. 80 rd. 3 yd. 1 ft.

9. Express 29 rd. 1 ft. 6 in. as the fraction of a mile.

10. Express 60 rd. 1 yd. 2 ft. 3.36 in. as the decimal of a mile.

11. What fraction of 4 yd. is 1 yd. 2 ft. 6 in.?

12. What fraction of a mile is 125 yd. 2 ft. $1\frac{5}{7}$ in.?

13. Divide 44 cu. yd. 21 cu. ft. 138 cu. in. by 7.

14. Find $\frac{5}{7}$ of 25 bu. 3 pk. 2 qt.

15. What fraction of a hundredweight is 0.006 of a ton?

16. Divide 2 common yr. 135 dy. 17 hr. by 36.

17. Express 3.345 dy. in days, hours, etc.

18. The latitude of New York is 40° 42′ 43″ N. The latitude of Boston is 42° 21′ 30″ N. Find the difference.

19. In a carload of 30,000 lb. of pork how many barrels are there?

20. Find the weight of 105 bu. $3\frac{1}{2}$ pk. of potatoes.

21. The radius of a circle is 0.1591549 of its circumference. The circumference contains 360°. Find the angle at the centre whose intercepted arc is equal in length to the length of the radius.

Measurements.

EXERCISE 198. — ORAL.

How many feet board measure in:

1. A board 16 ft. long, 12 in. wide, 1 in. thick?

2. A board 18 ft. long, 8 in. wide, $\frac{3}{4}$ in. thick?

3. A board 12 ft. long, 10 in. wide, $\frac{1}{2}$ in. thick?

4. A joist 3 in. by 4 in. and 11 ft. long?

5. A joist 2 in. by 9 in. and 14 ft. long?

6. A 2-in. plank 12 ft. long, 15 in. wide?

7. A 3-in. plank 12 ft. long, 10 in. wide?

8. A 4-in. plank 18 ft. long, 10 in. wide?

9. A stick of square timber 10 in. by 12 in. and 14 ft. long?

10. How many feet board measure in a cubic foot?

11. How many cubic feet in a stick of square timber 12 in. wide, 9 in. thick, and 10 ft. long?

12. How many cubic yards in a rectangular ditch 21 ft. long, 3 ft. wide, and 3 ft. deep?

13. How many cords of wood in a pile 24 ft. long, 4 ft. wide, and 4 ft. high?

14. How many cubic feet in a rectangular cistern that is 5 ft. square and 6 ft. deep?

15. What length must be cut off an inch board 8 in. wide to obtain 4 ft. board measure?

16. At $30 a thousand, what is the value of a stick of timber 25 ft. long and 1 ft. square at the end?

17. Find the entire surface of a cubical block of marble 2 ft. on an edge.

18. What must be the length of a walk $2\frac{1}{4}$ ft. wide to contain 72 sq. ft.?

19. The floor of a rectangular room is 12 ft. by 15 ft. Find its perimeter and its area.

20. A square field is 40 rd. on a side. Find its area in acres, and the cost of fencing it at $1.50 per rod.

21. Find the area of a triangle if its base is 8 ft. and its altitude 6 ft.

22. At 20 cents a square meter, what will it cost to paint a rectangular surface 10 meters long and 8 meters wide?

23. How many sters of wood in a pile of wood 4 meters long, 2 meters wide, and 1 meter high?

24. If a metric ton of raw sugar costs $70, what is the cost of a kilogram?

25. At 20 cents a liter, what is the price of a hektoliter of olive oil?

26. A rectangular lot contains $\frac{1}{4}$ of an acre. If its width is 5 rd., what is its length?

27. The diameter of a bicycle wheel is 28 in. What is its circumference?

28. How many yards of carpeting 1 yd. wide will be required for a room 18 ft. by 18 ft., if there is no waste in matching the pattern?

29. A rectangular cistern 7 ft. long and 5 ft. wide contains 105 cu. ft. What is its depth?

30. How many boards each 12 ft. long and 10 in. wide will be required to cover 300 sq. ft.?

31. At 10 cents a cubic foot, what is the cost of a block of stone 3 ft. long, 2 ft. wide, and 2 ft. thick?

32. What must be the length of a driveway 3 meters wide to contain an ar?

33. If a cubic centimeter of water weighs a gram, how many kilograms will 4 liters weigh?

EXERCISE 199. — WRITTEN.

1. How many cubic yards of earth will be cut out of a drain 120 ft. long, 2 ft. wide, 3 ft. deep?

2. Find the entire surface of a rectangular block of granite 8 ft. long, 2 ft. wide, 1 ft. thick.

3. How many revolutions will be made by a wheel $3\frac{3}{4}$ yd. in circumference in passing over 99 miles?

4. At 15 cents a foot, what will be the cost of fencing a rectangular field 320 yd. long and 210 yd. wide?

5. What is the length of a rectangular field containing $7\frac{1}{2}$ A., if the breadth is 121 yd.?

6. What must be paid for a pile of wood 24 ft. long, 4 ft. wide, and 4 ft. high at $3.75 a cord?

7. Find the cost of plastering a ceiling 18 ft. by 15 ft., at 30 cents a square yard.

8. Allowing 1000 shingles for 110 sq. ft., how many thousand will be required to cover the pitched roof of a barn 40 ft. long, if the width of each side of the roof is 28 ft.?

9. How many slates at three to the square foot will be required to cover 25 squares of roof?

10. Find the circumference and the area of a circle 10 ft. in diameter.

11. Find the area in acres of a rectangular field which measures 10 ch. by 8 ch.

12. A rectangular cistern is 5 ft. by 4 ft. by 3 ft. How many gallons of water will it hold, reckoning $7\frac{1}{2}$ gal. to the cubic foot?

13. Find the number of gallons in a round cistern 8 ft. in diameter and 7 ft. deep, allowing $7\frac{1}{2}$ gal. to a cubic foot.

14. At $8 a cord, what must be paid for a load of white oak wood 8 ft. long, 4 ft. wide, and $3\frac{1}{2}$ ft. high?

15. What should be paid for a pile of 4-ft. wood at $5 a cord, if the pile is 100 ft. long and averages 5 ft. high?

16. If a short ton of soft coal measures 42 cu. ft., how many tons will a rectangular bin hold 10 ft. long, 6 ft. wide, and 5 ft. deep?

17. How many square inches on the surface of a globe whose diameter is 12 in.?

18. How many cubic inches in the volume of a globe whose diameter is 12 in.?

19. How many cubic feet in a stone roller 6 ft. long and 20 in. in diameter?

20. Into how many lots of 3.75^{a} may $8^{ha}\ 40^{a}$ be divided?

21. How much would it cost at $1.20 a square yard to pave a street 90 rd. long and 63 ft. wide?

22. How many bricks will be required for the walls of a house 40 ft. long, 30 ft. front, and 18 ft. high, deducting three doors 7 ft. 6 in. by 4 ft. and 10 windows 5 ft. by 4 ft., the walls to be 1 ft. thick? (Reckon 22 bricks to the cubic foot.)

23. How many feet board measure will be required to plank a rectangular room 100 ft. by 60 ft. with planks 4 in. thick?

24. What must be the length of a pile of cord wood 6 ft. high to contain 10 cords?

25. How many gallons of water will pass under a bridge every 10 minutes, if the stream is 26 ft. wide and $3\frac{1}{2}$ ft. deep on the average, and flows at the rate of 2 mi. per hour?

26. Find the number of cubic inches in a cylindrical measure 8 in. deep and $10\frac{1}{2}$ in. in diameter.

Percentage.

Exercise 200. — Oral.

What is:

1. 20% of 400 sheep?
2. 25% of 160 horses?
3. $12\frac{1}{2}$% of 400 feet?
4. $6\frac{1}{4}$% of 160 acres?
5. $33\frac{1}{3}$% of 90 days?
6. $66\frac{2}{3}$% of 300 apples?
7. $37\frac{1}{2}$% of 320 rods?
8. $\frac{1}{2}$ of 1% of $100?

What is the number of which:

9. 5 is $62\frac{1}{2}$%?
10. 60 is 75%?
11. 49 is $87\frac{1}{2}$%?
12. 70 is $8\frac{1}{3}$%?
13. $\frac{1}{2}$ is $16\frac{2}{3}$%?
14. 680 is 170%?
15. $\frac{3}{4}$ is 75%?
16. $12\frac{1}{2}$ is 200%?

What per cent of:

17. 100 is $12\frac{1}{2}$?
18. 150 is 25?
19. 800 is 72?
20. 25 is 15?
21. 2 is $\frac{1}{2}$?
22. 3 is $\frac{2}{3}$?
23. $\frac{1}{8}$ is $\frac{1}{2}$?
24. $\frac{3}{4}$ is $\frac{1}{2}$?

25. 81 is $12\frac{1}{2}$% more than what number?
26. 120 is $6\frac{1}{4}$% less than what number?
27. 56 is $33\frac{1}{3}$% more than what number?
28. $\frac{5}{16}$ is 25% less than what number?

Find the net amount of:

29. A bill of $160 with $\frac{1}{4}$ and $16\frac{2}{3}$% off.
30. A bill of $300 with $\frac{1}{3}$ and 25% off.
31. A bill of $120 with $12\frac{1}{2}$% and $\frac{1}{7}$ off.
32. A bill of $24 with $\frac{1}{2}$ and $33\frac{1}{3}$% off.

33. Velvet is sold for $10.75 per yard at a gain of 25%. Find the cost of the velvet.

34. A cord of wood costing $4.50 is sold for $6. What is the gain per cent?

35. What number diminished by 10% of itself equals 81?

36. Find the commission at 1% on $6000.

37. A man received $32 as his commission at 4% for collecting a debt. What was the amount of the debt?

38. A man sells wood for $6500, and receives $130 commission. What is the rate of his commission?

39. How many barrels of flour at $3.50 can an agent buy for $364, if this sum includes his commission at 4%?

40. What is the premium at $1\frac{1}{5}$% on $3000 insurance?

Find the interest on:

41. $700 for 2 yr. at 4%.

42. $900 for 5 yr. at 6%.

43. $200 for 2 yr. at $4\frac{1}{2}$%.

44. $500 for 5 yr. at 5%.

45. $3000 for 4 mo. at 6%.

46. $2000 for 3 mo. at 6%.

47. $600 for 2 mo. at 4%.

48. $2000 for 6 mo. at $4\frac{1}{2}$%.

49. $5000 for 30 dy. at 6%.

50. $1200 for 90 dy. at 6%.

51. $2000 for 60 dy. at $4\frac{1}{2}$%.

52. $400 for 30 dy. at $4\frac{1}{2}$%.

At what rate per cent:

53. Will $160 produce $24 interest in 3 yr.?

54. Will $800 amount to $880 in 2 yr.?

55. Will $500 amount to $510 in 4 mo.?

56. Will $600 produce $$7\frac{1}{2}$ interest in 90 dy.?

Find the time in which:

57. The interest on $250 will be $25 at 5%.

58. $300 will double itself at 6%.

59. $1000 will amount to $1200 at 4%.

60. $400 will amount to $436 at $4\frac{1}{2}$%.

Find the principal that will:

61. Produce $180 interest in 3 yr. at 6%.

62. Produce $100 interest in 8 yr. at 5%.

63. Amount to $560 in 3 yr. at 4%.

64. Amount to $1180 in 4 yr. at $4\frac{1}{2}$%.

65. Find the bank discount for 63 days on $600 at 6%.

66. Find the bank discount for 93 days on $800 at 5%.

EXERCISE 201. — WRITTEN.

1. A house worth \$7500 is damaged by fire \$1920. What is the rate per cent of the loss?

2. A man received from a bankrupt \$468.75, which was $37\frac{1}{2}\%$ of the sum due. What was the sum due?

3. A dealer sold 10% of his stock of sugar, and then $33\frac{1}{3}\%$ of the remainder, after which he had 3120 lb. left. How much sugar had he at first?

4. A horse and wagon are valued together at \$225, and the horse is valued at 25% more than the wagon. What is the value of the horse?

5. For what price per pair must shoes be sold to gain 25%, if 15% is lost when they are sold at \$2.55 per pair?

6. If 14% of a ton of butter costs \$84, what per cent of a ton can be bought for \$114?

7. An agent makes 20% by selling a book for \$1.44. If he had sold it for \$2, what per cent would he have made?

8. If 98 sq. rd. is 20% of the area of a field 30 rd. in length, what is the width of the field?

9. A sea captain paid \$690 at $1\frac{1}{2}\%$ for insuring $\frac{2}{3}$ the value of his ship. What is the value of the ship?

10. A town has to raise \$64,000 for expenses. If 4% is allowed for collecting, how much money must be voted?

11. A merchant sent \$15,375 to his agent for the purchase of cotton. Find the sum spent for cotton, if the agent charges $2\frac{1}{2}\%$ commission for buying.

12. How many barrels of flour can be bought for \$12,600 at \$5 a barrel, commission 5% for buying?

13. What insurance must be placed upon goods valued at \$4017, that the value of the goods and the premium of $2\frac{1}{2}\%$ may be recovered in the case of loss by fire?

Find the interest on:

14. $1200 for 1 yr. 7 mo. 18 dy. at 6%.

15. $2000 for 7 mo. 21 dy. at 5%.

16. $650 for 1 yr. 3 mo. 15 dy. at 4½%.

17. $868 for 1 yr. 1 mo. 12 dy. at 7%.

18. $1584 for 3 yr. 7 mo. 17 dy. at 6½%.

19. $2957.50 for 63 dy. at 6%.

20. $5167 for 90 dy. at 5½%.

21. $679 for 1 yr. 6 mo. at 1% a month.

22. $850 for 6 yr. 5 mo. 24 dy. at 3½%.

Find the rate per cent:

23. When the interest on $1500 for 3 yr. is $315.

24. When the interest on $4500 for 2 yr. is $517.50.

25. When the interest on $2557 for 2 yr. 6 mo. is $255.70.

26. When a sum of money is doubled in 12 yr.

27. When $3516 amounts to $3738.68 in 8 mo.

Find the time:

28. When $1200 at 3⅛% amounts to $1320.

29. When the interest on $835.20 at 3¾% is $100.92.

30. When the interest on $78 at 4% is $6.24.

31. When the principal at 7% is doubled.

Find the principal that will:

32. Produce $212.47 interest in 3 yr. at 5½%.

33. Produce $150.80 interest in 3 yr. 4 mo. at 4½%.

34. Produce $15.01 interest in 3 mo. 2 dy. at 4¼%.

35. Amount to $2551.50 in 1 yr. 1 mo. at 5%.

36. Amount to $1111.06 in 2 yr. 3 mo. 18 dy. at 5%.

Find the day of maturity, the time to run, the discount, and the proceeds of the following notes, without grace and with grace:

	Face of Note.	Date of Note.	Time.	Day of Discount.	Rate of Discount.
37.	$500,	Sept. 3, 1896,	30 dy.,	Sept. 3, 1896,	6%.
38.	$1880,	Oct. 26, 1896,	60 dy.,	Nov. 2, 1896,	7%.
39.	$1975,	Sept. 12, 1896,	3 mo.,	Sept. 26, 1896,	$5\frac{1}{2}$%.
40.	$5786,	Sept. 28, 1896,	4 mo.,	Oct. 6, 1896,	$4\frac{1}{2}$%.
41.	$8339,	Oct. 30, 1896,	90 dy.,	Nov. 17, 1896,	5%.

42. Find the proceeds of the following note discounted in Concord, Oct. 28, 1896, at 7%:

$300. Concord, N. H., Oct. 14, 1896.

Ninety days after date, without grace, I promise to pay John James, or order, three hundred dollars at the Suffolk Bank, Boston. Value received.

Samuel Brown.

43. Find the proceeds of the following note discounted in New York, Dec. 12, 1896, at 6%, exchange being $0.15 per $1000:

$10,000. Boston, Mass., Oct. 17, 1896.

Four months after date, without grace, we jointly and severally promise to pay to John S. Pierce & Co., or order, ten thousand dollars. Value received.

James Johnson,
Thomas Williams.

44. Find the proceeds of the following draft, discounted at 6%, exchange $\frac{1}{4}$%:

$400. Newfield, Me., Nov. 4, 1896.

At thirty days' sight pay to Brown & Co., of Portland, or order, four hundred dollars, value received, and charge to the account of

S. V. Durgin.

To Casco Bank, Portland.

45. Find the compound interest on $2000 for 2 yr. 6 mo. at 5% per annum.

46. Find the compound interest on $1250 for 1 yr. 3 mo. at 6% per annum, interest compounded semi-annually.

47. Find the compound interest on $5000 for 2 yr. 3 mo. at 4% per annum, interest compounded quarterly.

48. Find the annual interest on $1600 for 4 yr. 8 mo. at 6%.

49. A note of $3140 dated May 10, 1895, and drawing interest at 6%, had payments endorsed on it as follows: Oct. 2, 1895, $315; Feb. 28, 1896, $25; Apr. 4, 1896, $1540; Aug. 10, 1896, $610. Find the amount due Dec. 10, 1896.

50. Find the cost of five United States 4% bonds, $1000 each, at 116½.

51. Find the cost of 62 shares of Concord and Montreal R.R. at 163.

52. How many shares ($100 each) of Old Colony R.R. stock at 175 may be bought for $3675?

53. At 91⅛, what will be the cost of 100 shares of stock, brokerage being ⅛?

54. What is the price of stock when $4600 stock is bought for $4485?

55. What is the income from $7000 of 6% stock?

56. A man receives $630 as his annual dividend from 7% stock. How many shares of $100 each does he hold?

57. What is the rate of dividend paid by a railroad when a holder of 123 shares receives $861?

58. If an 8% stock is worth $150, what rate of interest does a purchaser receive on his money?

59. What must be the price of a 5% stock, that a buyer may receive 6% on his investment?

60. What must be the price of a 6% stock, that a buyer may receive 5% on his investment?

CHAPTER XVII.

MISCELLANEOUS EXAMPLES.

EXERCISE 202. — WRITTEN.

1. The charge for sending a telegram from Boston to New York is 25 cents for ten words, and 2 cents for every additional word. Find the charge for sending 40 words.

2. John has $19.19 and James has $31. How much more money has James than John?

3. A man bought a cargo of coal for $500. He sold it for $5.75 a ton and gained $75. How many tons did he buy?

4. Find the prime factors of 20,790.

5. If a man walks $3\frac{3}{4}$ miles an hour, how many hours will it take him to walk $37\frac{1}{2}$ miles?

6. Find the cost of a piece of land that is 20 yards square, at 25 cents a square foot.

7. At $1\frac{1}{2}\%$ commission an agent received $194.58 for selling goods. Find the amount of the sale.

8. Find the interest on $1500 for 1 yr. 5 mo. 12 dy. at $4\frac{1}{2}\%$.

9. Find the bank discount on a note of $1000 for 60 days, without grace, at 5%.

10. How many 4 per cent bonds $1000 each will produce an income of $480?

11. A man bought 57 shares of mining stock. He paid \$192.80 cash, and gave his note for \$400. How much a share did he pay for the stock?

12. A man has a salary of \$1200 a year. He pays for rent \$192, for groceries \$312.30, for meats \$211.50, for clothing \$197.63, for other expenses \$97.37. How much of his salary does he save?

13. A grocer sold 7960 pounds of sugar at \$4.75 a hundred pounds. How much did he receive?

14. Find the circumference and the area of a circle whose radius is 2 ft. 6 in.

15. If a 9% stock is bought at 150, what rate of interest is received on the investment?

16. Find the number of feet board measure in 15 boards 16 feet long, 10 inches wide, and 1 inch thick.

17. Find the number of feet board measure in 18 joists 16 feet long, 4 inches wide, and 3 inches thick.

18. How many tons of Lehigh coal will a bin hold that is 9 ft. long, 8 ft. wide, and 5 ft. 6 in. deep, if it requires 33 cubic feet for a ton?

19. Multiply 1 day 14 hours 57 minutes 33 seconds by 225.

20. A rectangular lot contains $\frac{1}{4}$ of an acre. If its width is 80 feet, find its length.

21. A broker charged $\frac{1}{4}$ brokerage for buying stocks, and received from James Johnson \$10.75. How many shares did the broker buy for Johnson?

22. The annual catch of fish in the great northern lakes of America averages a hundred million pounds, and brings to the fishermen two and a half million dollars. Find the average price per pound the fishermen receive.

EXERCISE 203. — WRITTEN.

1. The flow of water over the Falls of Niagara averages 265,000 cubic feet a second. Find the average number of gallons carried over the Falls a second.

2. Great Britain paid $1.25 a bushel on the average for wheat in 1880, and only 67 cents in 1894. Find the decrease in price per bushel and the decrease per cent.

3. Great Britain paid $7.01 a barrel for flour on the average in 1880, and only $3.28 in 1894. Find the decrease in price per barrel and the decrease per cent.

4. In 1894 the imports of wheat into Great Britain from the United States amounted to 46 million bushels; from Russia 31 million bushels; from Argentina 25 million bushels; and from all other countries 6 million bushels. Find the per cent of the sum total furnished by each of the three great wheat exporting countries.

5. Find the number of feet board measure in 60 flooring timbers 9″ by 2″ and 16 ft. long.

6. How many cubic yards of earth will be thrown out in digging a cellar $38' \times 30' \times 6'\ 6''$?

7. A man owning $\frac{2}{3}$ of a factory sold $\frac{5}{7}$ of his share for $31,500. What was the factory worth at the same rate?

8. If $6\frac{1}{4}$ pounds of coffee cost $2, find the cost of a bag of coffee containing 61 lb. 8 oz.

9. Find the quotient of $6 \times 12 \times 15 \times 16 \times 17 \times 21 \times 24$ divided by $2 \times 7 \times 8 \times 9 \times 10 \times 36 \times 51$.

10. Find the difference between $1\frac{2}{3} \times 2\frac{5}{8}$ and 0.019 of 220.

11. If $\frac{3}{4}$ of an inch on a map represents an actual distance of $7\frac{1}{2}$ miles, what distance on the map represents $67\frac{1}{2}$ miles?

12. How much will a creditor lose on a debt of $5344, if he receives only $62\frac{1}{2}$ cents on a dollar?

13. If 7 gallons and 1 quart of alcohol cost $17.40, what will 1 pint cost at the same rate?

14. A man spent 0.555 of his money for a house and 0.27 of it for land. He had $2187.50 left. How much did he pay for the house? How much for the land?

15. How many yards of carpet 30 in. wide will be needed for a room 20′ by 17′ 6″, if the strips run lengthwise?

16. Find the number of rolls of paper required for a room 16′ × 15′ × 10′, having two doors 3′ 10″ wide 7′ 6″ high, two windows 4′ wide 6′ 6″ high, if the base board of the room is 10″ and the border for the paper is 14″ wide.

17. How many loads of gravel averaging 1 cubic yard will be required to grade 2 miles of road, the gravel to be laid 12 feet wide and 6 inches deep?

18. What is the width of a 2-inch plank 15 feet long that contains 40 feet board measure?

19. Find the cost of insuring for a year property worth $10,000 at $\frac{1}{2}\%$ if only $\frac{3}{8}$ of its value is insured.

20. For what sum should property worth $18,624 be insured at 3%, so that in case of loss both the value of the property and the premium paid should be recovered?

21. If $10,250 is the sum paid for wool and $2\frac{1}{2}\%$ commission to the purchasing agent, how much is the amount of the commission?

22. A commission of $40.43 was paid for selling $622 worth of property. What was the rate of commission?

Exercise 204. — Written.

1. The sugar contained in the sugar beet is $6\frac{1}{4}\%$ of the weight of the beet. How many pounds of sugar will be obtained from an acre of beets, if it produces 400 bushels?

2. The weight of ashes obtained from burning dry white oak wood is $2\frac{3}{4}\%$ of the weight of the wood. A cubic foot of dry white oak weighs 52 pounds. Find the weight of ashes obtained from burning a cord of dry white oak.

3. A tailor bought four pieces of cloth of equal lengths at $2.50 a yard. He sold three of the pieces for $405, and gained $67.50 on the three pieces together. How many yards were there in each piece?

4. If 97 bushels and 12 quarts of wheat are raised from 7 acres and 40 square rods of land, what is the average yield per acre?

5. If a wood chopper cuts $15\frac{3}{4}$ cords of wood in 9 days, how many cords does he cut on the average each day?

6. Find the least number of dollars which, if divided into 18, 24, or 30 equal parts, will give $17 remainder in each case.

7. A commission of $50 was charged for selling $2000 worth of goods. Find the rate of commission.

8. A man paid $25 for having his house insured for three years at $1\frac{1}{4}\%$ on $\frac{2}{3}$ of its value. Find the value of the house.

9. If $\frac{6}{7}\%$ of a number exceeds $\frac{4}{5}\%$ of the number by 200, what is the number?

10. If brokerage at $\frac{1}{8}$ for buying stocks amounts to $20.25, what number of shares is purchased?

11. One half of a stock of goods valued at $306.30 is sold for $\frac{2}{3}$ of the value of the whole stock. What was the gain per cent?

12. A cotton broker receives $75 commission at $\frac{3}{4}\%$ for buying cotton. What sum is required to pay for the cotton and the commission?

13. Find the interest on a note of $1510 for 3 months at 5%.

14. Find the interest on $1520 for 2 yr. 10 mo. 27 dy. at $4\frac{1}{2}\%$.

15. Find the rate per cent, when a sum of money at simple interest is doubled in 15 years.

16. Find the bank discount on a note of $1000 for 4 mo., without grace, at 6%.

17. Find the net amount of a bill of $765 with 25, 10, and 5 off.

18. A lady receives $1200 annual income from the U. S. 4's that she owns. Find the face value of the U. S. bonds that she owns.

19. A man directs his broker to buy a certain stock at $84\frac{1}{2}$. If the total cost of the stock including $\frac{1}{8}$ brokerage is $10,155, what is the number of shares bought?

20. A rectangle is 243 yards long and 108 yards wide. Find the side of a square that has the same area as the rectangle.

21. Find the number of feet board measure in 70 planks 16 ft. long, 8 in. wide, and $1\frac{3}{4}$ in. thick.

22. Find the number of feet board measure in 48 joists 16 ft. long, 10 in. wide, and 3 in. thick.

23. In what time will $492 produce $144.32 interest, at 4%?

EXERCISE 205. — WRITTEN.

1. How many cubic yards of earth will be thrown out in digging a cellar 40 ft. long, 30 ft. wide, and 6 ft. deep?

2. What length of 2-inch plank 18 in. wide will contain 48 feet board measure?

3. How many feet board measure are there in 40 joists 18 ft. long, 10 in. wide, and 3 in. thick?

4. How many yards of carpet 27 in. wide will be needed for a room 18 ft. long and 17 ft. 6 in. wide, if the strips run lengthwise, and if there is a waste of 9 in. on each strip in matching the pattern?

5. The U. S. sub-treasury in New York has a vault $15' \times 12' \times 10'$ to hold six million silver dollars. A silver dollar weighs $412\frac{1}{2}$ grains. Find in tons avoirdupois the weight of the six million dollars.

6. The imports into Mexico from the United States in 1893 amounted to $33,555,099, and from Great Britain, France, and Germany combined, to $8,734,388. What per cent of the total imports from these four countries were the imports from the United States?

7. Exports from the United States to Canada increased from $38,246,134 in 1888 to $51,432,359 in 1893. Find the total increase and the increase per cent.

8. A merchant burned in his store 125,120 cubic feet of gas in a year. What was his gas bill at $1.25 a thousand?

9. From a piece of cloth containing $37\frac{5}{8}$ yards there were sold nineteen yards and three quarters. How many yards were left?

10. A man paid $7888.30 for 89 acres 90 square rods of land. What was the price per acre?

11. If 1869 sovereigns weigh 40 pounds troy, how many grains does one sovereign weigh?

12. The area of the walls of Rome, erected by Aurelian (A.D. 271), was 1396.5 hektars. A hektar is 2.47110 acres. Find the area in acres.

13. A liquid quart contains 57.75 cubic inches. Find to the nearest hundredth of an inch the edge of a cube that will hold a liquid quart.

14. A liter contains 61.027 cubic inches. Find to the nearest hundredth of an inch the edge of a cube that will hold a liter.

15. The Victoria Bridge across the St. Lawrence River at Montreal is $1\frac{3}{4}$ miles long. In how many minutes will a train cross it at the rate of 22 feet a second?

16. Find the area in acres of a rectangular field 13 chains 17 links long and 10 chains 40 links wide.

17. A rectangular lot 150 feet deep fronts on a certain street. The lot contains an acre, and is sold for $20 a front foot. How much is received for the lot?

18. Find the total length of the edges of a rectangular block $12'' \times 8'' \times 4''$. Find the total surface of the block.

19. How many boards 15 feet long will be required to build a straight fence 4 boards high about a rectangular field 60 rods long and 30 rods wide?

20. If 13 bushels of barley are worth 9 bushels of wheat, how many bushels of barley are worth 2160 pounds of wheat?

21. If a train goes 400 miles in 9 hours and 20 minutes, how long will it be in going 300 miles at the same rate?

22. If $20,500 includes the amount paid for wool and $2\frac{1}{2}\%$ commission to the purchasing agent, how much money is laid out for wool by the agent?

EXERCISE 206. — WRITTEN.

1. The value of the imports into the United States for ten months, ending October 31, 1894, of carpets, cloths, and dress goods was \$11,956,465; and for ten months, ending October 31, 1895, the value was \$42,692,257. Find the total increase, and the increase per cent.

2. For the fiscal year, 1894, the value of the exports from the United States to foreign countries was \$869,204,937. The farmers furnished 73.3% of this great sum. Find the value of farm products exported in 1894 from the United States.

3. Find the compound interest of \$450 for 3 years 4 months at $4\frac{1}{2}$ per cent.

4. The ratio of the diameter to the circumference of a circle was given by Peter Metius as 113 to 355. Find by this ratio the circumference of the Ferris wheel, reckoning its diameter as 250 feet.

5. If 10 burners, consuming on the average 6 cubic feet of gas each per hour, are used 4 hours a day for 300 days, what is the gas bill at \$1.50 a thousand feet?

6. Find the area in acres of a rectangular field 15 chains 50 links long and 12 chains 25 links wide.

7. What length of 3-inch plank 10 inches wide will contain 35 feet board measure?

8. Find the number of feet board measure in 6 beams 16 ft. long and 16 in. square at the end.

9. Find the difference in acres between two fields, if one contains 100 square rods and the other is 100 rods square.

10. Find the cost of a stair carpet at \$1.25 a yard for a straight flight of 20 steps 12 inches wide and 6 inches rise,

allowing $\frac{3}{4}$ yard extra for the bottom of the stairs and the same for the top.

11. A certain wheel revolves 7040 times in passing over 19 miles. Find the circumference of the wheel.

12. Divide 253 into parts proportional to $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$.

13. A bankrupt has \$5345 assets, and owes \$22,750. The expense of settlement is \$795. How much will his estate pay on a dollar?

14. Flint glass contains by weight 72% sand, 14% soda, 12% lime, and 2% alumina. Find the number of pounds of each of these substances in a ton of flint glass.

15. A bell weighing 1000 pounds is composed of 77% copper, 21% tin, and the rest zinc. How many pounds of each metal does the bell contain?

16. A cup weighs $10\frac{1}{2}$ ounces, and is composed of gold and silver in the ratio 15 to 1. Find the weight of each metal in the cup.

17. Gun metal is composed by weight of 11 parts of copper to 2 parts of tin. How many pounds of tin must be put with 418 pounds of copper to make gun metal?

18. What per cent gain is made by buying a horse for \$180 and selling him for \$200?

19. Find the cost of 6% stock that pays $5\frac{1}{2}$% on the money invested.

20. Concord and Montreal Railroad stock pays 7% dividend, and sells for \$168 a share of \$100. Find the rate of interest an investment in this stock will receive.

21. A company with a capital of \$240,000 earned net \$17,500 for 1895. It paid 6% dividend, and carried the balance to surplus. How much did it add to its surplus?

Exercise 207. — Written.

1. A can do a piece of work in 6 days, and B can do it in 8 days. How many days will it take A and B together to do the work?

If A can do the work in 6 days, in 1 day he can do $\frac{1}{6}$ of the work. If B can do the work in 8 days, in 1 day he can do $\frac{1}{8}$ of the work.

Therefore, A and B together in 1 day can do $\frac{1}{6} + \frac{1}{8} = \frac{7}{24}$ of the work.

Hence, they together can do $\frac{1}{24}$ of the work in $\frac{1}{7}$ of a day, and therefore the whole work in 24 times $\frac{1}{7}$ of a day, or $\frac{24}{7}$ days, that is, $3\frac{3}{7}$ days. *Ans.*

2. A can mow a field in 10 hours, and B can mow it in 12 hours. How many hours will it take A and B together to mow it?

3. A can build a wall in 15 days, B in 18 days, and C in 12 days. How many days will it take all three working together to build the wall?

4. A and B can dig a ditch in 20 days, A and C in 18 days, and B and C in 16 days. What part of the ditch can all three together dig in 2 days? In how many days can all three working together dig the ditch? In how many days can each alone dig the ditch?

5. A and B can mow a field in 10 days, A and C in 12 days, and B and C in 15 days. In how many days can all three working together mow the field? In how many days can each alone mow the field?

6. A, B, and C working together can do a piece of work in 12 hours, and A and C working together can do the work in 16 hours. How many hours will it take B alone to do the work?

7. If 11 boys or 7 men can rake a field in 3 hours, how many hours will it take 5 boys and 5 men to rake the field?

8. If 8 boys or 5 men can do a piece of work in 10 days, in how many days can 8 boys and 5 men do the work?

9. 6 men or 8 women can do a piece of work in 12 hours. How many hours will it take 4 men and 10 women to do the work?

10. A cistern can be filled by three pipes, running separately, in 4 hours, 5 hours, 6 hours, respectively. In how many hours will the empty cistern be filled by all running together?

11. Two pipes running at the same time can empty a cistern in 40 minutes, and one of them alone can empty the cistern in 70 minutes. In how many minutes can the other pipe alone empty the cistern?

12. A man sold a horse for \$192 and lost 4% on the cost of the horse. At what price should he have sold him to gain 20%?

13. Find the interest on \$2900 at $4\frac{1}{2}$% for 6 months.

14. Find the premium on ten U. S. 4's of \$1000 each, if the bonds are bought at $116\frac{3}{4}$.

15. If \$4.88 is paid for £1, what will be the cost of a bill of exchange on London for £700?

16. Find the bank discount and the proceeds of a note of \$2000 due in 90 days, with grace, at 5%.

17. Find the net amount of a bill of \$720, discounts allowed being 55, 10, and 5.

18. Find the simple interest on \$1200 for 3 yr. 4 mo. at 6%.

19. A house worth \$5000 rents for \$360 a year. What per cent of the cost is the rental?

20. If 620 is $6\frac{1}{4}$% of a number, find the number.

EXERCISE 208. — WRITTEN.

1. The deflection of the earth's curvature is 8 inches for one mile, $2^2 \times 8$ inches for 2 miles, $3^2 \times 8$ inches for 3 miles, and so on, the deflection being 8 inches multiplied by the square of the distance expressed in miles. Find the height of a light above the sea level that is just visible by a man 20 miles away.

2. Bronze consists of $4\frac{1}{2}$ parts of copper by weight to 1 part of tin. How much copper must be used with 100 pounds of tin to make bronze?

3. A lunar month is 29 days 12 hours 44 minutes 3 seconds. Express a lunar month in days and the decimal fraction of a day.

4. A solar year is 365.242199 days. Express a solar year as a compound quantity in days, hours, minutes, and seconds.

5. Reduce to lowest terms $\frac{39}{143}$; $\frac{68}{102}$; $\frac{132}{231}$; $\frac{1072}{1200}$.

6. What fraction of the year 1896 is the time from April 2 to and including August 22?

7. On four successive days the barometer indicated 29.20 inches, on the fifth day 30.02 inches, on the sixth day 30.18 inches, and on the seventh day 30.20 inches. What was the average for the week?

8. The distance from Boston to Montreal is 342 miles, and from Montreal to Chicago 838 miles. What per cent of the distance from Montreal to Chicago is the distance from Boston to Montreal?

9. The driving wheels of a locomotive have a diameter of 6 ft. 6 in. How many revolutions a minute must each wheel make to travel 40 miles an hour?

10. The front wheels of a wagon are 3 ft. 8 in. in diameter, and the hind wheels are 4 ft. 2 in. How many more

turns will a front wheel make than a hind wheel in going a mile?

11. Find the cost of 1250 boards 16 ft. long, 5 in. wide, and $\frac{3}{4}$ in. thick at \$9.50 per M.

12. Find the number of dry quarts a cylindrical measure $18\frac{1}{2}$ in. in diameter and 8 in. high will hold.

13. An agent sold a house for \$7500, and sent to the owner \$7350. What was his rate of commission?

14. At what rate would \$250 yield \$30 interest in 2 years?

15. Find the proceeds and bank discount of a note for \$827.40 payable in 3 mo., without grace, rate of discount being 5% and exchange $\frac{1}{8}$%.

16. Find by the Merchants Rule the balance due June 1, 1896, on a note for \$1000 dated July 1, 1895, that bears interest at 6% and has the following endorsements: Aug. 1, \$600; Sept. 5, \$120; Oct. 20, \$50.

17. Find by the United States Rule the balance due July 8, 1896, on a note for \$1500, dated May 20, 1895, and bearing interest at 6%, which has the following endorsements: Oct. 2, 1895, \$200; Feb. 29, 1896, \$20; Apr. 6, 1896, \$540; June 1, 1896, \$300.

18. Find the compound interest of \$6000 for 2 yr. 3 mo. 15 dy. at 5%.

19. Find the annual interest of \$6000 for 2 yr. 3 mo. 15 dy. at 5%.

20. Find the simple interest of \$6000 for 2 yr. 3 mo. 15 dy. at 5%, and find how much it falls short of the compound interest, and also of the annual interest.

21. How much money must be invested in 7% stock at 168 to obtain an annual income of \$1400?

22. Find the cost of a draft for \$2000, payable in 30 days, without grace, if exchange is $\frac{1}{8}$% and the rate of interest is 6%.

EXERCISE 209. — WRITTEN.

1. 150 is 25% more than what number?

2. 150 is 25% less than what number?

3. 510 is $6\frac{1}{4}$% more than what number?

4. 60 is $16\frac{2}{3}$% of what number?

5. A man sold a house for $3400, which was 15% less than the house cost him. What did the house cost him?

6. If cloth is bought at $2.75 a yard and sold for $3 a yard, what is the gain per cent?

7. What per cent is made by buying coal by the long ton and selling it by the short ton at the same price?

8. A broker gained 17% by selling a quantity of cotton for $1894.23. What did he pay for the cotton?

9. An agent sold 550 bushels of oats at 30 cents a bushel and charged $4.95 commission. What rate of commission did he charge?

10. Find the interest on $200 for 2 yr. 2 mo. 10 dy. at 6%.

11. What principal will produce $325 yearly at 5%?

12. Find the day of maturity, the proceeds and bank discount of a note for $2500, dated July 1, 1896, payable in 4 months, with grace, discounted July 15 at $5\frac{1}{2}$%.

13. Find the present worth of $3225 due in 6 months, without grace, money being worth 6%.

14. Find the cost of 100 shares of stock at $137\frac{3}{8}$, brokerage $\frac{1}{8}$.

15. What number of shares of Old Colony Railroad stock does a man hold, if he receives $294 quarterly, the stock paying $1\frac{3}{4}$% quarterly?

16. A ship and cargo are insured at $4\frac{3}{8}\%$. The policy is for \$317,300 and covers the premium as well as the value of the ship and cargo. Find the value of the ship and cargo.

17. A cubic inch of iron weighs $4\frac{4}{9}$ oz. Find the weight in pounds of an iron bar 1 inch square at the end and 1 yard long.

NOTE. The answer to this problem gives the reason of the rule for finding the weight in pounds of an iron beam, namely, *multiply the number of square inches in a cross section by 10 and this product by the number of yards in the length of the beam.*

18. Find by this rule the weight of an iron beam 30 feet long, if its cross section contains 18 square inches.

19. If 80 men with the use of machinery can make as many boots and shoes as 500 men can make without machinery, what per cent of the labor of making boots and shoes is done by machinery?

20. If 7 men with machinery can make as much furniture as 11 men can make without machinery, what per cent of the labor of making furniture is done by machinery?

21. If 160 men with mining machines can mine as much coal as 500 men without machines, what per cent of the labor of mining coal is done by machinery?

22. If a workman can make three times as many tin cans by the use of machinery as he can make without machinery, what per cent of the labor of making tin cans is done by machinery?

23. If a weaver by a hand-loom can weave 70 picks of cotton a minute, and if he can tend 4 power-looms each weaving 180 picks a minute, what per cent of the labor of weaving cotton cloth is saved by the use of power-looms?

EXERCISE 210. — WRITTEN.

1. The cost of the necessaries of life at the present time (1896) is 10% lower than the cost in 1860, while wages are on the average 68.6% higher. If a man now receives $100 a month, how much money can he save in a year, supposing that he has the same occupation and the same standard of living as in 1860, and that he saved nothing in 1860?

2. If we take into account the number of hours men work now for an average day in comparison with the number in 1860, the advance in wages is 76.8%. If a carpenter received $1.50 a day in 1860, what would he receive now for a day of the same number of hours?

3. If a mason received $1.98 a day in 1860, how much will he receive for a day of the same length in 1896, reckoning the advance in wages at 76.8%?

4. When water is heated from the freezing-point to the boiling-point it expands $\frac{1}{24}$ in volume. If a cubic foot of water at the freezing-point weighs $62\frac{1}{2}$ pounds, what does a cubic foot of boiling water weigh?

5. If water expands $7\frac{1}{2}\%$ in freezing, find to the nearest pound the weight of a cubic foot of ice.

6. Find to the nearest pound the weight of the air in a room $36' \times 30' \times 16'$, if water is 770 times as heavy as air and weighs 1000 ounces a cubic foot.

7. There are 22 parts by weight of oxygen in every 100 parts of air. Find the weight of the oxygen in the room described in Example 6.

8. Find the proceeds of a bank note for $1250, payable in 90 days, without grace; discount $5\frac{1}{2}\%$.

9. By Mr. Atkinson's Tables, given in his "Science of Nutrition," 5 pounds of potatoes are equivalent in food

value to 1 pound of wheat flour. On this basis, when flour is $5 a barrel, what should be paid for a bushel of potatoes?

10. One pound of good meat (beef, mutton, or lean pork) is not greater in food value than $1\frac{1}{2}$ pounds of wheat flour. What ought to be paid for a pound of meat when flour is $5 a barrel, if only economy of living is regarded?

11. One hundred barrels of good flour can be bought in Boston for $400. The cost of converting this flour into good bread is $420. Each barrel will make 300 pounds of bread. If the bread is in one pound loaves and is sold for 4 cents a loaf, what per cent profit is made on the bread?

12. The average charge for transporting a ton a mile by the railroads of the United States is now $\frac{3}{4}$ of a cent; in 1860 the average charge was $2\frac{3}{4}$ cents. What per cent of the charge in 1860 is the charge now?

13. A shipowner insures his ship so that in case of loss he may recover the value of the ship and the premium. The vessel is valued at $128,700, and the premium is $2\frac{1}{2}$%. Find the amount for which he insures and the amount of the premium he pays.

14. Water weighs $62\frac{1}{2}$ pounds a cubic foot and milk $64\frac{1}{2}$ pounds. The specific gravity of water is reckoned 1. Find the specific gravity of milk.

15. By buying 7% stock I get 4% on my money. At what price did I buy the stock?

16. Find the value of U. S. 4's that will produce an income of $1500 a year.

17. The base of the Great Pyramid of Egypt is a square whose side is 500 Egyptian cubits long. Find the area in acres of the ground covered by the pyramid, reckoning a cubit at 18.24 inches.

APPENDIX.

Remark. Spanish Land Measures are for pupils in Texas. They can be adapted to pupils in California by changing the length of the vara from 33⅓ inches to 33 inches. The Vermont rule for partial payments and the Vermont method of levying taxes are for pupils in Vermont; the New Hampshire rule for partial payments is for pupils in New Hampshire; and the Connecticut rule for partial payments is for pupils in Connecticut.

The other topics explained here are of general interest, but are put in an Appendix because they cannot be said to belong to a course in Practical Arithmetic as the phrase is commonly understood. They are, however, convenient for reference, and they may be taken up by any pupils who have time to do so.

Spanish Land Measures.

429. In those sections of the United States that formerly belonged to Mexico, the Spanish Measures are much used in Land Surveying.

The vara is taken as the unit of measure.

The length of the vara varies in Mexico and in different sections of the United States.

In Mexico the length of the vara used is 32.9927 inches, in California 33 inches, in Texas 33⅓ inches.

In the problems given below its length will be taken as 33⅓ inches.

A Labor is a square, each side of which is 1000 varas.

A Square League is a square, each side of which is 5000 varas.

TABLE I.

1 vara = $33\frac{1}{3}$ inches.
3 varas = 100 inches.
36 varas = 100 feet.
108 varas = 100 yards.
1900.8 varas = 1 mile.

TABLE II.

1,000,000 square varas are 1 Labor = 177.136 acres.
25 Labors, or 25,000,000 square varas, are 1 Square League = 4428.4 acres.
5645.376 square varas are 1 acre.

EXERCISE 211. — WRITTEN.

1. Change 540 varas to yards; 864 varas to feet.

2. Change 324 varas to feet; 936 varas to yards.

3. Change 8246 varas to miles, rods, etc.

4. Change 8 mi. 122 rd. 13 ft. 6 in. to varas.

5. How many square varas are there in a rectangular lot 150 ft. long and 100 ft. deep?

6. Find the number of square varas in a rectangular field 76 rd. long and 56 rd. wide.

7. A township contains 36 sq. mi. How many Labors are there in a township? How many Square Leagues?

8. The distance by railway from Houston to El Paso is 833 miles. Express the distance in varas.

9. The distance between two cities is 180,576 varas. What is the distance in miles?

10. A rectangular field is 420 varas long and 330 varas wide. How many acres does the field contain?

Special Rules for Partial Payments.

The New Hampshire Rule.

430. When a note is written *with interest annually* or *with annual interest* the following is the **New Hampshire Rule:**

If in any year, reckoning from the time the annual interest began to accrue, payments have been made, compute interest upon them to the end of the year in which they are made.

The amount of the payments is to be then applied; first, to cancel interest upon annual interest; secondly, to cancel annual interest; thirdly, to diminish the principal.

If, however, at the date of any payment there is no interest except the accruing annual interest, and the payment, or the sum of the payments, does not exceed the annual interest at the end of the year, deduct the payment, or the sum of the payments, without interest on the same.

The Vermont Rule.

431. If we omit the last paragraph of the New Hampshire Rule, we have the **Vermont Rule.**

The Connecticut Rule.

432. The **Connecticut Rule** for partial payments is:

Follow the United States Rule if a year's interest or more is due at the time of a payment, and in case of the last payment.

If less than a year's interest is due at the time of a payment except the last, find for a new principal the difference between the amount of the principal for an entire year and the amount of the payment for the remainder of that year.

If the interest due at the time of a payment exceeds the payment, find the interest on the principal only.

1. Find, by the New Hampshire Rule, the amount due Sept. 22, 1896, on a note for $500 dated July 16, 1893, with interest annually at 6%, on which the following payments have been made: Oct. 16, 1894, $200; Dec. 16, 1895, $20.

Principal,		$500.00		
1st annual interest,			$30.00	
Int. on 1st annual int. for 1 yr.,				$1.80
2d annual interest,			30.00	
		$500.00	$60.00	$1.80
Payment Oct. 16, 1894,	$200			
Int. on payment July 16, 1895,	9			
Amt. of payment July 16, 1895,	$209 =	147.20 +	60.00 +	1.80
Principal July 16, 1895,		$352.80		
3d annual interest,			$21.17	
Payment Dec. 16, 1895,			20.00	
		$352.80 +	$1.17	
Principal July 16, 1896,		$353.97		
4th annual interest,		3.89		
Amt. due Sept. 22, 1896,		$357.86. *Ans.*		

2. Find, by the Vermont Rule, the amount due Sept. 22, 1896, of the note in Example 1.

From Example 1, the principal July 16, 1895, is $352.80

Principal July 16, 1895,		$352.80	
3d annual interest,			$21.17
Payment Dec. 16, 1895,	$20.00		
Int. on payment July 16, 1895,	0.70		
Amt. of payment July 16, 1895,	$20.70 =		20.70
		$352.80 +	$0.47
Principal July 16, 1896,		$353.27	
4th annual interest,		3.89	
Amt. due Sept. 22, 1896,		$357.16. *Ans.*	

3. Find, by the Connecticut Rule, the amount due Sept. 22, 1896, on a note for $500 given Apr. 10, 1892, bearing interest at 6%, which has the following endorsements:

June 16, 1893, $100 ; Dec. 28, 1893, $100 ; May 10, 1895, $15 ; Mar. 4, 1896, $200.

Principal,		$500.00
Int. on principal to June 16, 1893,		35.50
Amt. of principal June 16, 1893,		$535.50
Payment June 16, 1893,		100.00
New principal June 16, 1893,		$435.50
Int. on principal to June 16, 1894,		26.13
Amt. of principal June 16, 1894,		$461.63
Payment Dec. 28, 1893,	$100.00	
Int. on payment to June 16, 1894,	2.80	
Amt. of payment June 16, 1894,		$102.80
New principal June 16, 1894,		$358.83
Int. on principal to June 16, 1895,		21.53
Amt. of principal June 16, 1895,		$380.36
Payment May 10, 1895 (less than interest),		15.00
New principal June 16, 1895,		$365.36
Int. on principal to Mar. 4, 1896,		15.71
Amt. of principal Mar. 4, 1896,		$381.07
Payment Mar. 4, 1896,		200.00
New principal Mar. 4, 1896,		$181.07
Int. on principal to Sept. 22, 1896,		5.98
Amt. due Sept. 22, 1896,		$187.05

Exercise 212. — Written.

1. Find by the New Hampshire Rule and also by the Vermont Rule the amount due Sept. 22, 1896, on a note for $1750, dated June 6, 1892, with interest annually at 6%, which has the following endorsements: Aug. 12, 1893, $300 ; Dec. 23, 1893, $200 ; Jan. 15, 1895, $50 ; Apr. 23, 1896, $800.

2. Find by the Connecticut Rule the amount due Sept. 22, 1896, on a note for $1500, dated Aug. 9, 1892, with interest at 6%, which has the following endorsements: Mar. 17, 1893, $250 ; Apr. 19, 1894, $50 ; Sept. 21, 1895, $500 ; June 26, 1896, $600.

Vermont Taxes.

433. The **Grand List** of every municipality in Vermont consists of 1% of the assessed value of the real estate, 1% of the assessed value of the personal property less specified debts, and $2 for each taxable poll.

Taxes are often voted at a stated number of cents on a dollar on the grand list.

Thus, a town may vote 90 cents on a dollar of its grand list instead of voting a gross sum, as $20,000.

The General Assembly fixes the state and county taxes, and the minimum highway tax (at present 20% of the grand list).

NOTE. A discount of 4% of the tax is made if the tax is paid within 90 days after notice.

1. The town of M. has a grand list of $10,577. A tax of 60% was voted for town purposes, and the highway tax required by law was 20%. Find the gross amount of the levy.

SOLUTION. $10,577 is the grand list of the town.
60% + 20% = 80%, and 80% of $10,577 = $8461.60. *Ans.*

2. Mr. B.'s taxable real estate is $5627.50, and his taxable personal estate $3240.50. He lives in M. and pays a poll tax. Find his grand list and his tax.

SOLUTION.
Real estate = $5627.50
Personal estate = 3240.50
$8868.00

1% of $8868 = $88.68;
and $88.68 + $2 poll = $90.68, B.'s grand list. *Ans.*
80% of $90.68 = $72.54, B.'s tax. *Ans.*

EXERCISE 213. — WRITTEN.

1. The city of R. raises taxes on its grand list of $91,450 as follows: 120% city tax, 18% state tax, 20% highway

tax. What is the amount of each tax? The total tax? What will the total tax amount to if a discount of 4% is allowed for prompt payment, and if 2% of the tax is uncollectable?

2. J. C. lives in R., pays no poll tax, has real estate of $35,000, personal estate of $17,500, and debts of $6500. What will his tax be if he takes advantage of the 4% discount? What is his grand list?

3. The grand list of the town of B. is $6328. Its taxes are 75% for town purposes, 5% special for highways. What is the gross amount of the town taxes?

4. Mrs. G. lives in B. Her personal property is $875, her real estate $570. What is her grand list? Her tax?

5. The grand list of H. is $39,551. A tax of 75% for town purposes was voted at the annual meeting, and the highway tax required by law was 20%. Find the gross amount of the tax.

Foreign Exchange.

434. Exchange for sight drafts, July 1, 1896, was quoted as follows:

On London at $4.89 for 1 pound sterling.

On Paris at 5.15 francs for $1.

On cities in Germany at 4 reichsmarks for $0.96.

On Amsterdam at 1 guilder for $0.40¼.

(Form of Draft.)

£500. New York, July 1, 1896.

At sight of this First of Exchange (Second and Third of the same tenor and date unpaid) pay to the order of James Patterson five hundred pounds, value received, and charge to account of

To Baring Brothers, London. BLISS AND MORTON.

NOTE. Foreign bills are generally drawn in sets of three, of the same tenor and date, called *First*, *Second*, and *Third* of Exchange. These are sent by different mails to avoid loss or delay. When one is accepted or paid the others are void.

1. Find the cost of a draft on London for £500 12s. 6d.

$$6\text{d.} = 0.5\text{s.},\ 12.5\text{s.} = £0.625. \quad \therefore £500\ 12\text{s.}\ 6\text{d.} = £500.625.$$
$$500.625 \times \$4.89 = \$2448.06. \quad \textit{Ans.}$$

2. Find the cost of a draft on Paris for 2800 francs.

$$2800 \div 5.15 = 543.69. \quad \$543.69. \quad \textit{Ans.}$$

3. Find the cost of a draft on Frankfort for 6400 reichsmarks.

$$4 \text{ reichsmarks} = \$0.96. \quad \therefore 1 \text{ reichsmark} = \$0.24.$$
$$6400 \times \$0.24 = \$1536. \quad \textit{Ans.}$$

4. Find the cost of a draft on Amsterdam for 3600 guilders.

$$3600 \times \$0.40\tfrac{1}{4} = \$1449. \quad \textit{Ans.}$$

EXERCISE 214. — WRITTEN.

Find the cost of a draft on :

1. London for £2500.
2. London for £1574 8s. 8d.
3. Paris for 3472 francs.
4. Hamburg for 4276 reichsmarks.
5. Amsterdam for 2765 guilders.
6. London for £472 16s. 6d.
7. Dresden for 5872 reichsmarks.
8. Liverpool for £4842 2s. 10d.
9. Rotterdam for 6857 guilders.
10. Paris for 8872 francs.
11. Berlin for 6487 reichsmarks.
12. Paris for 4679 francs.

Arithmetical Progression.

435. **An Arithmetical Progression** is a series of numbers that increase or decrease by a common difference.

Thus, the numbers 6, 9, 12, 15, form an arithmetical progression with a common difference 3.

The several numbers of a progression are called its **terms.**

436. To find any term of an arithmetical progression.

In the arithmetical progression

1st	2d	3d	4th	5th	6th
4,	7,	10,	13,	16,	19,

any term, as for example the 6th, is found by adding to the first term the product of the common difference by a number 1 less than the number of the term: $4 + (3 \times 5)$, or 19.

If the series is a decreasing series, we subtract from the first term the product of the common difference by a number 1 less than the number of the term. Thus, the 12th term of the series 50, 47, 44, 41, etc., is $50 - (3 \times 11)$, or 17.

Therefore, we have the following rule for finding any term of an arithmetical progression:

Multiply the common difference by a number that is 1 less than the number of the required term. Add this product to the first term if the series is an increasing series; subtract this product from the first term if the series is a decreasing series.

Exercise 215. — Written.

1. Find the 10th term of 2, 6, 10, 14, etc.
2. Find the 6th term of 63, 58, 53, 48, etc.
3. Find the 8th term of 2, 9, 16, 23, etc.
4. Find the 9th term of 1, 8, 15, 22, etc.
5. Find the 7th term of 100, 92, 84, 76, etc.
6. Find the 18th term of 9, 13, 17, 21, etc.

437. To find the sum of the terms of an arithmetical progression.

The sum of seven terms of the series 1, 3, 5, etc., is

$$\begin{array}{lr}
 & 1+\ 3+\ 5+\ 7+\ 9+11+13 \\
\text{in reverse order is} & \underline{13+11+\ 9+\ 7+\ 5+\ 3+\ 1} \\
\text{Hence, } \textit{twice the sum} \text{ is} & 14+14+14+14+14+14+14, \\
\text{or} & 7 \times 14. \qquad\qquad\qquad\qquad
\end{array}$$

Therefore, the *sum* is $\frac{1}{2}(7 \times 14)$, or 49.

Here 7 is the *number of terms,* and 14 is the *sum of the first and last terms.* Therefore, we have the following rule for finding the sum of an arithmetical progression:

Take half the product of the number of terms by the sum of the first and last terms.

Find the sum of nine terms of an arithmetical progression whose first term is 3 and last term 41.

$$\frac{9 \times (3 + 41)}{2} = 198. \ \textit{Ans.}$$

Exercise 216. — Written.

Find the sum of:

1. 3, 6, 9, 12, to 8 terms.
2. 7, 9, 11, 13, to 15 terms.
3. 5, 14, 23, 32, to 7 terms.
4. 1, 7, 13, 19, to 6 terms.
5. How many times in a day does a clock strike that strikes the hours only?
6. In a potato race 30 potatoes are placed in a straight line 3 ft. distant from each other. A boy, starting from a basket 3 ft. from the first potato, is required to pick them up one by one and carry them to the basket. To finish the race, how far must the boy run?
7. A body falls through a space of $16\frac{1}{12}$ ft. in the first second of its fall, and in each succeeding second $32\frac{1}{6}$ ft. more than in the second just before. How far will a body fall in 12 seconds?

8. A body falls through a space of 4.9^m in the first second of its fall, and in each succeeding second 9.8^m more than in the second just before. A stone dropped from a balloon was 42 seconds in reaching the ground. How high was the balloon?

Geometrical Progression.

438. A **Geometrical Progression** is a series of numbers in which each term after the first is obtained by multiplying the preceding term by a constant multiplier, called the *ratio* of the progression.

Thus, the numbers 2, 6, 18, 54, etc., form a geometrical progression, since each term after the first is three times the preceding term; and the numbers 27, 9, 3, 1, etc., form a geometrical progression, since each term after the first is one third of the preceding term.

439. To find any term of a geometrical progression.

In the geometrical progression

1st	2d	3d	4th	5th	
2,	6,	18,	54,	162,	etc.,

the 2d term is 2×3; the 3d term is 2×3^2; the 4th term is 2×3^3; the 5th term is 2×3^4; and so on. Therefore, we have the following rule for finding any term of a geometrical progression:

Raise the ratio to a power that is 1 less than the number of the required term, and multiply the result by the first term.

EXERCISE 217. — WRITTEN.

1. Find the 6th term of 1, 2, 4, etc.
2. Find the 5th term of 2, 8, 32, etc.
3. Find the 7th term of 5, 15, 45, etc.
4. Find the 5th term of 7, 35, 175, etc.
5. Find the 6th term of 6, 36, 216, etc.
6. Find the 6th term of 64, 32, 16, etc.

440. To find the sum of the terms of a geometrical progression.

In the geometrical progression 2, 6, 18, 54, 162, etc., the sum of five terms is $2+6+18+54+162$.

If we multiply this sum by the ratio 3, and from the product subtract the sum of the five terms we have :

$$\begin{array}{r} 6+18+54+162+486 \\ \underline{2+6+18+54+162} \\ 486-2, \end{array}$$ or three times the sum minus the sum.

That is, *twice* the sum $=486-2$.

Therefore, the sum $=\dfrac{486-2}{2}$.

The numerator of the fraction is the difference between the product of the last term by the ratio and the first term; the denominator is the difference between the ratio and 1.

Therefore, we have the following rule for finding the sum of a given geometrical progression:

Multiply the last term by the ratio, and subtract the first term from the product. Divide the remainder by the ratio minus 1.

If the ratio is less than 1,

Multiply the last term by the ratio, and subtract the product from the first term. Divide the remainder by 1 minus the ratio.

EXERCISE 218. — WRITTEN.

Find the sum of :

1. 2, 4, 8, to 8 terms.
2. 1, 3, 9, to 6 terms.
3. 3, 12, 48, to 7 terms.
4. 64, 32, 16, to 8 terms.

5. A person saved in one year \$36, and in each succeeding year for 6 years more $1\frac{1}{2}$ times as much as in the preceding year. Find the whole amount saved.

6. Find the sum of $\frac{1}{5}$, $\frac{1}{15}$, $\frac{1}{45}$, to 5 terms.

Compound Interest Problems.

Table I.

441. *Showing the amount of $1 at compound interest for:*

YR.	2 PER CENT.	2½ PER CENT.	3 PER CENT.	3½ PER CENT.	4 PER CENT.
1	1.02000	1.02500	1.03000	1.03500	1.04000
2	1.04040	1.05063	1.06090	1.07123	1.08160
3	1.06121	1.07689	1.09273	1.10872	1.12486
4	1.08243	1.10381	1.12551	1.14752	1.16986
5	1.10408	1.13141	1.15927	1.18769	1.21665
6	1.12616	1.15969	1.19405	1.22926	1.26532
7	1.14869	1.18869	1.22987	1.27228	1.31593
8	1.17166	1.21840	1.26677	1.31681	1.36857
9	1.19509	1.24886	1.30477	1.36290	1.42331
10	1.21899	1.28009	1.34392	1.41060	1.48024
11	1.24337	1.31209	1.38423	1.45997	1.53945
12	1.26824	1.34489	1.42576	1.51107	1.60103
13	1.29361	1.37851	1.46853	1.56396	1.66507
14	1.31948	1.41297	1.51259	1.61870	1.73168
15	1.34587	1.44830	1.55797	1.67535	1.80094
16	1.37279	1.48451	1.60471	1.73399	1.87298
17	1.40024	1.52162	1.65285	1.79468	1.94790
18	1.42825	1.55966	1.70243	1.85749	2.02582
19	1.45681	1.59865	1.75351	1.92250	2.10685
20	1.48595	1.63862	1.80611	1.98979	2.19112

YR.	4½ PER CENT.	5 PER CENT.	5½ PER CENT.	6 PER CENT.	7 PER CENT.
1	1.04500	1.05000	1.05500	1.06000	1.07000
2	1.09203	1.10250	1.11303	1.12360	1.14490
3	1.14117	1.15763	1.17424	1.19102	1.22504
4	1.19252	1.21551	1.23882	1.26248	1.31080
5	1.24618	1.27628	1.30696	1.33823	1.40255
6	1.30226	1.34010	1.37884	1.41852	1.50073
7	1.36086	1.40710	1.45468	1.50363	1.60578
8	1.42210	1.47746	1.53469	1.59385	1.71819
9	1.48610	1.55133	1.61909	1.68948	1.83846
10	1.55297	1.62889	1.70814	1.79085	1.96715
11	1.62285	1.71034	1.80209	1.89830	2.10485
12	1.69588	1.79586	1.90121	2.01220	2.25219
13	1.77220	1.88565	2.00577	2.13293	2.40985
14	1.85194	1.97993	2.11609	2.26090	2.57853
15	1.93528	2.07893	2.23248	2.39656	2.75903
16	2.02237	2.18287	2.35526	2.54035	2.95216
17	2.11338	2.29202	2.48480	2.69277	3.15882
18	2.20848	2.40662	2.62147	2.85434	3 37993
19	2.30786	2.52695	2.76565	3.02560	3.61653
20	2.41171	2.65330	2.91776	3.20714	3.86968

442. The table on the preceding page shows the amount of $1 at compound interest at various per cents for from 1 to 20 years. The compound interest on $1 is found by subtracting 1 from the proper number shown in the table.

1. What principal will in 10 yr. compound interest at 6% yield $1898.04 interest?

SOLUTION. The interest on $1 for 10 yr. at 6% is $0.79085.
Since $0.79085 is the interest on $1,
$1898.04 is the interest on $\$\frac{1898.04}{0.79085}$, or $2400. *Ans.*

2. In what time will $1600 at 4½% compound interest yield $1000 interest?

SOLUTION. Since $1600 yields $1000, $1 will yield $\frac{1}{1600}$ of $1000, or $0.625, in the same time, and $1 will amount to $1.625. By the table, $1 will in 11 yr. at 4½% amount to $1.62285, and in 12 yr. to $1.69588. Hence, the required time is a little more than 11 yr.

3. At what rate, compound interest, will $1500 yield $1201.41 interest in 15 yr.?

SOLUTION. Since $1500 yields $1201.41 interest in 15 yr., $1 in 15 yr. will yield $\frac{1}{1500}$ of $1201.41, or $0.80094, and $1 will amount in 15 yr. to $1.80094. In the table, opposite 15 yr., we find in the 4% column the amount of $1 is $1.80094. Therefore, the rate is 4%. *Ans.*

EXERCISE 219. — WRITTEN.

1. Find the compound interest at 6% for 15 yr. on $1287.62.

2. Find the compound interest at 3½% for 12 yr. 6 mo. on $686.70.

3. Find the amount of $960 for 9 yr. at 5%, interest compounded semi-annually.

4. Find the amount of $1216 for 5 yr. at 8%, interest compounded quarterly.

5. What principal will in 14 yr. at 5½% compound interest amount to $1880.78?

6. At what rate compound interest will $2200 yield $4749.40 interest in 17 yr.?

7. In what time will $1800 at 5% compound interest amount to $3000?

8. In what time will $962.44 yield $1080.72 interest at 6% compound interest?

9. What principal will in 20 yr. at 4% compound interest yield $2500 interest?

10. At what rate compound interest will $462.50 yield $277.98 interest in 12 yr.?

11. What principal will in 10 yr. at 6% amount to $3612.22, interest being compounded semi-annually?

12. In what time at 5% will $1250 amount to $2000, interest being compounded semi-annually?

13. At what rate per annum will $500 amount to $779.83 in 9 yr., interest being compounded semi-annually?

Annuities.

443. An **Annuity** is a sum of money to be paid at regular intervals of time, as years, half years, quarter years.

A **Perpetual Annuity** is an annuity that continues forever.

A **Certain Annuity** is an annuity that begins and ends at a specified time.

A **Contingent Annuity** is an annuity that depends upon some particular event, as the death of an individual. Life insurance, dowers, and pensions are examples of contingent annuities.

An **Annuity in Reversion** is an annuity that begins at some future time.

An **Annuity in Arrears** is an annuity on which the payments remain unpaid after they have become due.

The **Final Value** of an annuity is the sum to which all its payments at compound interest will amount at the end of the annuity.

The **Present Value** of an annuity is the sum which at compound interest will amount to its final value.

1. Find the present value of an annuity for $500 for 5 yr. at 4%.

SOLUTION. The present value of $1 for 5 yr. at 4% by Table II, page 353, is $4.45182; and of $500 is 500 × $4.45182, or $2225.91.

2. A person 41 years of age pays $9797.75 for a life annuity. If interest is reckoned at 4% find the amount of the annuity?

SOLUTION. According to Table III, page 354, the expectancy of life for a person 41 years of age is about 27 years.

The present value of an annuity of $1 for 27 yr. at 4% is by Table II, $16.32959. Hence, the annuity is $\$\frac{9797.75}{16.32959}$, or $600. *Ans.*

Therefore, we have the following rules:

To find the present value of a given annuity,

Multiply the present value of $1 for the given time and rate shown in Table II by the given annuity.

To find an annuity from its present value,

Divide its present value by the present value of an annuity of $1 for the given time and rate shown in Table II.

EXERCISE 220. — WRITTEN.

1. A person 22 years old has a life annuity of $600. Find its present value at 4%.

2. A person 35 years old has a life annuity of $1000. Find its present value at 4%.

3. A person 53 years old has a life annuity of $300. Find its present value at 4%.

4. A person 75 years old has a life annuity of \$2000. Find its present value at $3\frac{1}{2}\%$.

5. A person 22 years old pays \$4948.19 for a life annuity. If interest is 4%, find the amount of the annuity.

6. A person 29 years old pays \$7465.84 for a life annuity. If interest is 4%, find the amount of the annuity.

7. A person 35 years old pays \$9368.14 for a life annuity. If interest is $3\frac{1}{2}\%$, find the amount of the annuity.

8. A person 44 years old pays \$5933.35 for a life annuity. If interest is $3\frac{1}{2}\%$, find the amount of the annuity.

TABLE II.

Showing the present value of an annuity of \$1 per annum, at compound interest from 1 to 40 years at $3\frac{1}{2}\%$ and at 4%.

YR.	3½ PER CENT.	4 PER CENT.	YR.	3½ PER CENT.	4 PER CENT.
1	0.96618	0.96154	21	14.69797	14.02916
2	1.89969	1.88610	22	15.16713	14.45112
3	2.80164	2.77509	23	15.62041	14.85684
4	3.63708	3.62990	24	16.05837	15.24696
5	4.51505	4.45182	25	16.48152	15.62208
6	5.32855	5.24214	26	16.89035	15.98277
7	6.11454	6.00206	27	17.28537	16.32959
8	6.87396	6.73275	28	17.66702	16.66306
9	7.60769	7.43533	29	18.03577	16.98372
10	8.31661	8.11090	30	18.39205	17.29203
11	9.00155	8.76048	31	18.73628	17.58849
12	9.66333	9.38507	32	19.06887	17.87355
13	10.30274	9.98565	33	19.39021	18.14765
14	10.92052	10.56312	34	19.70068	18.41120
15	11.51741	11.11839	35	20.00066	18.66461
16	12.09412	11.65230	36	20.29049	18.90828
17	12.65132	12.16570	37	20.57053	19.14258
18	13.18968	12.65930	38	20.84109	19.36786
19	13.70984	13.13394	39	21.10250	19.58449
20	14.21240	13.59033	40	21.35507	19.79277

TABLE III.

Carlisle Table of expectancy of life.

AGE.	EX-PECTANCY.	AGE.	EX-PECTANCY.	AGE.	EX-PECTANCY.	AGE.	EX-PECTANCY.
0	38.72	26	37.14	52	19.68	78	6.12
1	44.68	27	36.41	53	18.97	79	5.80
2	47.55	28	35.69	54	18.28	80	5.51
3	49.82	29	35.00	55	17.58	81	5.21
4	50.76	30	34.34	56	16.89	82	4.93
5	51.25	31	33.68	57	16.21	83	4.65
6	51.17	32	33.03	58	15.55	84	4.39
7	50.80	33	32.36	59	14.92	85	4.12
8	50.24	34	31.68	60	14.34	86	3.90
9	49.57	35	31.00	61	13.82	87	3.71
10	48.82	36	30.32	62	13.31	88	3.59
11	48.04	37	29.64	63	12.81	89	3.47
12	47.27	38	28.96	64	12.30	90	3.28
13	46.51	39	28.28	65	11.79	91	3.26
14	45.75	40	27.61	66	11.27	92	3.37
15	45.00	41	26.97	67	10.75	93	3.48
16	44.27	42	26.34	68	10.23	94	3.53
17	43.57	43	25.71	69	9.70	95	3.53
18	42.87	44	25.09	70	9.18	96	3.46
19	42.17	45	24.46	71	8.65	97	3.28
20	41.46	46	23.82	72	8.16	98	3.07
21	40.75	47	23.17	73	7.72	99	2.77
22	40.04	48	22.50	74	7.33	100	2.28
23	39.31	49	21.81	75	7.01	101	1.79
24	38.59	50	21.11	76	6.69	102	1.30
25	37.86	51	20.39	77	6.40	103	0.83

The above table shows the average number of years of life due to persons at the ages indicated. This table is known as the Carlisle Table, because based upon the rate of mortality, as carefully observed at Carlisle, England.

Repeating Decimals.

444. If the denominator of a common fraction in its lowest terms contains only the factors 2 and 5, the fraction can be expressed *exactly* by a decimal fraction; otherwise it cannot.

Thus, if we take the fraction $\frac{3}{11}$ to express as a decimal, we have 0.27272727..... and the division will never end, however far it is carried.

445. A decimal that contains a *constantly recurring figure or series of figures* is called a **repeating decimal** or a **circulating decimal.**

Thus, the decimal 0.27272727..... is a repeating decimal, the figures constantly recurring being 27.

446. The figure or series of figures that constantly recurs is called the **repetend.**

447. If the repetend begins at the first place in the decimal, the decimal is called a *pure* repeating decimal; otherwise it is called a *mixed* repeating decimal.

448. In writing a repeating decimal, we stop with the last figure of the repetend and place dots over the first and last figures.

Thus, we write 0.272727..... $0.\dot{2}\dot{7}$, and we write 0.333..... $0.\dot{3}$.

449. The repetend may be made to begin at any place we please after the first repeating figure.

Thus, $4.\dot{1}3548548$..... may be written $4.1\dot{3}54\dot{8}$, or $4.13\dot{5}48\dot{5}$, or $4.135\dot{4}85\dot{4}$, etc.

450. Sometimes the repetend takes in one or more figures of the *integral* part of the number.

Thus, $76\dot{5}.4\dot{3}$ is the same as $765.\dot{4}3\dot{5}$, or as $765.4\dot{3}5\dot{4}$, etc.

To change a repeating decimal to a common fraction.

1. Change $0.\dot{2}\dot{7}$ to a common fraction.

SOLUTION. From 100 times the decimal, or 27.2727.....,
take 1 time the decimal, or 0.2727.....
Then 99 times the decimal is 27.

Therefore, the decimal is equal to $\frac{27}{99}$, or $\frac{3}{11}$. *Ans.*

2. Change $0.5\dot{2}4\dot{3}$ to a common fraction.

SOLUTION. $0.5\dot{2}4\dot{3} = 0.5\frac{243}{999} = 0.5\frac{9}{37} = \frac{5\frac{9}{37}}{10} = \frac{194}{370} = \frac{97}{185}$. *Ans.*

451. Hence, we have the following rule for changing a repeating decimal to a common fraction:

Write the repetend for the numerator and as many 9's for the denominator as there are figures in the repetend.

EXERCISE 221. — WRITTEN.

Express as a common fraction or a mixed number:

1. $0.\dot{7}\dot{2}$.	4. $0.27\dot{7}\dot{2}$.	7. $0.11\dot{3}\dot{6}$.
2. $0.\dot{3}5\dot{1}$.	5. $0.02\dot{0}\dot{3}$.	8. $2.1\dot{3}3\dot{6}$.
3. $2.0\dot{8}\dot{1}$.	6. $0.7\dot{2}8\dot{3}$.	9. $4.\dot{3}16\dot{2}$.

Scales of Notation.

452. The common mode of representing numbers is called the **common scale of notation,** and 10 is called its **radix** or **base.**

453. In the common or decimal scale every figure placed to the left of another represents *ten* times as much as if it were in the place of that other.

454. Instead of the radix number 10, any other integral number might be used as the base of a system of notation.

Thus, the number 6532 stands for:

In the scale of 10, $6 \times 10^3 + 5 \times 10^2 + 3 \times 10 + 2$.
In the scale of 8, $6 \times 8^3 + 5 \times 8^2 + 3 \times 8 + 2$.
In the scale of 7, $6 \times 7^3 + 5 \times 7^2 + 3 \times 7 + 2$.

455. A given number can be changed from one scale to another scale.

1. Express 6532 in the scale of 6.

SOLUTION. The quotients and remainders of the successive divisions by 6 are as follows:

$$\begin{array}{r|l}
6 & 6532 \\
6 & 1088 \text{ remainder } 4. \\
6 & 181 \text{ remainder } 2. \\
6 & 30 \text{ remainder } 1. \\
 & 5 \text{ remainder } 0.
\end{array}$$

Thus, 6532 expressed in the scale of 6 is 50,124.

2. Change 50,124 from the scale of 6 to the scale of 8.

SOLUTION.

8 | 50124
8 | 3440 remainder 4.
8 | 250 remainder 0.
8 | 20 remainder 6.
1 remainder 4.

Therefore, the number required is 14,604.

Since 50,124 is in the scale of 6, each figure is *six* times the value it would have one place to the right. Hence, at the beginning we have to divide $6 \times 5 + 0$ by 8, and we get 3 for the quotient and 6 for the remainder. The next partial dividend is $6 \times 6 + 1$, or 37, and this divided by 8 gives 4 for the quotient and 5 for the remainder. The next partial dividend is $6 \times 5 + 2$, or 32, and this divided by 8 gives 4 for the quotient and 0 for the remainder; and so on.

3. Change 14,604 from the scale of 8 to the scale of 10.

SOLUTION.

10 | 14604
10 | 1215 remainder 2.
10 | 101 remainder 3.
6 remainder 5.

Therefore, the required number is 6532.

4. Add 56,432 and 15,646 (scale of seven).

56432
15646
105411

SOLUTION. The process differs from that in the decimal scale only in that when a sum greater than *seven* is reached, we divide by *seven* (not ten), set down the remainder, and add the quotient with the next column.

5. Subtract 34,561 from 61,235 (scale of eight).

61235
34561
24454

SOLUTION. When the number of any order of units in the minuend is less than the number of the corresponding order in the subtrahend, we increase the number in the minuend by eight instead of ten as in the common scale.

6. Multiply 5732 by 428 (scale of nine).

5732
428
51477
12564
25238
2712127

SOLUTION. We divide each partial product by *nine*, set down the remainder, and add the quotient to the next partial product.

7. Divide 2,712,127 by 5732 (scale of nine).

$$\begin{array}{r} 428 \\ 5732\,\overline{)\,2712127} \\ \underline{25238} \\ 17722 \\ \underline{12564} \\ 51477 \\ \underline{51477} \end{array}$$

The operations of multiplication and subtraction involved in this problem are precisely the same as in the decimal notation. The only difference is that the radix number is 9 instead of 10.

Exercise 222. — Written.

Change 3863 of the common scale to:

1. The scale of 7.
2. The scale of 8.
3. The scale of 3.
4. The scale of 5.
5. The scale of 2.
6. The scale of 6.

Change to the common scale:

7. 41,543 of the scale of 7.
8. 41,543 of the scale of 6.
9. 41,543 of the scale of 8.
10. 41,543 of the scale of 9.

Perform the following arithmetical processes:

11. Add 2314, 4314, 3423 (scale of five).
12. Add 6438, 6347, 6725 (scale of nine).
13. Subtract 37,541 from 75,243 (scale of eight).
14. Subtract 15,542 from 31,423 (scale of six).
15. Multiply 6341 by 53 (scale of seven).
16. Multiply 5864 by 483 (scale of nine).
17. Divide 12,432 by 36 (scale of seven).

Average of Payments, when the Terms of Credit begin at Different Dates.

456. Find the average time of payment of the following debts: April 14, 1896, \$200 due in 2 months; June 17, 1896, \$300 due in 3 months; June 22, 1896, \$350 due in 3 months.

SOLUTION.

\$200 due June 14,		\$200		
\$300 due Sept. 17,	95 ×	300	=	\$28,500
\$350 due Sept. 22,	100 ×	350	=	35,000
		\$850		\$63,500

$$63{,}500 \div 850 = 74\tfrac{60}{85}.$$

June 14, 1896 + 75 dy. = Aug. 28, 1896.

The earliest date that any debt is due is June 14, \$200. The \$300 is due 95 days after, and the \$350 is due 100 days after. Proceeding as in § 381, we find the average term of credit to be $74\frac{60}{85}$ days, or 75 days after June 14; that is, Aug. 28. *Ans.*

Therefore, we have the following rule:

Select the earliest date that any debt becomes due as the standard date.

Multiply each of the other debts by the number of days from the standard date that it becomes due and divide the sum of the products by the sum of the debts.

The quotient is the number of days that must be added to the standard date to find the average time of the payments.

EXERCISE 223. — WRITTEN.

Find the average time of payment of the following debts:

1. Mar. 12, 1896, \$150 due in 3 months; Apr. 16, 1896, \$100 due in 2 months; May 19, 1896, \$125 due in 4 months.

2. Apr. 7, 1896, \$625 due in 30 days; Apr. 17, 1896, \$450 due in 90 days; Apr. 27, 1896, \$575 due in 60 days.

3. May 5, 1896, \$375 due in 4 months; May 28, 1896, \$115 due in 1 month; June 27, 1896, \$225 due in 2 months.

Savings Banks Accounts.

457. Savings banks receive money on deposit, and pay depositors compound interest, adding the interest to the principal every three months, six months, or twelve months.

458. The interval between the dates at which interest is computed is called an **Interest Term.**

Interest is added at the end of every interest term, computed on the smallest balance on deposit at any time during the whole interest term.

Each depositor has a bank book, in which is recorded every sum deposited, every sum withdrawn, and the interest due at the end of each interest term.

Find the balance on deposit Oct. 1, 1896, on the following account, interest being 4%, reckoned quarterly:

Deposited Jan. 1, 1896, $50; Feb. 4, 1896, $40; May 6, 1896, $60; Aug. 4, 1896, $40.

Withdrawn Mar. 3, 1896, $20; Apr. 22, 1896, $30; June 19, 1896, $25; Sept. 22, 1896, $40.

STATEMENT.

DATE.	DEPOSITED.		WITHDRAWN.		INTEREST.		BALANCE.	
1896.								
Jan. 1,	$50	00					$50	00
Feb. 4,	40	00					90	00
Mar. 3,			$20	00			70	00
Apr. 1,					$0	50	70	50
Apr. 22,			30	00			40	50
May 6,	60	00					100	50
June 19,			25	00			75	50
July 1,					0	40	75	90
Aug. 4,	40	00					115	90
Sept. 22,			40	00			75	90
Oct. 1,					0	76	76	66

The smallest sum on deposit during the first interest term was $50. The interest on $50 for 3 mo. at 4% is $0.50, which, added to the balance on deposit, makes $70.50.

The smallest sum on deposit during the second interest term was $40.50. The interest on $40.50 for 3 mo. at 4% is $0.40, which, added to the balance on deposit, makes $75.90.

The smallest sum on deposit during the third interest term was $75.90. The interest on $75.90 for 3 mo. at 4% is $0.76, which, added to the balance on deposit Oct. 1, 1896, makes $76.66.

Exercise 224. — Written.

Find the balance on deposit Jan. 1, 1897, on the following account:

1. Interest being 4%, computed quarterly. Deposited Jan. 1, 1896, $100; Mar. 14, 1896, $25; May 11, 1896, $28.50; Sept. 10, 1896, $85; Oct. 14, 1896, $75. Withdrawn Apr. 15, 1896, $62; June 19, 1896, $40; Aug. 3, 1896, $20; Oct. 28, 1896, $17; Dec. 18, 1896, $75.

2. Interest being 3%, computed quarterly. Deposited Jan. 1, 1896, $75; Feb. 24, 1896, $50; June 23, 1896, $40; Oct. 24, 1896, $60; Nov. 18, 1896, $48. Withdrawn Apr. 24, 1896, $30; July 2, 1896, $80; Aug. 4, 1896, $27; Dec. 3, 1896, $12; Dec. 22, 1896, $72.

3. Interest being 4%, computed semi-annually. Deposited Jan. 1, 1896, $150; Mar. 23, 1896, $80; May 30, 1896, $65; Aug. 11, 1896, $80; Oct. 5, 1896, $90. Withdrawn Apr. 22, 1896, $40; Sept. 22, 1896, $60; Nov. 5, 1896, $18; Nov. 25, 1896, $53; Dec. 23, 1896, $108.

4. Interest being 4%, computed annually. Deposited Jan. 1, 1896, $60; May 11, 1896, $82; Aug. 3, 1896, $50; Aug. 29, 1896, $50; Sept. 12, 1896, $100. Withdrawn Mar. 2, 1896, $8; Oct. 17, 1896, $16; Nov. 18, 1896, $25; Dec. 10, 1896, $40; Dec. 24, 1896, $38.

Settlement of Accounts.

459. Find the balance due of the following account on Sept. 10, 1896, by computing the interest at 6% on each item from its date to the day of settlement, reckoning the time in days:

1896.		Dr.	Int.	1896.		Cr.	Int.
June 29.	To Mdse.	$250	$3.04	July 3.	By Cash.	$200	$2.30
July 13.	"	400	3.93	" 17.	"	125	1.15
" 27.	"	500	3.75	" 31.	"	350	2.39
				Sept. 10.	By bal. acct.	475	
Settled Sept. 10, '96.				" 10.	" int.		4.88
		$1150	$10.72			$1150	$10.72

Hence, the cash balance is $475 + $4.88, or $479.88.

Note. When the balance of account and the balance of interest fall on *opposite* sides, the cash balance is their *difference*.

Exercise 225. — Written.

Find the cash balance, Sept. 10, 1896, of the following accounts, reckoning interest at 6%:

1.

1896.		Dr.	1896.		Cr.
May 12.	To Mdse.	$250.00	May 26.	By Cash.	$200.00
" 28.	"	610.00	June 22.	"	500.00
June 16.	"	300.00	" 30.	"	400.00

2.

1896.		Dr.	1896.		Cr.
Mar. 7.	To Mdse.	$350.00	Apr. 3.	By Cash.	$150.00
Apr. 10.	"	98.50	May 2.	"	150.00
May 25.	"	300.00	June 4.	"	200.00

3.

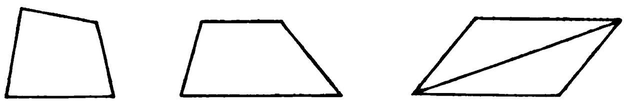
1896.		Dr.	1896.		Cr.
May 8.	To Mdse.	$250.00	June 22.	By Cash.	$200.00
June 5.	"	670.00	July 21.	"	500.00
July 3.	"	200.00	Aug. 19.	"	300.00

Supplementary Measurements.

460. Two lines are **parallel** if they do not meet however far produced.

461. A **Quadrilateral** is a plane figure bounded by four straight lines.

462. The **Diagonal** of a quadrilateral is a straight line joining two opposite corners.



Trapezium. Trapezoid. Parallelogram.

463. A **Trapezium** is a quadrilateral with no two of its sides parallel.

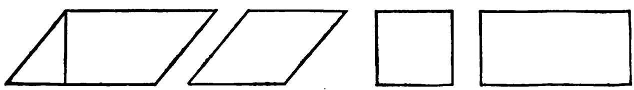
464. A **Trapezoid** is a quadrilateral with two of its sides parallel, but the other two sides not parallel.

465. A **Parallelogram** is a quadrilateral with its opposite sides parallel.

466. A **Rhomboid** is a parallelogram with its angles not right angles.

467. A **Rhombus** is a parallelogram with its angles not right angles, but with all its sides equal.

Parallelograms.



Rhomboid. Rhombus. Square. Rectangle.

468. The **altitude of a parallelogram or of a trapezoid** is the shortest distance between its parallel sides regarded as *bases.*

469. The **area of any parallelogram** is the product of its base by its altitude.

470. The **area of a rhombus** is also half the product of its diagonals.

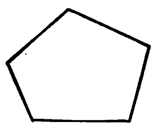
471. The **area of a trapezoid** is half the product of its altitude by the sum of its bases.

472. The **area of a trapezium** is the sum of the areas of the two triangles formed by drawing a diagonal.

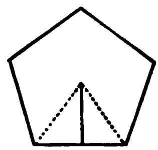
473. A **Polygon** is a plane figure bounded by straight lines.

A polygon of *three* sides is a *triangle;* of *four* sides, a *quadrilateral;* of *five* sides, a *pentagon;* of *six* sides, a *hexagon;* of *eight* sides, an *octagon;* of *ten* sides, a *decagon;* of *twelve* sides, a *dodecagon;* and so on.

474. The **area of any polygon** is found by dividing it into triangles and finding the sum of their areas.



Polygon.



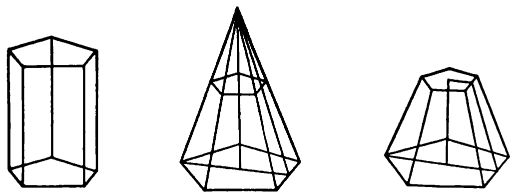
Regular Polygon.

475. A **Regular Polygon** is a polygon with all its sides equal and all its angles equal. The *centre* of a regular polygon is a point equidistant from the corners and also equidistant from the sides. Hence, lines drawn from the centre to the corners of a regular polygon divide the polygon into equal *isosceles* triangles; that is, into equal triangles having two sides equal. The *apothem* of a regular polygon is the distance from its centre to any side.

476. The **area of a regular polygon** is half the product of its perimeter by its apothem.

477. A **Right Prism** is a solid bounded by two equal and parallel polygons, called the *bases,* and by rectangles, called the *lateral faces.* The *altitude* of a prism is the shortest distance between its bases.

478. A **Regular Pyramid** is a solid bounded by a regular polygon, called the *base*, and by isosceles triangles, called the *lateral faces*. These isosceles triangles all terminate in a point called the *vertex* of the pyramid. The *altitude* of a pyramid is the shortest distance from its vertex to its base.



Right Prism. Regular Pyramid. Frustum of Regular Pyramid.

479. A **Frustum of a regular pyramid** is the part of a regular pyramid left after the top has been cut off by a plane parallel to its base. The *bases* of a frustum of a pyramid are the base of the pyramid and the section made by the cutting plane. The *altitude* of the frustum of a pyramid is the shortest distance between its bases. The *lateral faces* of a frustum of a pyramid are trapezoids.

480. The **area of the lateral surface** of a right prism, of a regular pyramid, or of a frustum of a regular pyramid, is the sum of the areas of its lateral faces.

481. The **area of the lateral surface of a right prism** is the product of the perimeter of its base by its altitude.

482. The **volume of a right prism** is the product of the area of its base by its altitude.

483. The **area of the lateral surface of a regular pyramid** is half the product of the perimeter of its base by the altitude of its lateral faces.

484. The **volume of a regular pyramid** is one third the product of the area of its base by its altitude.

485. The **area of the lateral surface of a frustum of a regular pyramid** is half the product of the altitude of its lateral faces by the sum of the perimeters of its bases.

486. The **volume of a frustum of a regular pyramid** is found by adding to the sum of the areas of its bases the square root of their product and multiplying this result by one third the altitude of the frustum.

Exercise 226. — Written.

Find the area of:

1. A parallelogram whose base is 12 in. and whose altitude is 8 in.

2. A rhomboid whose base is 24 ft. and whose altitude is 15 ft.

3. A rhombus whose diagonals are 16 ft. and 12 ft.

4. A trapezoid whose parallel sides are 12 in. and 8 in. and whose altitude is 7 in.

5. A parallelogram whose base is 36 ft. and whose altitude is 20 ft.

6. A parallelogram whose base is 100 ft. and whose altitude is 50 ft.

7. A trapezoid whose parallel sides are 12 ft. and 8 ft. and whose altitude is 10 ft.

8. A rhombus whose side is 6 in. and whose altitude is 5 in.

9. One diagonal of a trapezium is 10 in. and the perpendiculars from the opposite corners are 6 in. and 8 in. Find the area.

Find the area of a regular:

10. Pentagon whose side is 8 in. and whose apothem is 5.5056 in.

11. Hexagon whose side is 12 ft. and whose apothem is 10.3920 ft.

12. Octagon whose side is 6 in. and whose apothem is 7.2426 in.

13. Decagon whose side is 7 in. and whose apothem is 10.7716 in.

14. Triangle whose side is 9 in.

15. Quadrilateral whose side is 10 rd.

Find the lateral surface and the volume of a right prism, having:

16. Square base, side of base 9 in., altitude 14 in.

17. Rectangular base, 6 ft. by 4 ft., altitude 8 ft.

18. Triangular base, sides 3 ft., 4 ft., and 5 ft., altitude 12 ft.

19. Base a regular hexagon, side of base 6 ft., apothem of base 5.196 ft., altitude of prism 9 ft.

Find the lateral surface and the volume of a regular pyramid, having:

20. Base a square 6 ft. on a side, altitude of pyramid 4 ft., and altitude of lateral faces 5 ft.

21. Base a square 24 in. on a side, altitude of pyramid 16 in., and altitude of lateral faces 20 in.

Find the lateral surface and the volume of a frustum of a regular pyramid, having:

22. Bases, squares, respectively 12 in. and 8 in. on a side, altitude of frustum 6 in., and altitude of lateral faces 6.3246 in.

23. Bases, squares, respectively 4 in. and 3 in. on a side, altitude of frustum 4 in., and altitude of lateral faces 4.0311 in.

Division by Factors.

487. If it is required to divide one number by another which can be separated into factors, it is often easier and more convenient to divide the number by one of the factors of the divisor, the quotient by another factor, the second quotient by a third factor, and so on until all the factors have been used as divisors.

Divide 22,756 by 1155.

$$\begin{array}{r|l}
3 & 22756 \\
5 & 7585 \ldots\ldots 1 \\
7 & 1517 \\
11 & 216 \ldots\ldots 5 \\
 & \quad 19 \ldots\ldots 7
\end{array}$$

SOLUTION. $1155 = 3 \times 5 \times 7 \times 11$.

$22,756 \div 3 = 7585$ *threes* and 1 *unit* over. 7585 *threes* $\div 5 = 1517$ *fifteens*. 1517 *fifteens* $\div 7 = 216$ *one hundred fives* and 5 *fifteens* over. 216 *one hundred fives* $\div 11 = 19$ *eleven hundred fifty-fives* and 7 *one hundred fives* over. Therefore, the quotient is 19 and the remainder is $1 + 5 \times 15 + 7 \times 105$, or 811.

Therefore, to divide one number by another,

Divide the number by one of the factors of the divisor; divide the quotient by another factor of the divisor, and so on until all the factors have been used as divisors.

Multiply each remainder by all the divisors that precede the divisor that produced it. The sum of these products is the complete remainder.

NOTE. It is best to use the largest factors possible, not exceeding 12.

EXERCISE 227. — WRITTEN.

Divide by using factors:

1. 2875 by 48.
2. 4792 by 49.
3. 8463 by 75.
4. 9648 by 121.
5. 8762 by 81.
6. 7645 by 144.
7. 6589 by 99.
8. 8431 by 64.
9. 7865 by 128.
10. 8475 by 125.
11. 9175 by 88.
12. 37,560 by 616.
13. 87,542 by 495.
14. 92,653 by 504.
15. 88,475 by 378.
16. 75,555 by 1152.
17. 82,643 by 891.
18. 97,682 by 396.
19. 65,877 by 252.
20. 85,789 by 192.
21. 84,602 by 539.

The Greatest Common Measure of Fractions.

488. The fraction that will exactly divide a series of fractions is greatest when its numerator is as large as possible and its denominator is as small as possible.

The greatest common measure of a series of fractions, therefore, is the fraction

Whose numerator is the greatest common measure of the numerators of the given fractions, and whose denominator is the least common multiple of the denominators of the given fractions.

Find the G. C. M. of $\frac{5}{36}$, $\frac{25}{12}$, $\frac{35}{108}$.

The G. C. M. of 5, 25, 35 is 5.

The L. C. M. of 36, 12, 108 is 108.

Therefore, the G. C. M. of $\frac{5}{36}$, $\frac{25}{12}$, $\frac{35}{108}$ is $\frac{5}{108}$. *Ans.*

Exercise 228. — Written.

Find the G. C. M. of:

1. $\frac{7}{18}$, $\frac{14}{9}$, $\frac{35}{36}$.	6. $\frac{13}{60}$, $\frac{91}{120}$, $\frac{143}{150}$.
2. $\frac{11}{68}$, $\frac{22}{51}$, $\frac{33}{85}$.	7. $\frac{14}{51}$, $\frac{98}{85}$, $\frac{112}{119}$.
3. $\frac{4}{21}$, $\frac{8}{7}$, $\frac{16}{35}$.	8. $\frac{5}{7}$, $\frac{15}{8}$, $\frac{25}{9}$.
4. $\frac{57}{28}$, $\frac{19}{42}$, $\frac{95}{56}$.	9. $\frac{28}{9}$, $\frac{7}{12}$, $\frac{35}{18}$.
5. $\frac{27}{44}$, $\frac{45}{77}$, $\frac{9}{110}$.	10. $\frac{22}{27}$, $\frac{77}{36}$, $\frac{11}{45}$.

11. What is the length of the longest pole that will measure exactly the sides of a triangular lot whose sides are respectively $63\frac{3}{4}$ ft., $89\frac{1}{4}$ ft., and 102 ft.?

12. A man finds that he takes an exact number of steps in walking 29 ft. $6\frac{1}{4}$ in., 38 ft. $7\frac{1}{4}$ in., and 43 ft. $1\frac{3}{4}$ in. Find in inches the greatest possible length of his step.

13. A man has a triangular lot that measures on the sides $3\frac{1}{2}$, $2\frac{3}{4}$, and $2\frac{7}{8}$ chains, respectively. Find the greatest fraction of a chain that will exactly measure the three sides.

The Least Common Multiple of Fractions.

489. The fraction that will exactly contain a series of fractions is least when its numerator is as small as possible and its denominator is as large as possible.

The least common multiple of a series of fractions, therefore, is the fraction

Whose numerator is the least common multiple of the numerators of the given fractions, and whose denominator is the greatest common measure of the denominators of the given fractions.

Find the L. C. M. of $\frac{5}{36}$, $\frac{25}{12}$, $\frac{35}{108}$.

The L. C. M. of 5, 25, 35 is 175.
The G. C. M. of 36, 12, 108 is 12.
Therefore, the L. C. M. of $\frac{5}{36}$, $\frac{25}{12}$, $\frac{35}{108}$ is $\frac{175}{12}$, or $14\frac{7}{12}$. *Ans.*

EXERCISE 229. — WRITTEN.

Find the L. C. M. of:

1. $\frac{7}{18}$, $\frac{14}{9}$, $\frac{35}{36}$.
2. $\frac{11}{68}$, $\frac{22}{51}$, $\frac{33}{85}$.
3. $\frac{4}{21}$, $\frac{8}{7}$, $\frac{16}{35}$.
4. $\frac{57}{28}$, $\frac{19}{42}$, $\frac{95}{56}$.
5. $\frac{27}{44}$, $\frac{45}{77}$, $\frac{9}{110}$.
6. $\frac{13}{60}$, $\frac{91}{120}$, $\frac{143}{150}$.
7. $\frac{14}{51}$, $\frac{98}{85}$, $\frac{112}{119}$.
8. $\frac{5}{7}$, $\frac{15}{8}$, $\frac{25}{9}$.
9. $\frac{28}{9}$, $\frac{7}{12}$, $\frac{35}{18}$.
10. $\frac{22}{27}$, $\frac{77}{36}$, $\frac{11}{45}$.

11. Find in feet the shortest distance that can be exactly measured by three poles whose lengths are respectively $9\frac{5}{6}$ ft., $14\frac{3}{4}$ ft., and $11\frac{4}{5}$ ft.

12. What is the least number of yards of carpet in a roll that can be cut into lengths of $26\frac{2}{3}$ yd., 16 yd., or $11\frac{3}{7}$ yd.?

13. If a man steps $2\frac{1}{4}$ ft. and a horse $2\frac{2}{3}$ ft., find the smallest number of feet that is an exact number of steps for the man and for the horse.

Proofs by Casting out Nines.

490. If the sum of the digits of a number is subtracted from the number, the remainder is exactly divisible by 9. Hence any number divided by 9 will have the same remainder as the sum of its digits will have when divided by 9.

491. This truth may be applied to test the accuracy of the work in the simple processes of arithmetic.

NOTE. In finding the remainder from dividing the sum of the digits by 9, that is, in casting out the nines, we may, of course, omit the nines, or any two or three digits which we see at a glance will make 9.

Thus, in casting out the nines from 1,926,754, we see at once that 1, 2, 6, and 5, 4, make nines, and the single 7 will be the remainder. So in 254,786, we reject 5, 4, and 2, 7, and add 8 + 6; from the sum we reject 9, and have 5 left.

Proof of Addition.

The remainder after the nines are cast out:

From	81,364	is	4	Add.
From	27,632	is	2	
From	38,507	is	5	
From	67,549	is	4	
From	215,052 is **6**.		From 15 is **6**.	

Since the remainder in each case is **6**, the work may be assumed to be correct.

Proof of Subtraction.

The remainder after the nines are cast out:

From	7543	is	1	Subtract.
From	5674	is	4	
From	1869 is **6**.		**6**.	

Since the remainder of the minuend is less than the remainder of the subtrahend, 9 must be added to the remainder of the minuend before the subtraction can be performed.

Proof of Multiplication.

The remainder after the nines are cast out:

$$\left.\begin{array}{llll}\text{From} & 47 & \text{is} & 2\\ \text{From} & \underline{61} & \text{is} & \underline{7}\end{array}\right\}\text{Multiply.}$$

From 2867 is **5**. From 14 is **5**.

The product of the two *numbers* has 5 remaining and the product of the two *remainders* has 5 remaining after the nines are cast out. Therefore, the work may be assumed to be correct.

Proof of Division.

$$498\overline{)1348708}\quad 2708 \text{ with remainder } 124.$$

The product of the divisor and quotient plus the remainder is equal to the dividend.

The remainder after the nines are cast out:

From the dividend is **4.**

$$\left.\begin{array}{lll}\text{From the divisor} & \text{is} & 3\\ \text{From the quotient} & \text{is} & \underline{8}\end{array}\right\}\text{Multiply.}$$

$$\left.\begin{array}{ll}\text{From the product} & 24 \text{ is } 6\\ \text{From the remainder} & \text{is } \underline{7}\end{array}\right\}\text{Add.}$$

From the sum 13 is **4.**

Therefore, the work may be assumed to be correct.